Modular Termination Proofs of Recursive Java Bytecode Programs by Term Rewriting

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Termination Analysis for Imperative Programs

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  Termination Analysis by Abstraction & Model Checking
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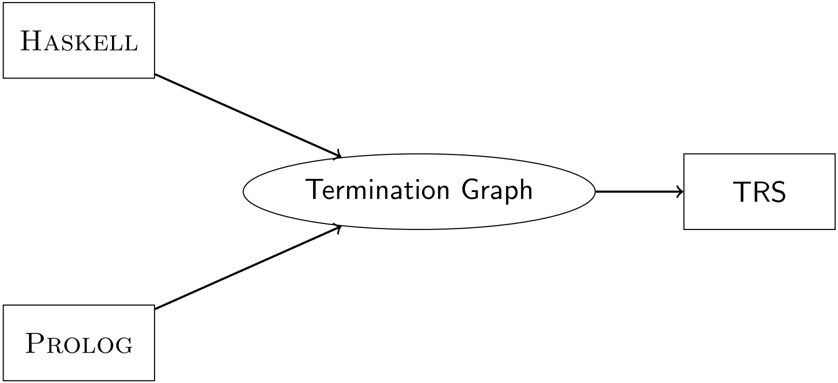
- Julia & COSTA
  Termination Analysis of JAVA BYTECODE (JBC)
  Fixed abstraction, via Constraint Logic Programs
  (Spoto, Mesnard, Payet, 10)
  (Albert, Arenas, Codish, Genaim, Puebla, Zanardini, 08)

Rewriting-based approaches



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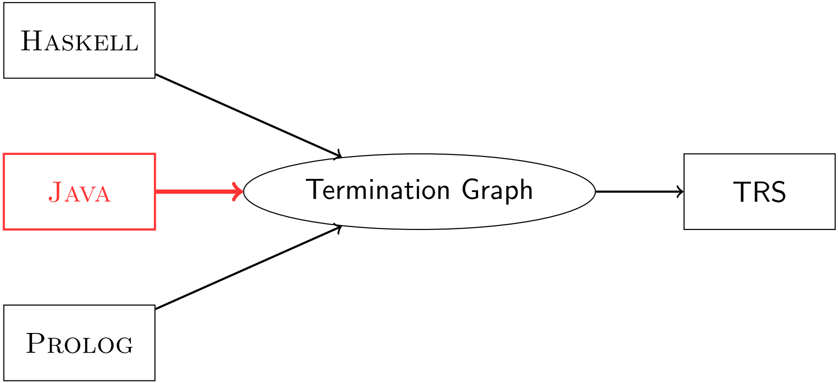
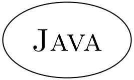


Rewriting-based approach: Structure



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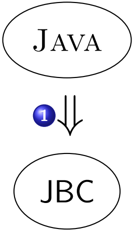

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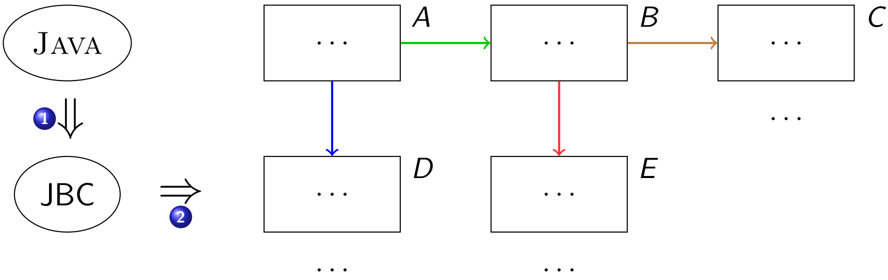

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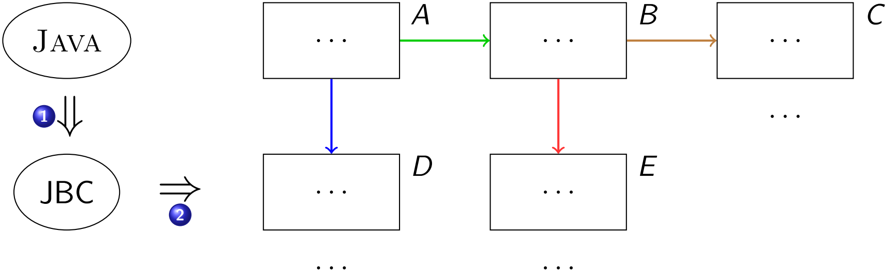

1. Sun/Oracle `javac`
2. Contribution 1
3. Contribution 2

$$f_A(\ldots) \longrightarrow f_B(\ldots) \qquad f_B(\ldots) \longrightarrow f_C(\ldots)$$

$$f_A(\ldots) \longrightarrow f_D(\ldots) \qquad f_B(\ldots) \longrightarrow f_E(\ldots)$$

Rewriting-based approach: Advantages

Handling of user-defined data structures:

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public class List {
  int value;
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- List [2, 4, 6] abstracted to
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- Our technique:
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- List [2, 4, 6] becomes
  List(2, List(4, List(6, null)))

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- **TRS techniques** search for suitable orders automatically

⇒ Complex orders for user-defined data structures possible

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- Aliasing, sharing, cyclicity, side-effects
  $\Rightarrow$ **Heap annotations** in **graph construction** (RTA '10)

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  ⇒ **Compile to Java Bytecode**, analyze that
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- Aliasing, sharing, cyclicity, side-effects
  ⇒ **Heap annotations** in **graph construction** (RTA '10)

Improvement over RTA '10:

- Recursion: Implies unbounded number of variables
- Modularity: Reusing termination proofs: How to handle side-effects?

The example

```
class List {
  List n;

  public void appE(int i) {
    if (n == null) {
     if (i <= 0) return;
     n = new List();
     i--;
    }
    n.appE(i);
  }
}
```

1. Run to end of list.
2. Append `i` elements.

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01: getfield n    // load this.n to opstack
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o_1: $\mathtt{List}(\mathtt{n} = o_2)$ $\quad i_3: \mathbb{Z}$

o_2: $\mathtt{List}(?)$

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o_1: `List(n` $=o_2$`)` $i_3: \mathbb{Z}$
o_2: `List(?)`

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Only explicit sharing

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A

State A:

- Do all calls of appE terminate?
- this is some acyclic list
- i some integer

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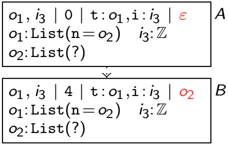

State *B*:

- aload_0 loads o_1 from this to opstack
- getfield n loads field n of o_1 to opstack
- *Evaluation* from *A* to *B*

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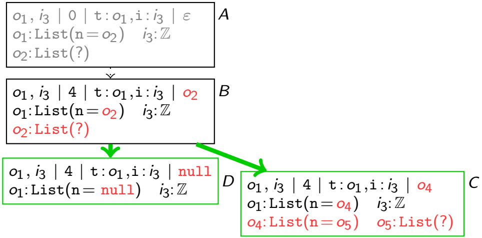

States C, D:

- `ifnonnull` branches depending on nullness of o_2
- Nullness of o_2 not known

$\Rightarrow$ Refine information:

- In C o_1 replaced by $o_4 : \texttt{List}(\texttt{n} = o_5)$
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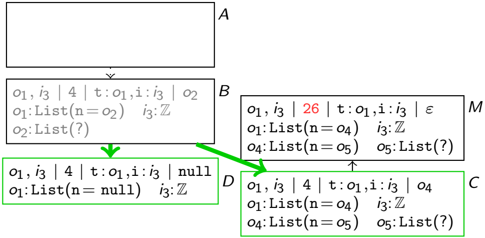

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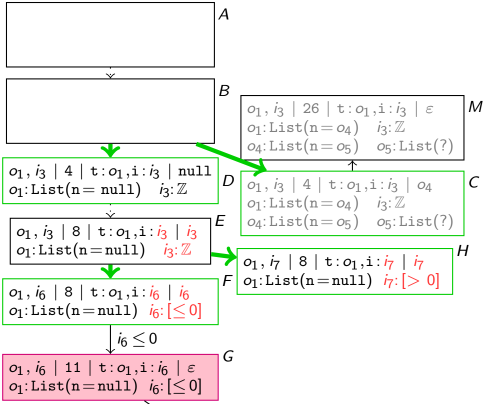

States *E*, *F*, *G*, *H*:

- *D* evaluates to *E*
- ifgt branches depending on relation of $i_3 > 0$

⇒ Refine information about i_3 (F, H)

- *Evaluation* to return state *G*

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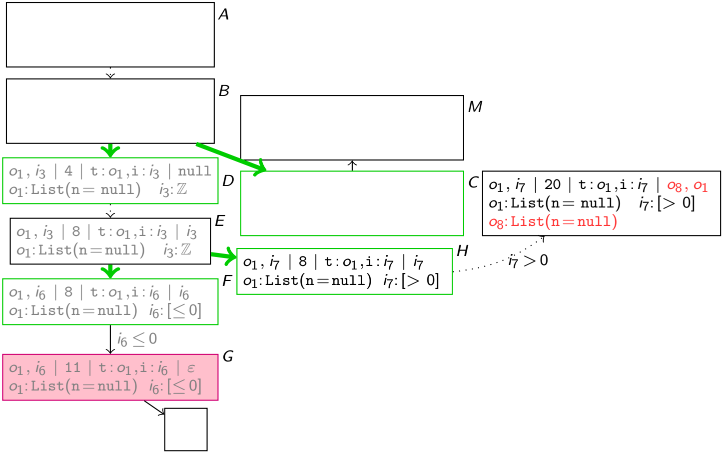

