Better termination proving through cooperation

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Termination Analysis: Invariants and Rank Functions

Example

```
$y := 1;$
**while** $x > 0$ **do**
    $x := x - y;$
    $y := y + 1;$
**done**
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- Invariant $y > 0$ and rank function x prove termination
- How do we know that we need $y > 0$? ↷ x requires it

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- Invariant $y > 0$ and rank function x prove termination
- How do we know that we need $y > 0$? ↷ x requires it
- How do we know that x is a RF? ↷ $y > 0$ proves it

Termination by iterative strengthening: Idea

1. Safety: Provide samples (Counterexamples)
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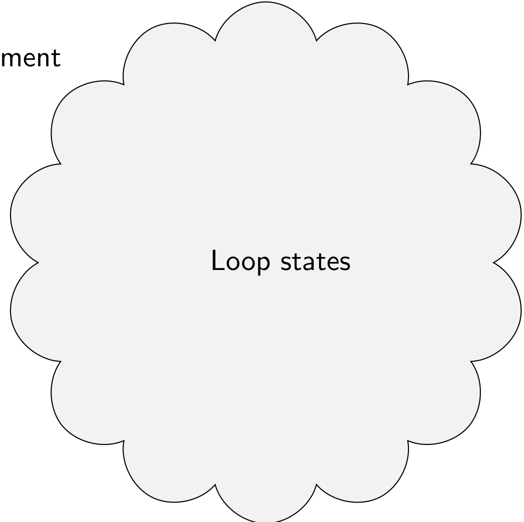