State *J*:

- Created new (empty) List object

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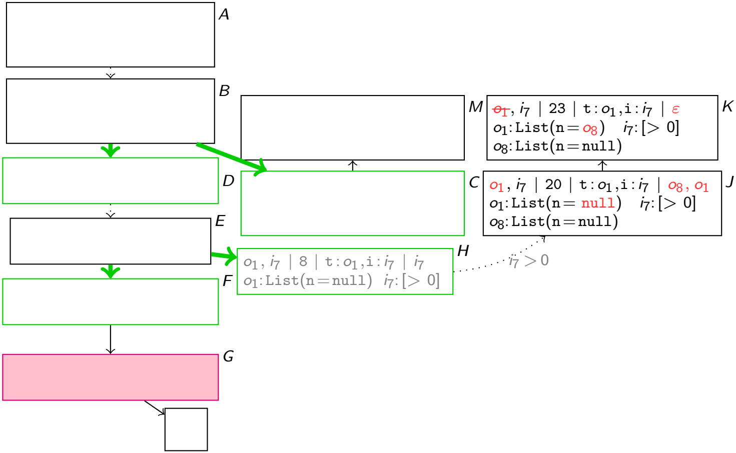
```

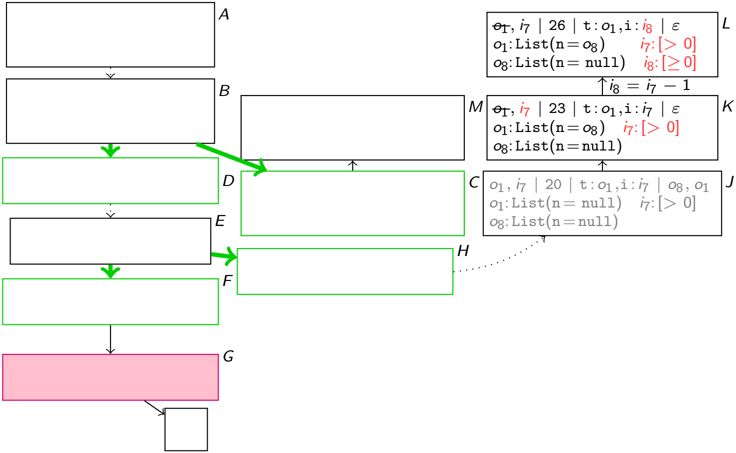


State K:

- putfield n changes field n of o_1
- *side effect* on input argument o_1

  ($\not{o_1}$ marks this)

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```

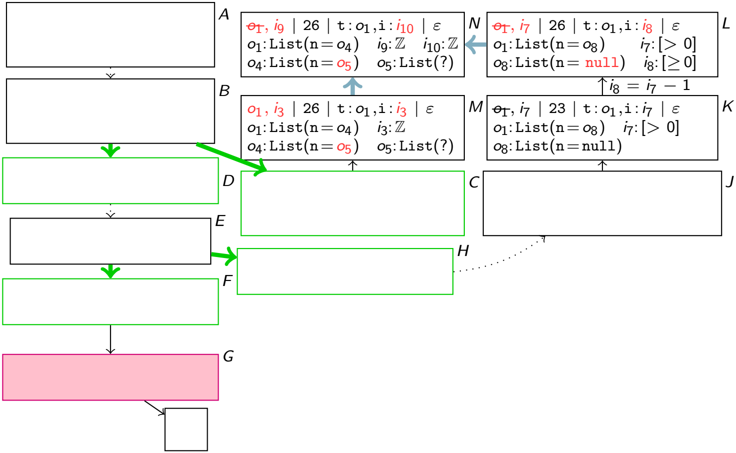


State L:

- Decrement i_7 by 1
- *No sharing* for primitive types!

  (copy on write to new value i_8)

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```

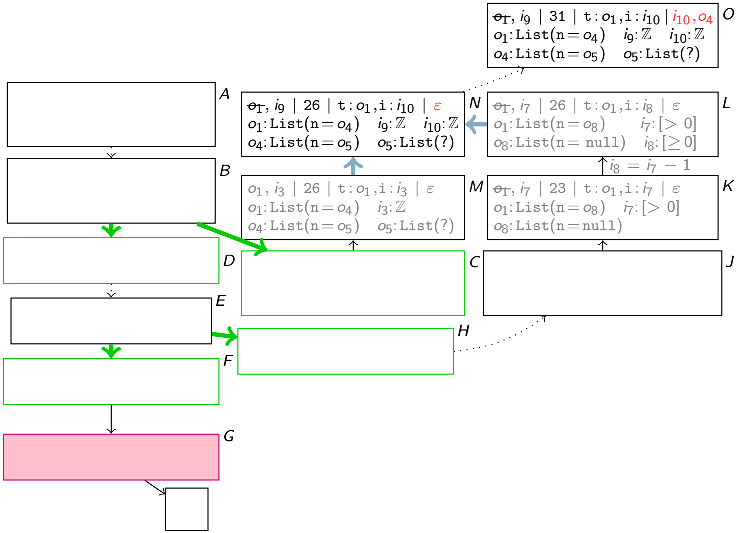


State N:

- States L and M very similar

$\Rightarrow$ N represents both ("L, M are instances of N")

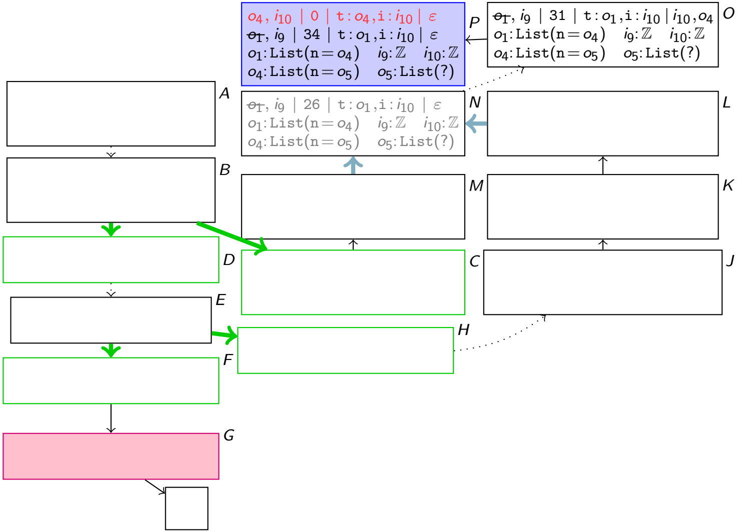
```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```



State *O*:

- Load `i` and `this.n` to opstack

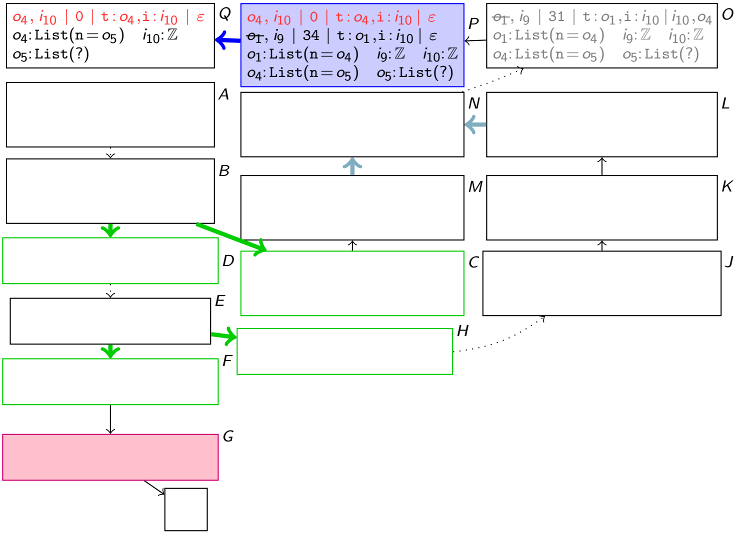
```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```



State *P*:

- Recursive call to appE
- New stackframe on top at position 0
- *P* call state

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```

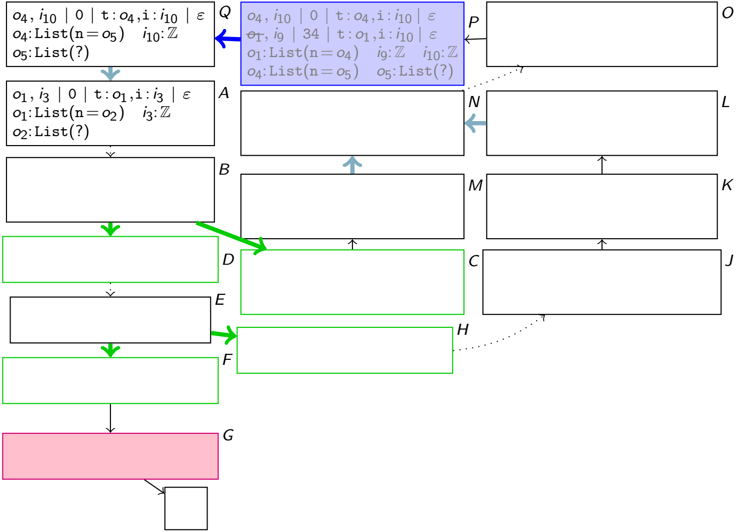


State Q:

- Avoid unbounded call stack growth

$\Rightarrow$ Split call stack (leave only top frame)

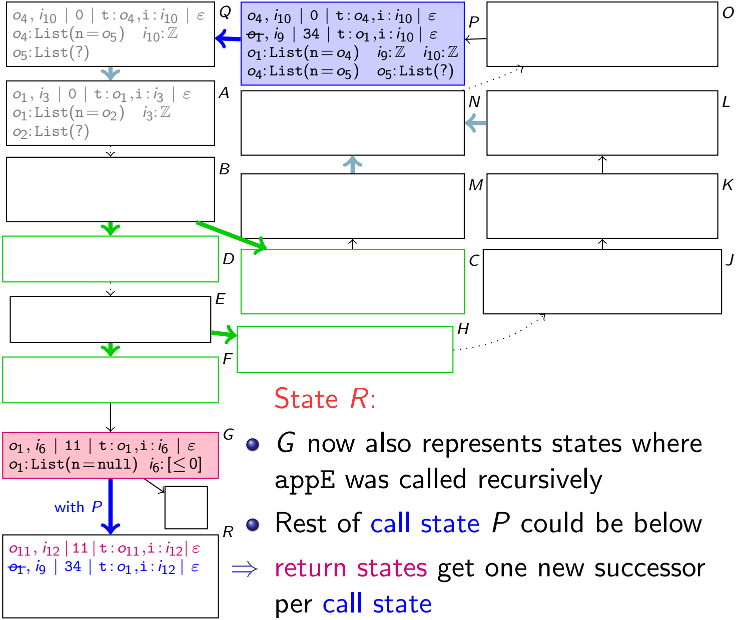
```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```