Find counterexample
then strengthen argument



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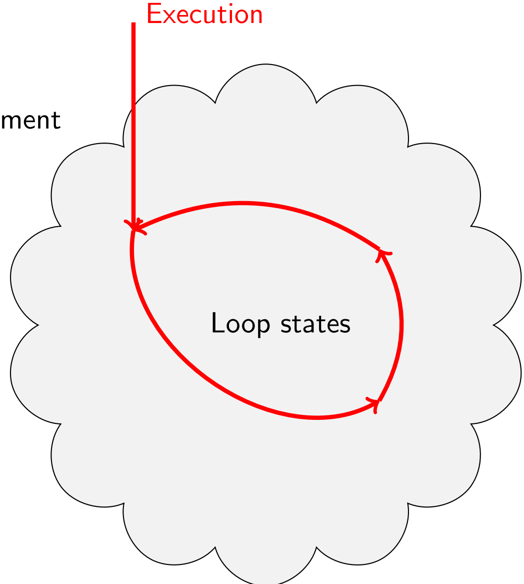

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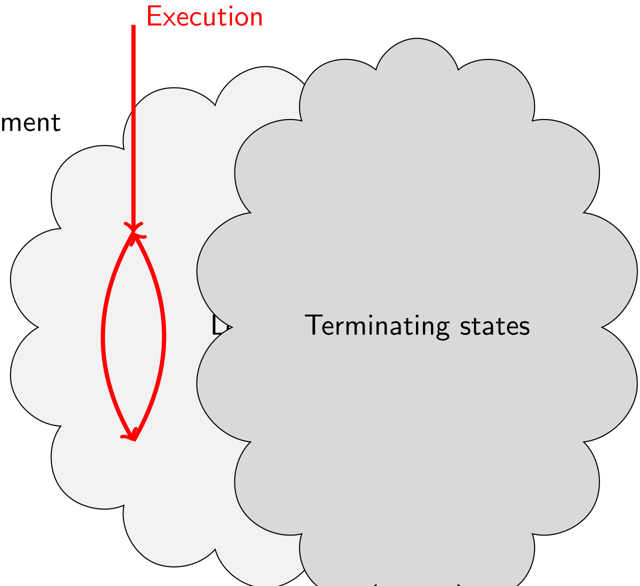

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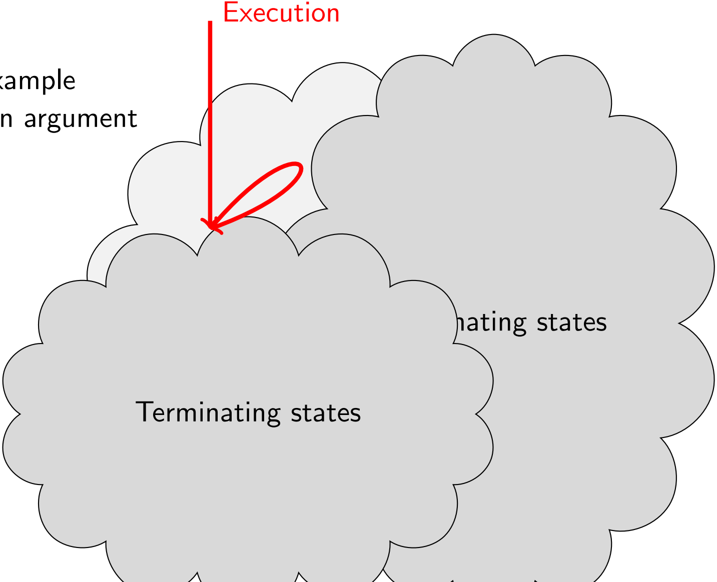

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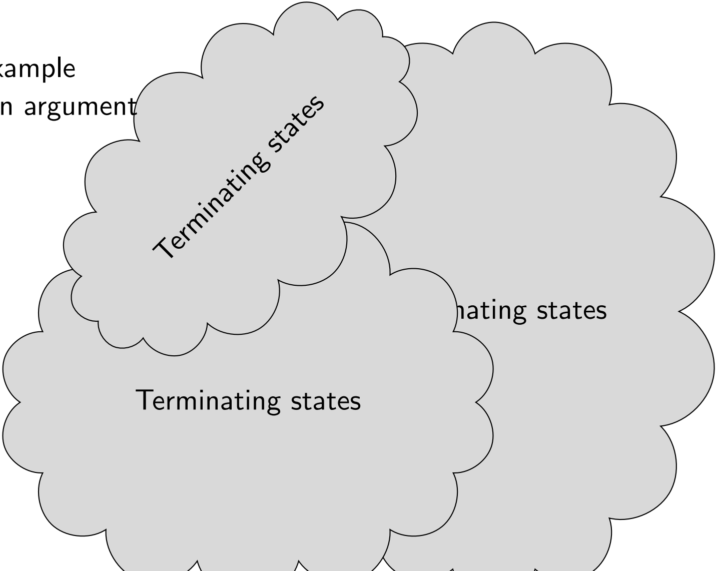

Termination by iterative strengthening: Worst case

1. Safety: Look at everything, then return old sample
2. Rank tool: Find **too** specific termination argument
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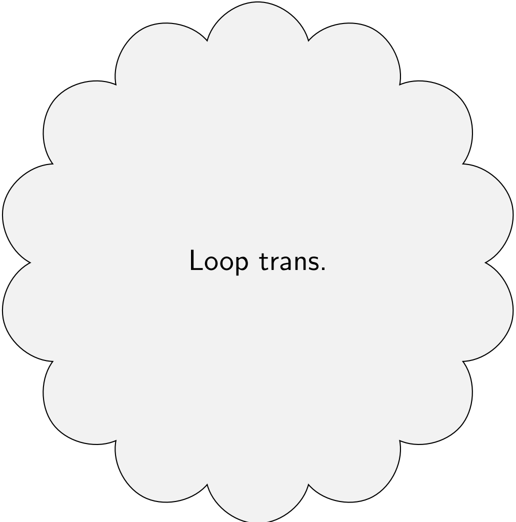
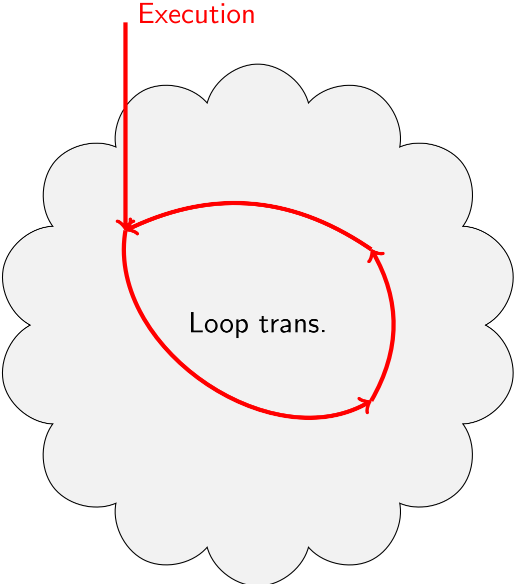


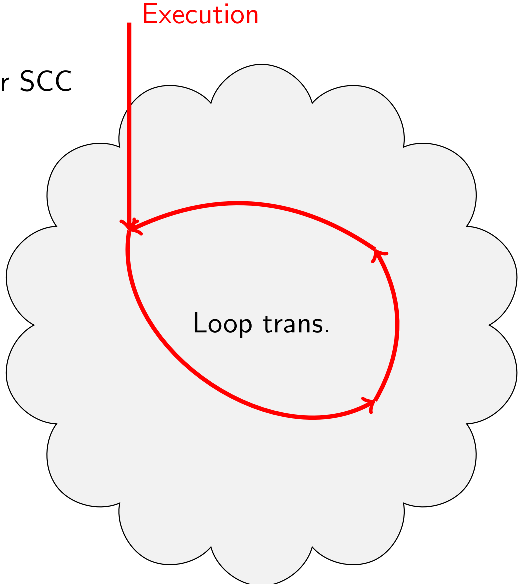
Loop trans.