- Q renaming of A

$\Rightarrow$ Q instance of A

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```

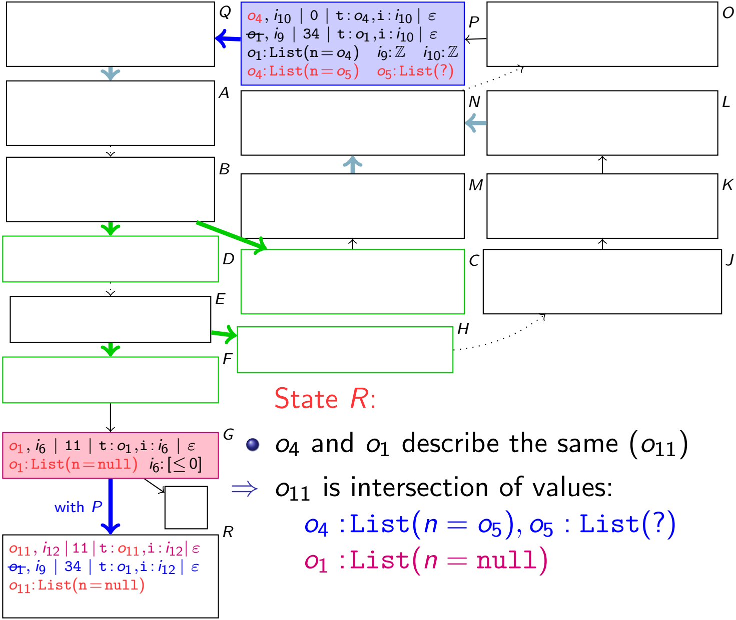


State *R*:

- *G* now also represents states where appE was called recursively
- Rest of call state *P* could be below

⇒ return states get one new successor per call state

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```



State R:

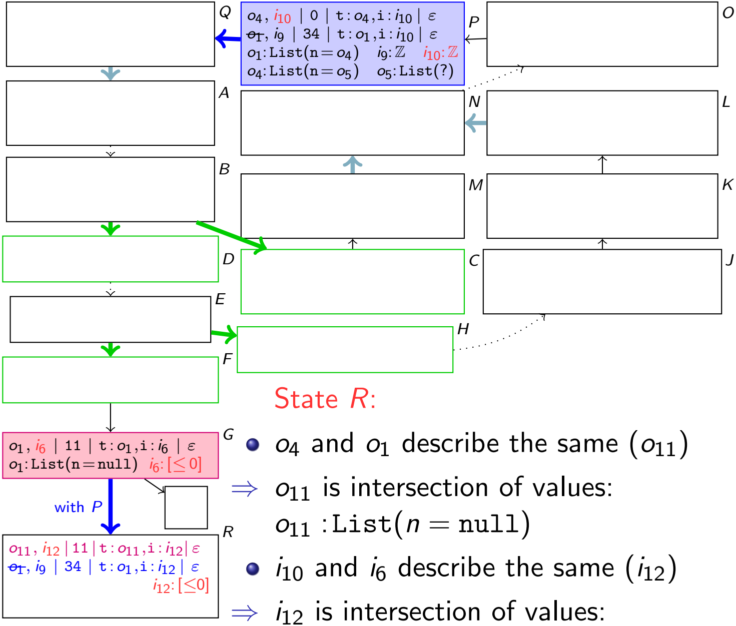
- o_4 and o_1 describe the same (o_{11})

⇒ o_{11} is intersection of values:

o_4 : List($n = o_5$), o_5 : List(?)

o_1 : List(n = null)

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```



State R:

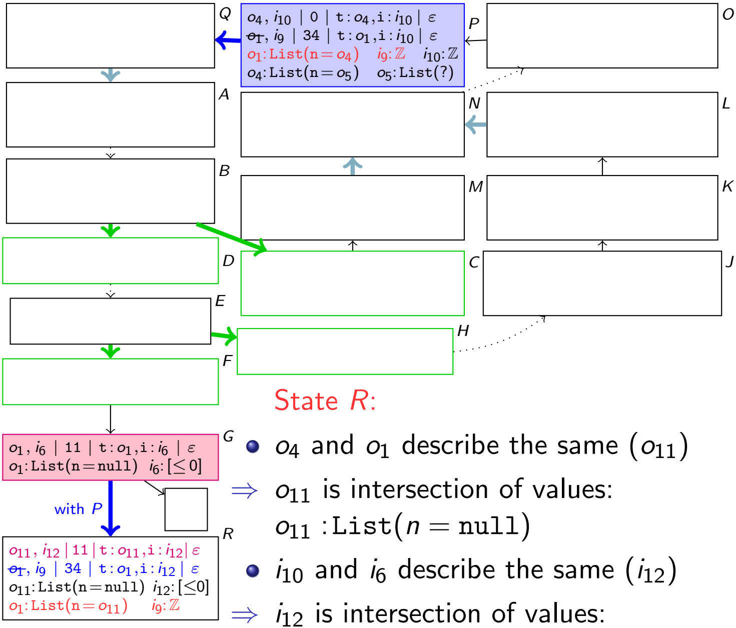
- o_4 and o_1 describe the same (o_{11})

$\Rightarrow$ o_{11} is intersection of values:
o_{11} : List(n = null)

- i_{10} and i_6 describe the same (i_{12})

$\Rightarrow$ i_{12} is intersection of values:
i_{10} : $\mathbb{Z}$
i_6 : $[\leq 0]$

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```



State R:

- o_4 and o_1 describe the same (o_{11})

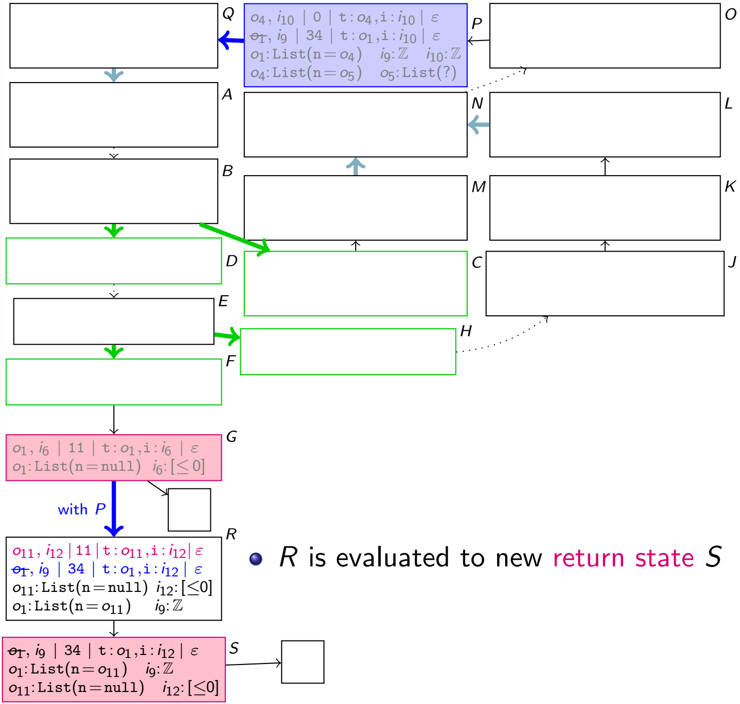
$\Rightarrow$ o_{11} is intersection of values:
o_{11} : List(n = null)

- i_{10} and i_6 describe the same (i_{12})

$\Rightarrow$ i_{12} is intersection of values:
i_{12} : [$\leq$ 0]