Termination by iterative simplification



Termination by iterative simplification

Find rank function for SCC



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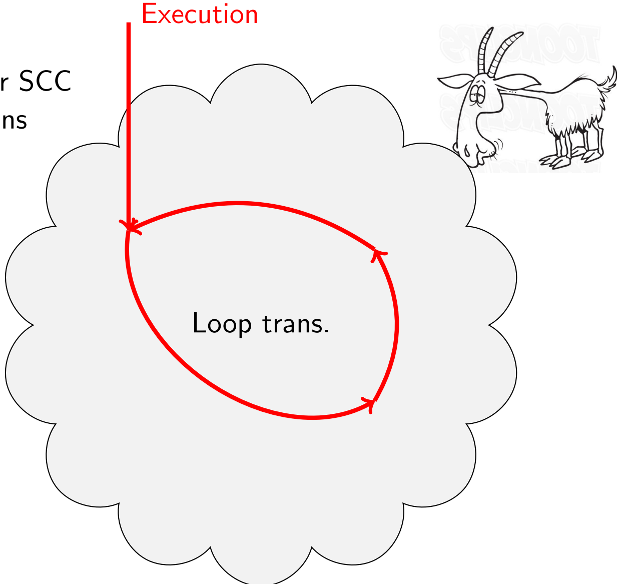

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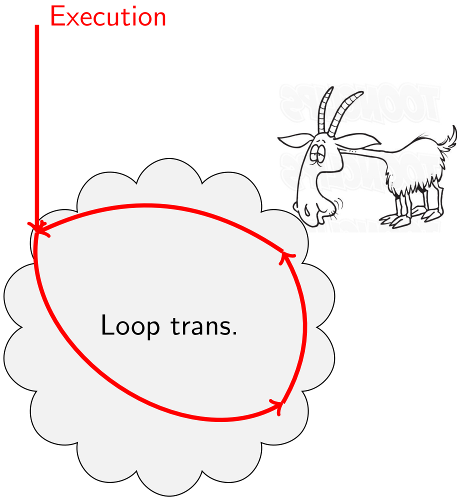

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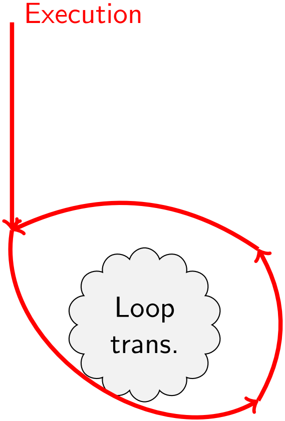
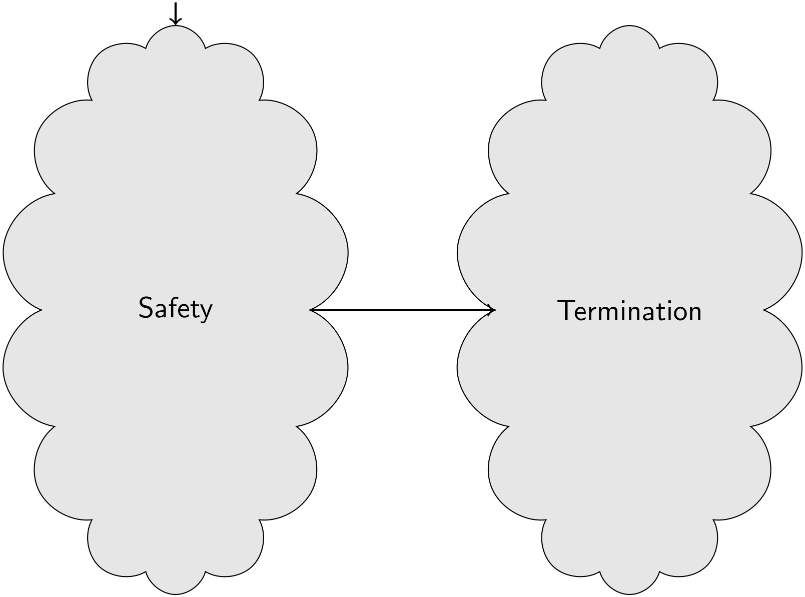


Termination by cooperation

1. Safety: Provide samples (Counterexamples)
2. Rank tool: Find termination argument **in context**
3. Rank tool: Mark definitely terminating parts
4. Safety: Prove generality for rest, or 1

Cooperation: High-level view



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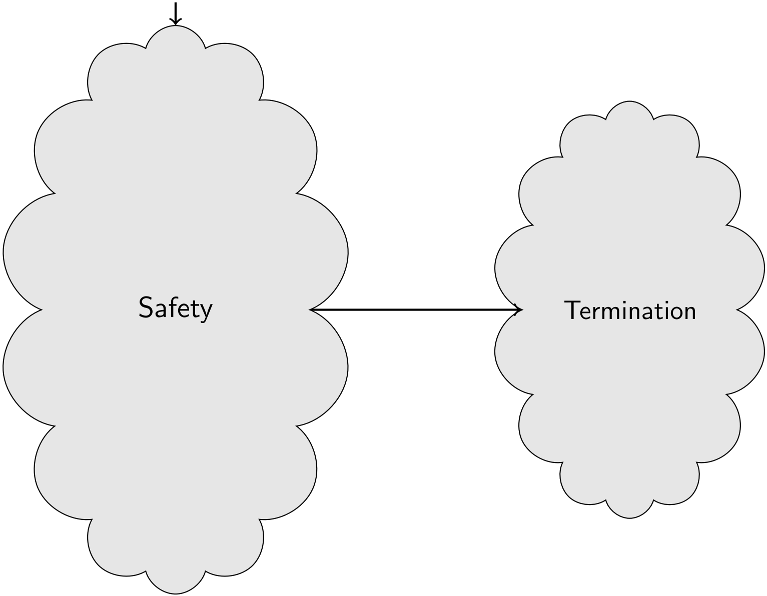

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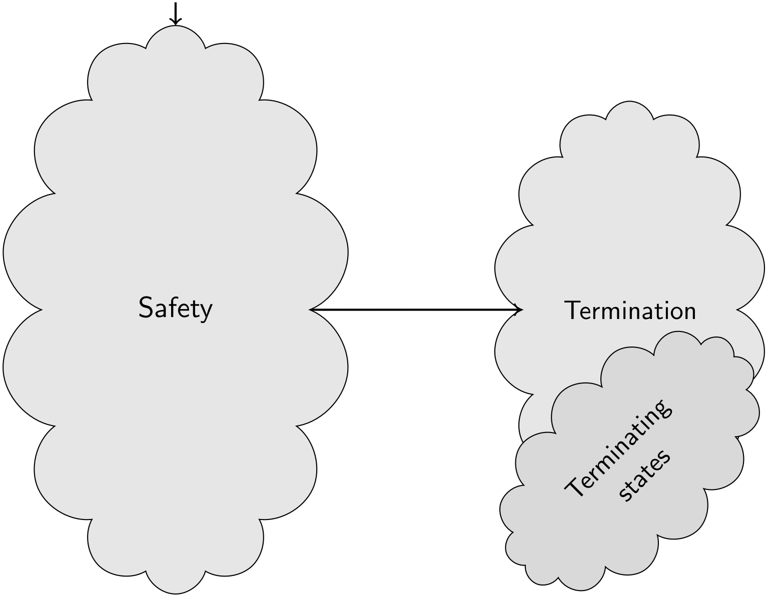

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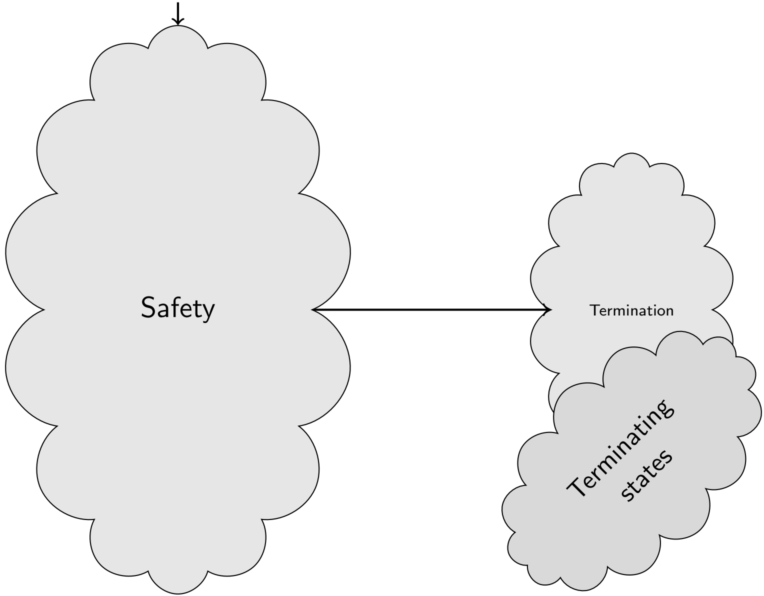