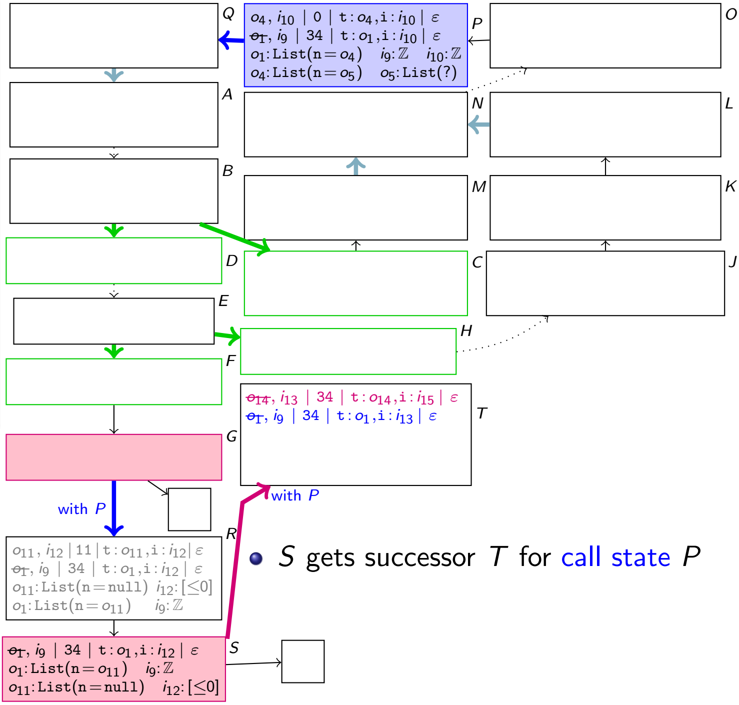
- Other values are copied

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```



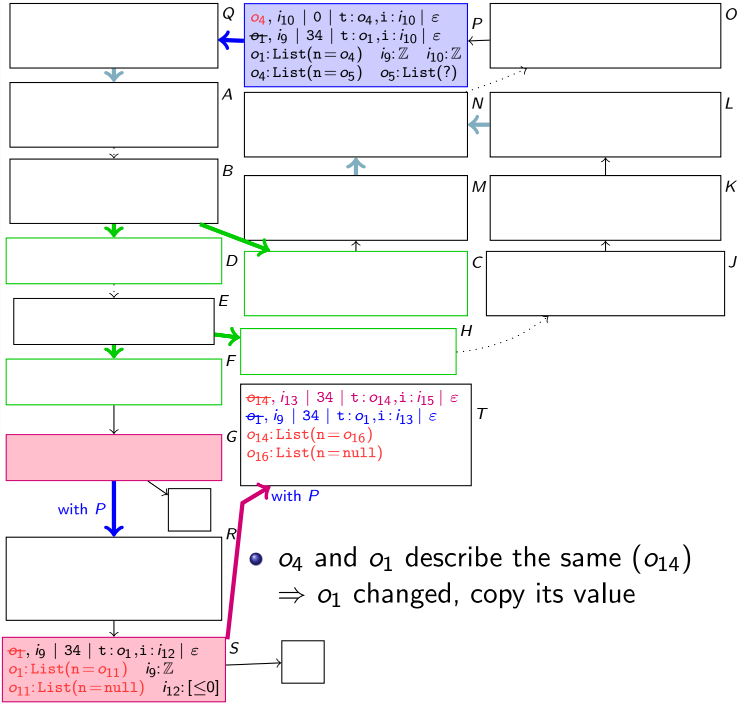
- *R* is evaluated to new return state *S*

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```



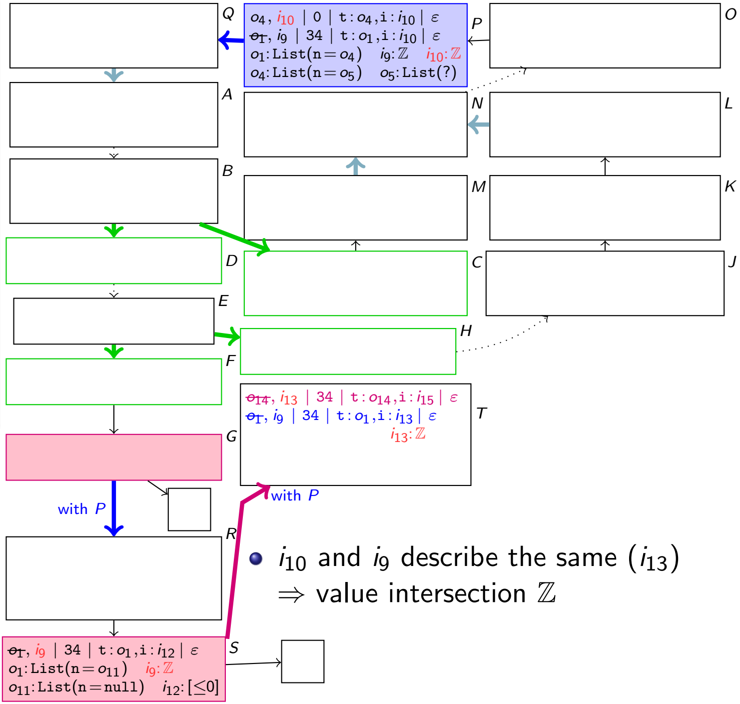
- S gets successor T for call state P

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```



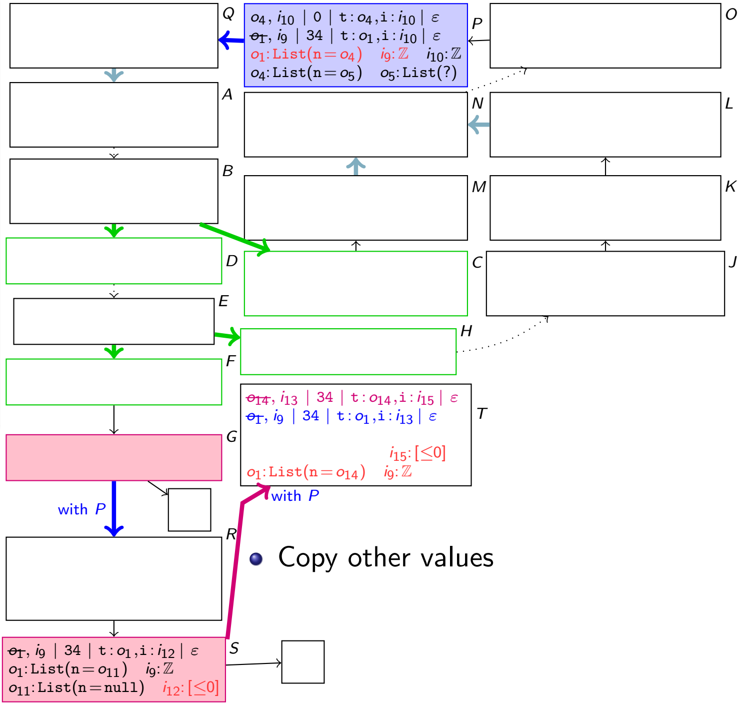
- o_4 and o_1 describe the same (o_{14})
  $\Rightarrow$ o_1 changed, copy its value

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```



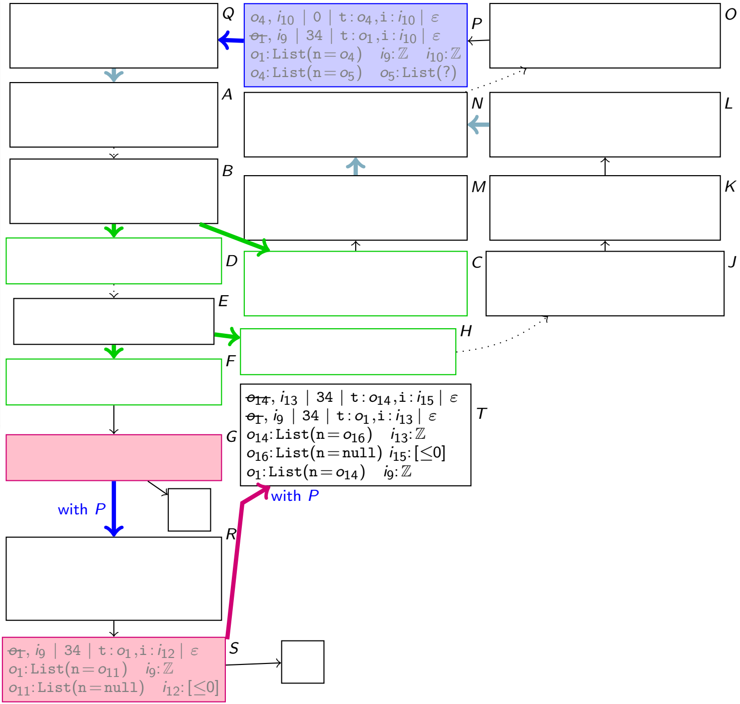
- i_{10} and i_9 describe the same (i_{13}) $\Rightarrow$ value intersection $\mathbb{Z}$

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```

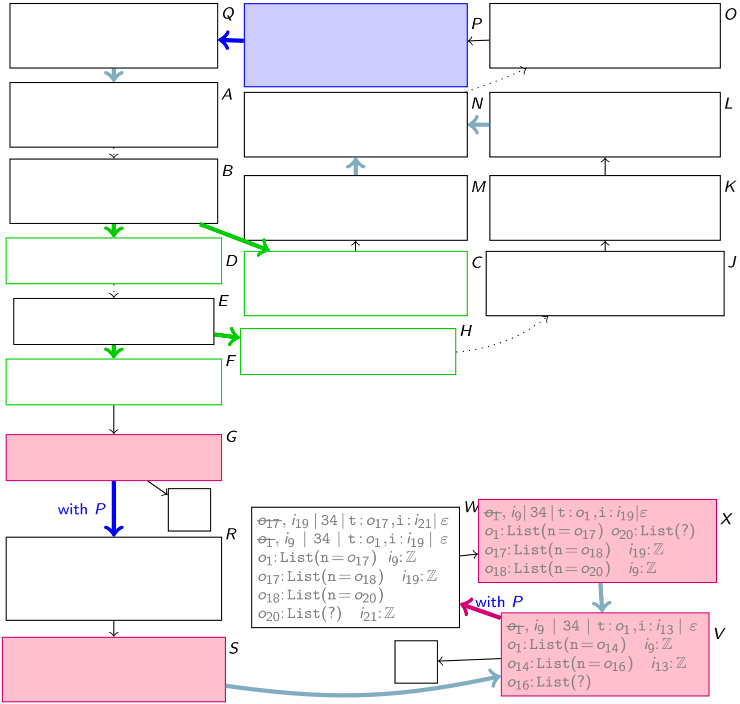


- Copy other values

```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```



```
00: aload_0
01: getfield n
04: ifnonnull 26
07: iload_1
08: ifgt 12
11: return
12: aload_0
13: new List
16: dup
17: invoke <init>
20: putfield n
23: iinc 1, -1
26: aload_0
27: getfield n
30: iload_1
31: invoke appE
34: return
```



Termination graphs

- Termination Graph from symbolic evaluation
- Repeating states are *generalized*

$\Rightarrow$ Graphs finite

Termination graphs

- Termination Graph from symbolic evaluation
- Repeating states are *generalized*

⇒ Graphs finite

- Reusal of Termination Graphs possible.

Termination graphs of several methods

```
public void appE(int i) {
  if (n == null) {
   if (i <= 0) return;
   n = new List();
   i--;
  }
  n.appE(i);
}
```

```
static void cappE(int j) {
  List a = new List();
  if (j > 0) {
   a.appE(j);
   while (a.n == null) {}
} }
```

- `cappE` creates a new `List a`
- If $j > 0$, j elements are appended
- Enters infinite loop if `a.n` is `null` afterwards

Termination graphs of several methods

```
public void appE(int i) {
  if (n == null) {
   if (i <= 0) return;
   n = new List();
   i--;
  }
  n.appE(i);
}
```

```
static void cappE(int j) {
  List a = new List();
  if (j > 0) {
   a.appE(j);
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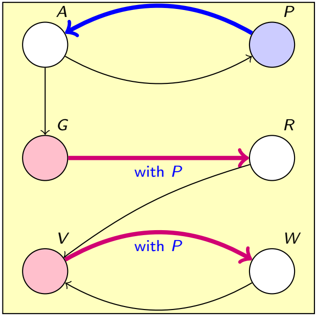
} }
```

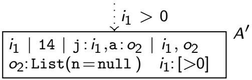


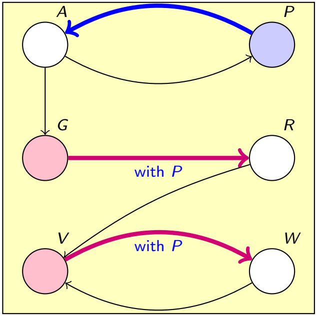
- cappE creates a new List a
- If $j > 0$, j elements are appended
- Enters infinite loop if a.n is null afterwards

Termination graphs of several methods

```
public void appE(int i) {
  if (n == null) {
   if (i <= 0) return;
   n = new List();
   i--;
  }
  n.appE(i);
}
```

```
static void cappE(int j) {
  List a = new List();
  if (j > 0) {
   a.appE(j);
   while (a.n == null) {}
} }
```





- cappE creates a new List a
- If $j > 0$, j elements are appended
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Termination graphs of several methods

```
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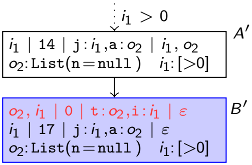
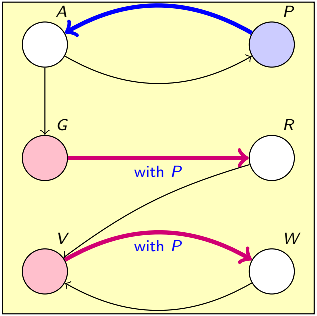
  if (n == null) {
   if (i <= 0) return;
   n = new List();
   i--;
  }
  n.appE(i);
}
```

```
static void cappE(int j) {
  List a = new List();
  if (j > 0) {
   a.appE(j);
   while (a.n == null) {}
} }
```



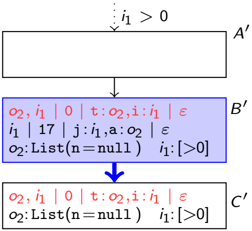


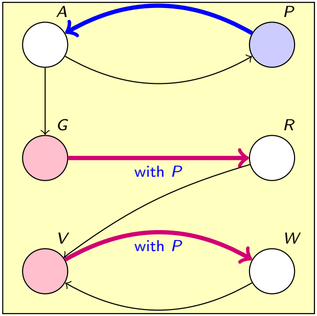
- State B' call state

Termination graphs of several methods

```
public void appE(int i) {
  if (n == null) {
   if (i <= 0) return;
   n = new List();
   i--;
  }
  n.appE(i);
}
```

```
static void cappE(int j) {
  List a = new List();
  if (j > 0) {
   a.appE(j);
   while (a.n == null) {}
} }
```



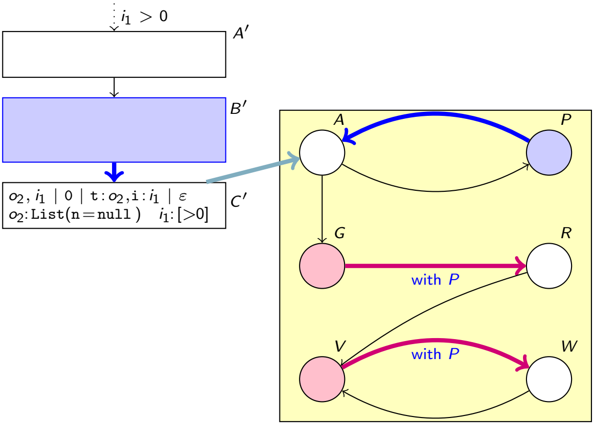


- State B' call state
- State C' call stack split result

Termination graphs of several methods

```
public void appE(int i) {
  if (n == null) {
   if (i <= 0) return;
   n = new List();
   i--;
  }
  n.appE(i);
}
```

```
static void cappE(int j) {
  List a = new List();
  if (j > 0) {
   a.appE(j);
   while (a.n == null) {}
} }
```

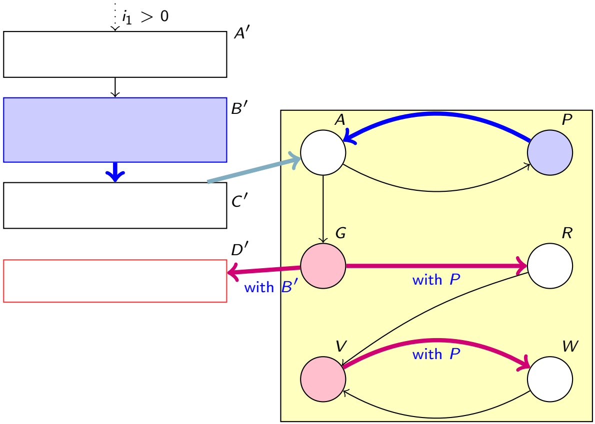


- State B' call state
- State C' call stack split result
- C' instance of A from appE

Termination graphs of several methods

```
public void appE(int i) {
  if (n == null) {
   if (i <= 0) return;
   n = new List();
   i--;
  }
  n.appE(i);
}
```

```
static void cappE(int j) {
  List a = new List();
  if (j > 0) {
   a.appE(j);
   while (a.n == null) {}
} }
```



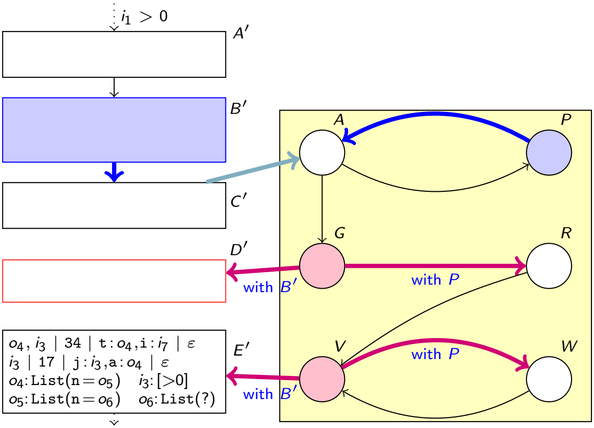
New successors for return states G, V:

- Successor D' of G does not exist:
  B' has $\texttt{i} > 0$, G has $\texttt{i} \leq 0$, intersection empty

Termination graphs of several methods

```
public void appE(int i) {
  if (n == null) {
   if (i <= 0) return;
   n = new List();
   i--;
  }
  n.appE(i);
}
```

```
static void cappE(int j) {
  List a = new List();
  if (j > 0) {
   a.appE(j);
   while (a.n == null) {}
} }
```



New successors for return states G, V:

- Successor D' of G does not exist
- Successor E' of V exists, a has length ≥ 1

Termination graphs of several methods

```
public void appE(int i) {
  if (n == null) {
   if (i <= 0) return;
   n = new List();
   i--;
  }
  n.appE(i);
}
```