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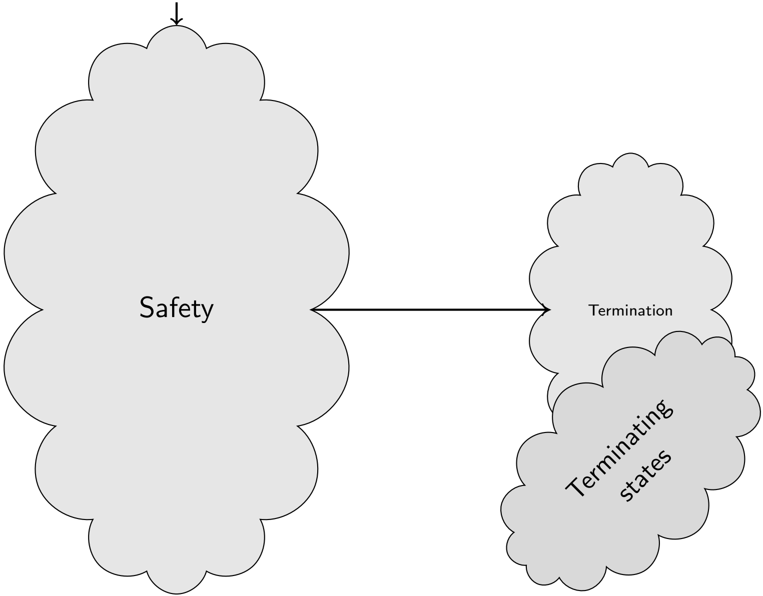

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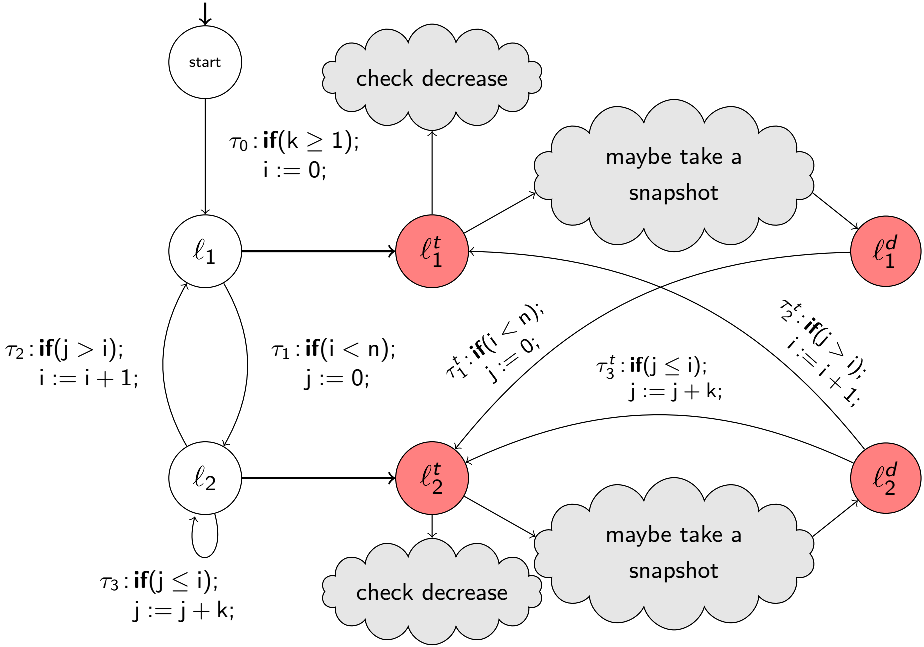
start
ℓ_1
ℓ_2
ℓ_1^t
ℓ_2^t
ℓ_1^d
ℓ_2^d
check decrease
maybe take a snapshot
maybe take a snapshot
check decrease
τ_0 : **if**(k $\geq$ 1); i := 0;
τ_1 : **if**(i < n); j := 0;
τ_2 : **if**(j > i); i := i + 1;
τ_3 : **if**(j $\leq$ i); j := j + k;
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Approach:

- Analyze whole SCC, not counterexample slice
- Remove transitions after proof

Cooperation: Evaluation

Evaluated on 449 termination proving benchmarks
260 known terminating, 181 known non-terminating, 8 unknown
Sources: Windows drivers, Apache, PostgreSQL, . . .

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	Term (#)	Term (avg. s)
Cooperating-T2	245	3.42
AProVE	197	2.21
KITTeL	196	4.65
T2	189	5.15
AProVE+Interproc	185	1.53
Terminator	177	4.99
Size-Change/MCNP	156	17.50
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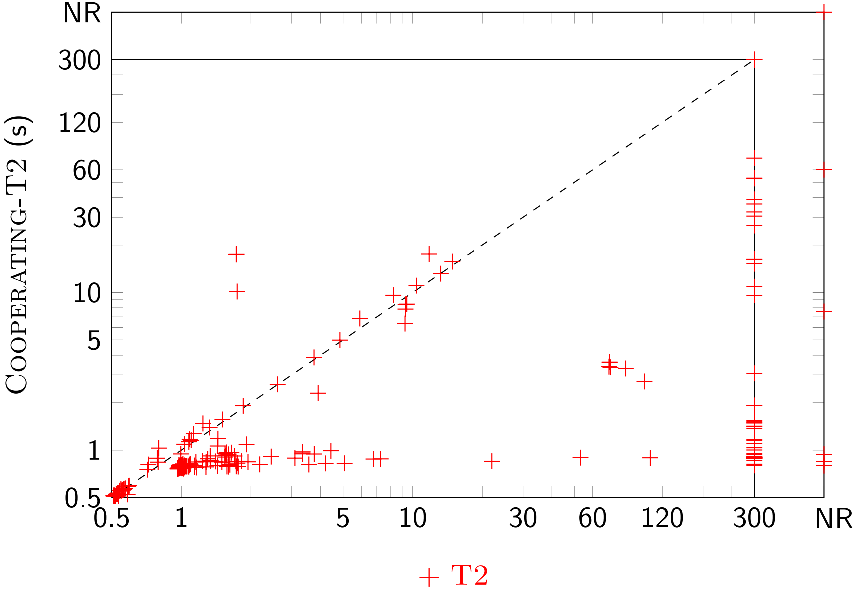

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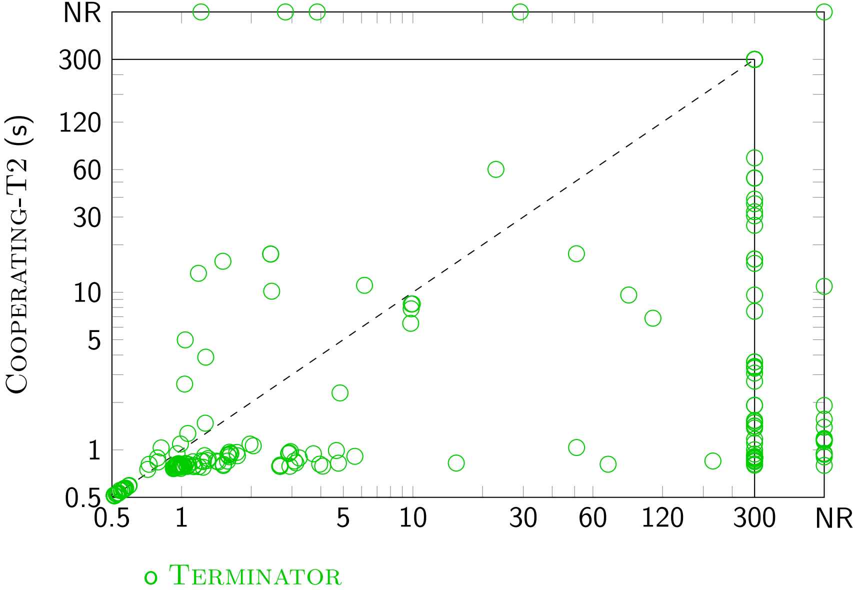

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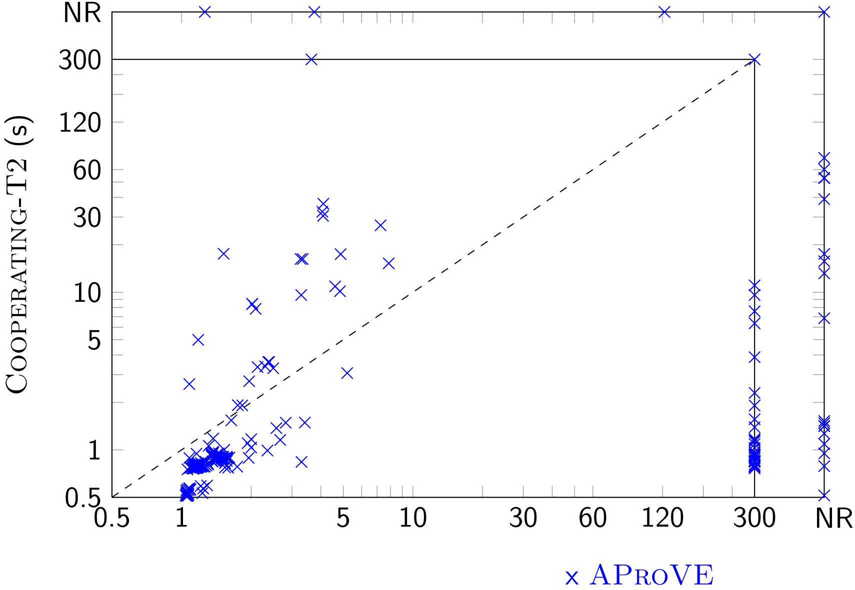

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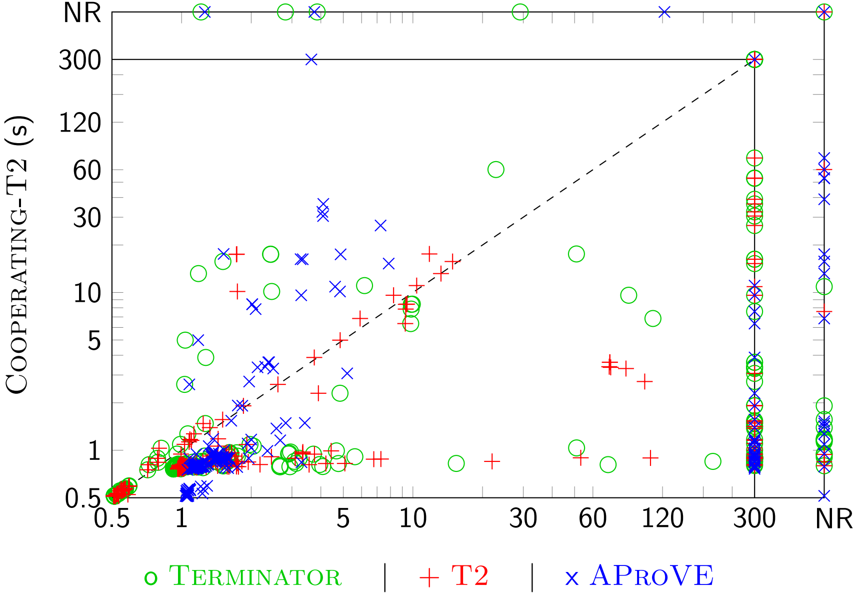

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Sources available: `http://research.microsoft.com/en-us/projects/t2/`