```
static void cappE(int j) {
  List a = new List();
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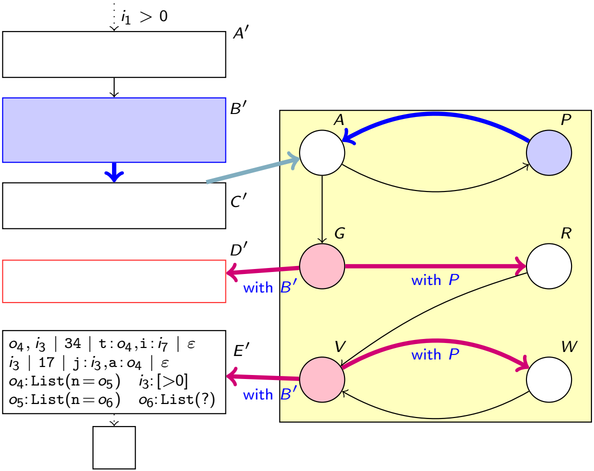
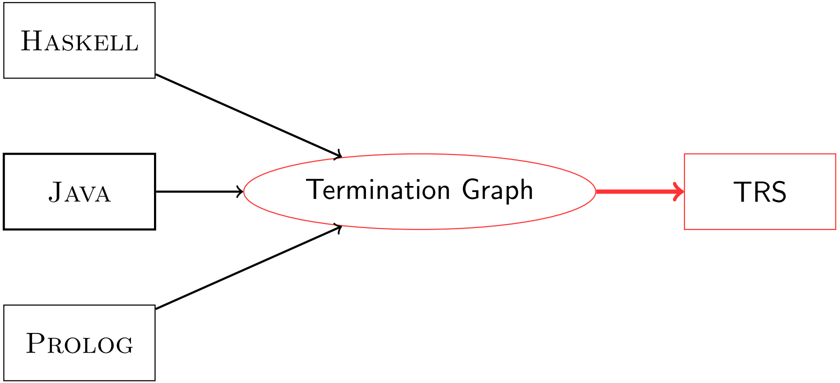
   while (a.n == null) {}
} }
```



New successors for return states G, V:

- Successor D' of G does not exist
- Successor E' of V exists, a has length ≥ 1
- Program terminates

From Termination Graphs to TRSs



Transforming values to terms

$$o_4, i_{10} \mid 0 \mid \texttt{t}: o_4, \texttt{i}: i_{10} \mid \varepsilon$$
$$\not{o_1}, i_9 \mid 34 \mid \texttt{t}: o_1, \texttt{i}: i_{10} \mid \varepsilon$$
$$o_1{:}\texttt{List}(\texttt{n}{=}o_4) \quad i_9{:}\mathbb{Z} \quad i_{10}{:}\mathbb{Z}$$
$$o_4{:}\texttt{List}(\texttt{n}{=}o_5) \quad o_5{:}\texttt{List}(?)$$

P

- Data structures transformed to nested constructor terms

$$\mathsf{List}(\,\mathsf{List}(o_5))$$

Transforming values to terms

$o_4, i_{10} \mid 0 \mid \texttt{t}: o_4, \texttt{i}: i_{10} \mid \varepsilon$

$\not{o_1}, i_9 \mid 34 \mid \texttt{t}: o_1, \texttt{i}: i_{10} \mid \varepsilon$

$o_1{:}\texttt{List}(\texttt{n}{=}o_4) \quad i_9{:}\mathbb{Z} \quad i_{10}{:}\mathbb{Z}$

$o_4{:}\texttt{List}(\texttt{n}{=}o_5) \quad o_5{:}\texttt{List}(?)$

P

- Data structures transformed to nested constructor terms
- Unknown values transformed to variable

$\text{List}(\text{List}(o_5))$

Transforming values to terms

$$
\begin{array}{l}
o_4, i_{10} \mid 0 \mid \mathtt{t}: o_4, \mathtt{i}: i_{10} \mid \varepsilon \\
\not{o_1}, i_9 \mid 34 \mid \mathtt{t}: o_1, \mathtt{i}: i_{10} \mid \varepsilon \\
o_1{:}\mathtt{List}(\mathtt{n}{=}o_4) \quad i_9{:}\mathbb{Z} \quad i_{10}{:}\mathbb{Z} \\
o_4{:}\mathtt{List}(\mathtt{n}{=}o_5) \quad o_5{:}\mathtt{List}(?)
\end{array}
\quad P
$$

- Data structures transformed to nested constructor terms
- Unknown values transformed to variable
- For each class `C` with n fields
  introduce one function symbol C of arity n

$$\mathrm{List}(\underbrace{\mathrm{List}(o_5)}_{o_4})$$

Transforming values to terms

$o_4, i_{10} \mid 0 \mid \texttt{t}: o_4, \texttt{i}: i_{10} \mid \varepsilon$

~~o_1~~$, i_9 \mid 34 \mid \texttt{t}: o_1, \texttt{i}: i_{10} \mid \varepsilon$

$o_1{:}\texttt{List}(\texttt{n} = o_4) \quad i_9{:}\mathbb{Z} \quad i_{10}{:}\mathbb{Z}$

$o_4{:}\texttt{List}(\texttt{n} = o_5) \quad o_5{:}\texttt{List}(?)$

P

- Data structures transformed to nested constructor terms
- Unknown values transformed to variable
- For each class `C` with n fields
  introduce one function symbol C of arity n
- Encode field values recursively

$$\underbrace{\mathsf{List}(\underbrace{\mathsf{List}(o_5)}_{o_4})}_{o_1}$$

Transforming states to terms

$$\begin{array}{l} o_4, i_{10} \mid 0 \mid \mathtt{t}: o_4, \mathtt{i}: i_{10} \mid \varepsilon \\ \not{o_1}, i_9 \mid 34 \mid \mathtt{t}: o_1, \mathtt{i}: i_{10} \mid \varepsilon \\ o_1: \mathtt{List}(\mathtt{n} = o_4) \quad i_9: \mathbb{Z} \quad i_{10}: \mathbb{Z} \\ o_4: \mathtt{List}(\mathtt{n} = o_5) \quad o_5: \mathtt{List}(?) \end{array} \quad P$$

- For stack frame at position pp in s, introduce $\mathsf{f}_{s,pp}$

$$\mathsf{f}_{P,34}(\mathsf{f}_{P,0}(\mathrm{eos}, \mathrm{List}(o_5), i_{10}, \mathrm{List}(o_5), i_{10}), \mathrm{List}(\mathrm{List}(o_5)), i_9, \mathrm{List}(\mathrm{List}(o_5)), i_{10})$$

Transforming states to terms

$$\begin{array}{l} o_4, i_{10} \mid 0 \mid \texttt{t}: o_4, \texttt{i}: i_{10} \mid \varepsilon \\ \not{o_1}, i_9 \mid 34 \mid \texttt{t}: o_1, \texttt{i}: i_{10} \mid \varepsilon \\ o_1{:}\texttt{List}(\texttt{n}{=}o_4) \quad i_9{:}\mathbb{Z} \quad i_{10}{:}\mathbb{Z} \\ o_4{:}\texttt{List}(\texttt{n}{=}o_5) \quad o_5{:}\texttt{List}(?) \end{array} \quad P$$

- For stack frame at position pp in s, introduce $\mathsf{f}_{s,pp}$
- Nest terms for stack frames (topmost is innermost)
  Topmost has symbol eos to mark **e**nd **o**f **s**tack

$$\mathsf{f}_{P,34}(\mathsf{f}_{P,0}(\mathsf{eos}, \mathsf{List}(o_5), i_{10}, \mathsf{List}(o_5), i_{10}), \mathsf{List}(\mathsf{List}(o_5)), i_9, \mathsf{List}(\mathsf{List}(o_5)), i_{10})$$

Transforming states to terms

$o_4, i_{10} \mid 0 \mid \texttt{t}: o_4, \texttt{i}: i_{10} \mid \varepsilon$
$\not{o_1}, i_9 \mid 34 \mid \texttt{t}: o_1, \texttt{i}: i_{10} \mid \varepsilon$
$o_1{:}\texttt{List}(\texttt{n}{=}o_4) \quad i_9{:}\mathbb{Z} \quad i_{10}{:}\mathbb{Z}$
$o_4{:}\texttt{List}(\texttt{n}{=}o_5) \quad o_5{:}\texttt{List}(?)$

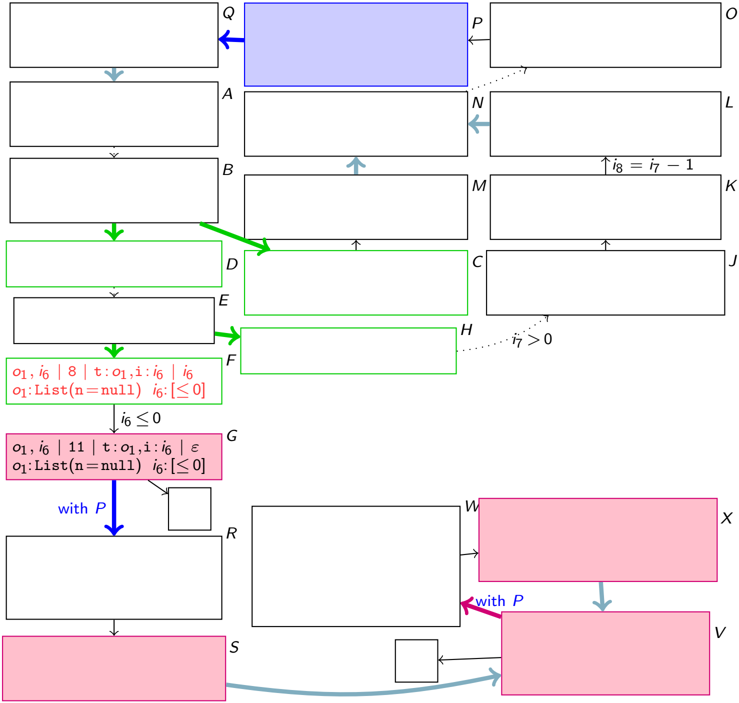
P

- For stack frame at position pp in s, introduce $\mathsf{f}_{s,pp}$
- Nest terms for stack frames (topmost is innermost)
  Topmost has symbol eos to mark **e**nd **o**f **s**tack
- Each input argument, local variable and opstack entry is encoded:

$$\mathsf{f}_{P,34}(\mathsf{f}_{P,0}(\mathsf{eos}, \overbrace{\mathsf{List}(o_5)}^{o_4}, i_{10}, \overbrace{\mathsf{List}(o_5)}^{o_4}, i_{10}),\ \underbrace{\mathsf{List}(\mathsf{List}(o_5))}_{o_1}, i_9, \underbrace{\mathsf{List}(\mathsf{List}(o_5))}_{o_1}, i_{10})$$

Converting Evaluation edges

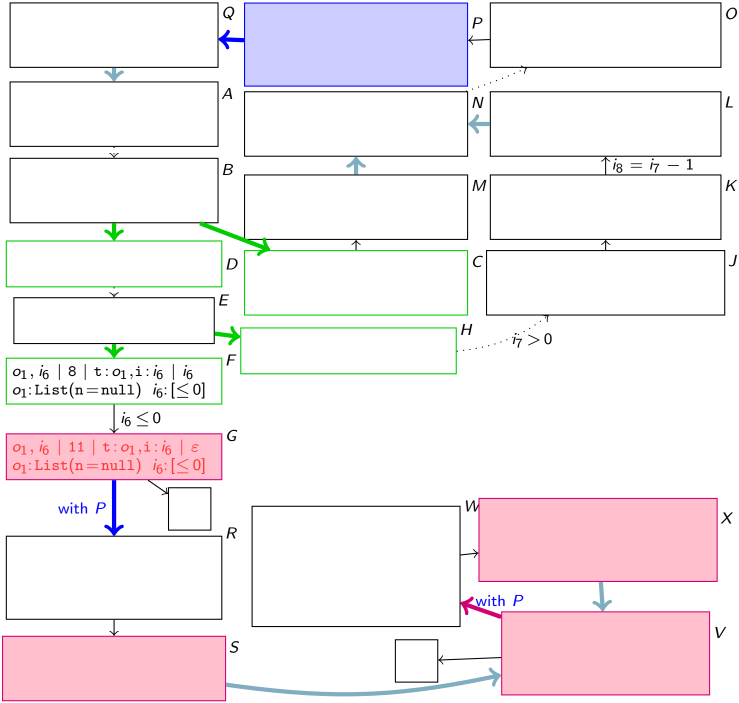
$f_{F,8}(\mathrm{eos},$
$\quad \mathrm{List}(\mathrm{null}),$
$\quad i_6,$
$\quad \mathrm{List}(\mathrm{null}),$
$\quad i_6)$



Converting Evaluation edges

$f_{F,8}(\mathsf{eos}, \mathsf{List}(\mathsf{null}), i_6, \mathsf{List}(\mathsf{null}), i_6)$

$f_{G,11}(\mathsf{eos}, \mathsf{List}(\mathsf{null}), i_6, \mathsf{List}(\mathsf{null}), i_6)$

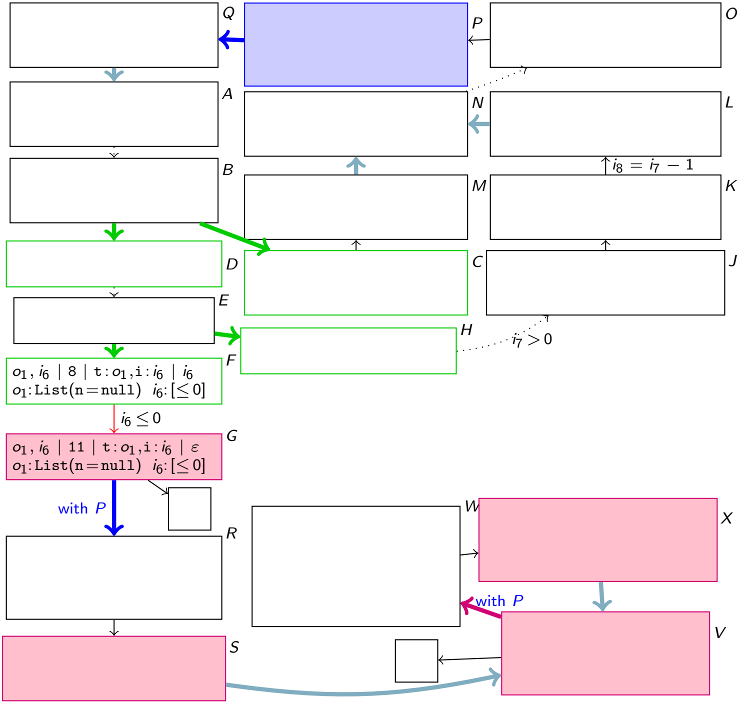


Converting Evaluation edges

$$f_{F,8}(\text{eos}, \text{List}(\text{null}), i_6, \text{List}(\text{null}), i_6)$$

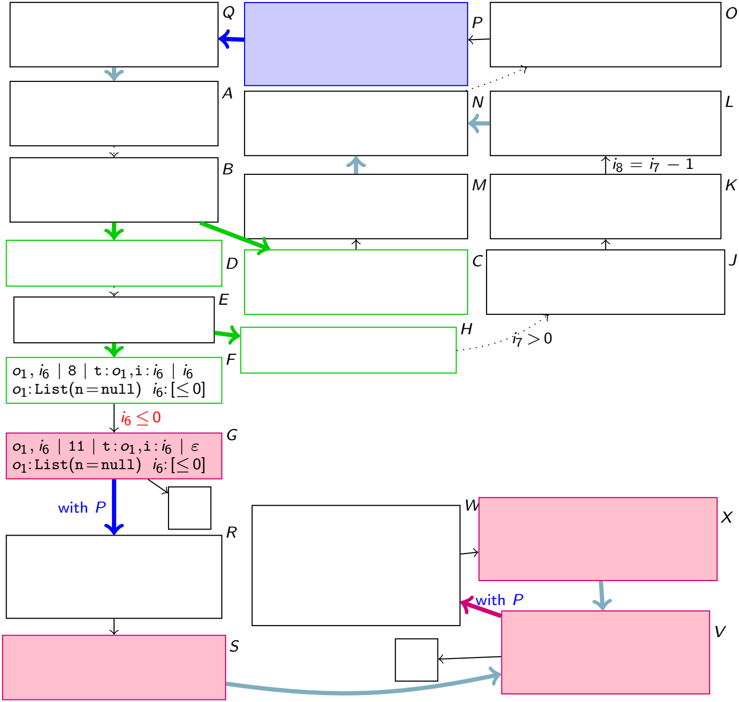
$$\longrightarrow$$

$$f_{G,11}(\text{eos}, \text{List}(\text{null}), i_6, \text{List}(\text{null}), i_6)$$



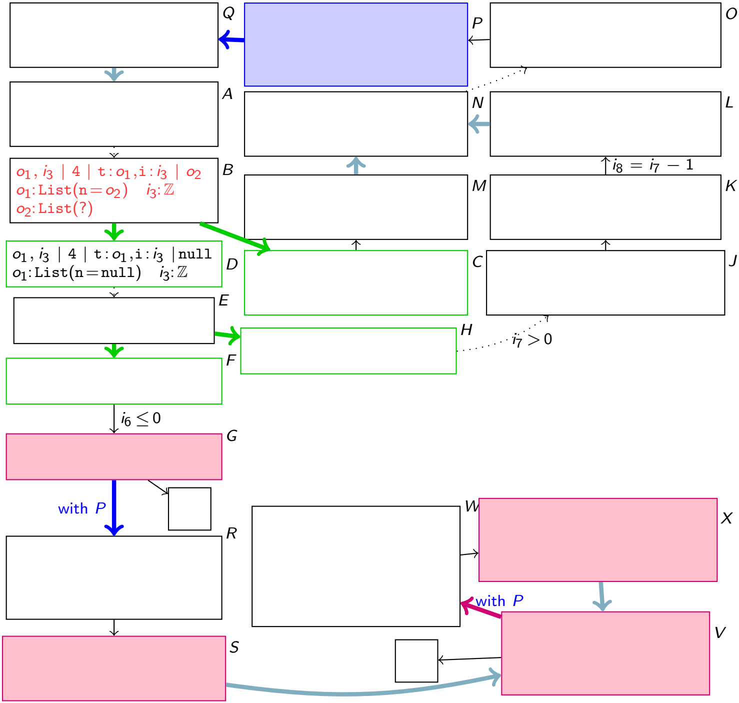
Converting Evaluation edges

$$f_{F,8}(\text{eos}, \text{List}(\text{null}), i_6, \text{List}(\text{null}), i_6) \longrightarrow f_{G,11}(\text{eos}, \text{List}(\text{null}), i_6, \text{List}(\text{null}), i_6) \mid i_6 \leq 0$$



Converting Refinement edges

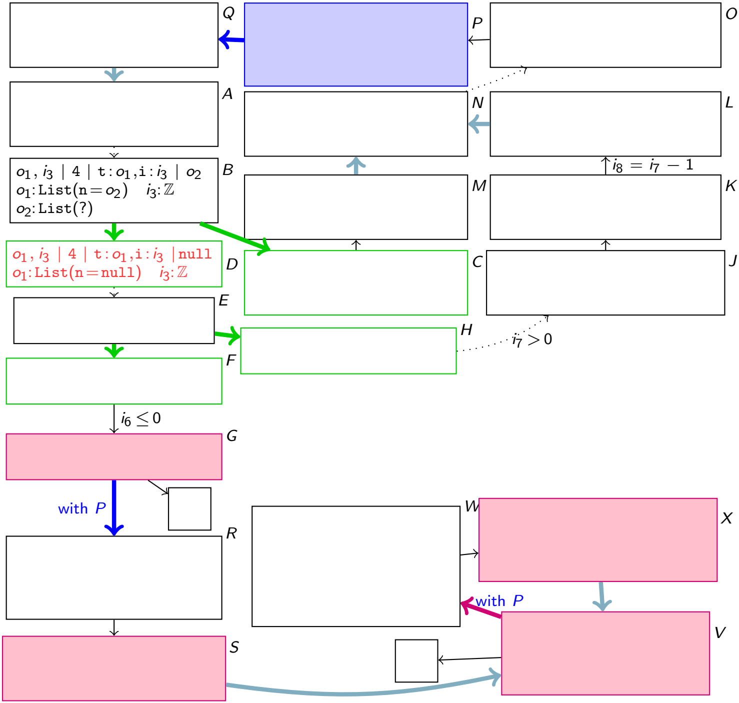
$\mathsf{f}_{B,4}(\mathsf{eos},$
$\mathsf{List}(o_2),$
$i_3,$
$\mathsf{List}(o_2),$
$i_3,$
$o_2)$



Converting Refinement edges

$f_{B,4}($eos,
List(o_2),
i_3,
List(o_2),
i_3,
$o_2)$

$f_{D,4}($eos,
List(null),
i_3,
List(null),
i_3,
null$)$

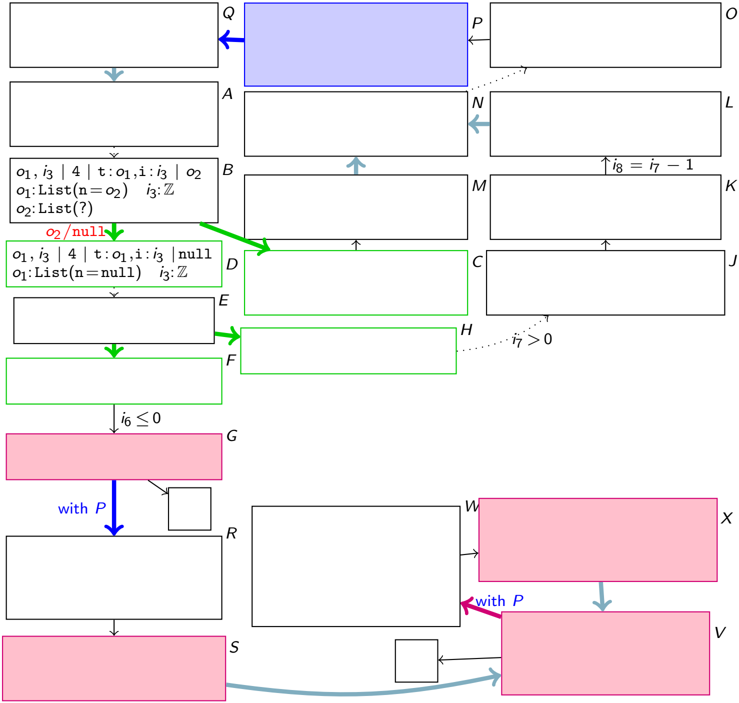


Converting Refinement edges

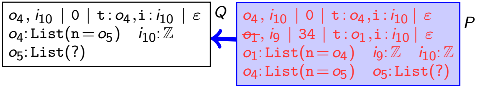
$\mathsf{f}_{B,4}(\mathsf{eos},$
$\quad \mathsf{List}(\mathsf{null}),$
$\quad i_3,$
$\quad \mathsf{List}(\mathsf{null}),$
$\quad i_3,$
$\quad \mathsf{null})$

$\longrightarrow$

$\mathsf{f}_{D,4}(\mathsf{eos},$
$\quad \mathsf{List}(\mathsf{null}),$
$\quad i_3,$
$\quad \mathsf{List}(\mathsf{null}),$
$\quad i_3,$
$\quad \mathsf{null})$

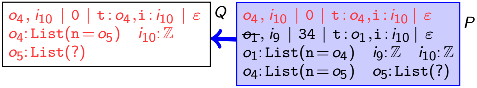


Converting stack split edges



$$\mathsf{f}_{P,34}(\mathsf{f}_{P,0}(\mathsf{eos}, \mathsf{List}(o_5), i_{10}, \mathsf{List}(o_5), i_{10}), \mathsf{List}(\mathsf{List}(o_5)), i_9, \mathsf{List}(\mathsf{List}(o_5)), i_{10})$$

Converting stack split edges



$$\mathsf{f}_{P,0}(\mathsf{eos}, \mathsf{List}(o_5), i_{10}, \mathsf{List}(o_5), i_{10})$$

$$\longrightarrow$$

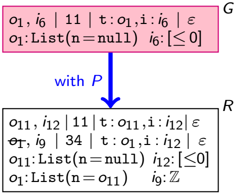
$$\mathsf{f}_{Q,0}(\mathsf{eos}, \mathsf{List}(o_5), i_{10}, \mathsf{List}(o_5), i_{10})$$

Converting call return edges

o_4, i_{10} | 0 | $\texttt{t}: o_4, \texttt{i}: i_{10}$ | ε
~~o_1~~, i_9 | 34 | $\texttt{t}: o_1, \texttt{i}: i_{10}$ | ε
$o_1: \texttt{List}(\texttt{n} = o_4)$ $i_9: \mathbb{Z}$ $i_{10}: \mathbb{Z}$
$o_4: \texttt{List}(\texttt{n} = o_5)$ $o_5: \texttt{List}(?)$

P

$$\mathsf{f}_{P,34}(\mathsf{f}_{P,0}(\mathsf{eos}, \mathsf{List}(o_5), i_{10}, \mathsf{List}(o_5), i_{10}), \mathsf{List}(\mathsf{List}(o_5)), i_9, \mathsf{List}(\mathsf{List}(o_5)), i_{10}))$$



Converting call return edges

P

$o_4, i_{10} \mid 0 \mid \texttt{t}: o_4, \texttt{i}: i_{10} \mid \varepsilon$
$\not{o_1}, i_9 \mid 34 \mid \texttt{t}: o_1, \texttt{i}: i_{10} \mid \varepsilon$
$o_1: \texttt{List}(\texttt{n} = o_4) \quad i_9: \mathbb{Z} \quad i_{10}: \mathbb{Z}$
$o_4: \texttt{List}(\texttt{n} = o_5) \quad o_5: \texttt{List}(?)$

$$\mathsf{f}_{P,34}(\mathsf{f}_{P,0}(\mathsf{eos}, \mathsf{List}(o_5), i_{10}, \mathsf{List}(o_5), i_{10}), \mathsf{List}(\mathsf{List}(o_5)), i_9, \mathsf{List}(\mathsf{List}(o_5)), i_{10}))$$

$$\mathsf{f}_{G,11}(\mathsf{eos}, \mathsf{List}(\mathsf{null}), i_6, \mathsf{List}(\mathsf{null}), i_6)$$

G

$o_1, i_6 \mid 11 \mid \texttt{t}: o_1, \texttt{i}: i_6 \mid \varepsilon$
$o_1: \texttt{List}(\texttt{n} = \texttt{null}) \quad i_6: [\leq 0]$

↓ with P

R

$o_{11}, i_{12} \mid 11 \mid \texttt{t}: o_{11}, \texttt{i}: i_{12} \mid \varepsilon$
$\not{o_1}, i_9 \mid 34 \mid \texttt{t}: o_1, \texttt{i}: i_{12} \mid \varepsilon$
$o_{11}: \texttt{List}(\texttt{n} = \texttt{null}) \quad i_{12}: [\leq 0]$
$o_1: \texttt{List}(\texttt{n} = o_{11}) \quad i_9: \mathbb{Z}$

From intersection:

- o_5 is `null`
- i_6, i_{10} renamed to i_{12}
- i_9 renamed to i_{13}

Converting call return edges

P

$o_4, i_{10} \mid 0 \mid \mathtt{t}: o_4, \mathtt{i}: i_{10} \mid \varepsilon$
$\cancel{o_1}, i_9 \mid 34 \mid \mathtt{t}: o_1, \mathtt{i}: i_{10} \mid \varepsilon$
$o_1: \mathtt{List}(\mathtt{n}{=}o_4) \quad i_9: \mathbb{Z} \quad i_{10}: \mathbb{Z}$
$o_4: \mathtt{List}(\mathtt{n}{=}o_5) \quad o_5: \mathtt{List}(?)$

$$\mathsf{f}_{P,34}(\mathsf{f}_{G,11}(\mathsf{eos}, \mathsf{List}(\mathsf{null}), i_{12}, \mathsf{List}(\mathsf{null}), i_{12}), \mathsf{List}(\mathsf{List}(\mathsf{null})), i_{13}, \mathsf{List}(\mathsf{List}(\mathsf{null})), i_{12})$$

G

$o_1, i_6 \mid 11 \mid \mathtt{t}: o_1, \mathtt{i}: i_6 \mid \varepsilon$
$o_1: \mathtt{List}(\mathtt{n}{=}\mathtt{null}) \quad i_6: [\leq 0]$

↓ with P

R

$o_{11}, i_{12} \mid 11 \mid \mathtt{t}: o_{11}, \mathtt{i}: i_{12} \mid \varepsilon$
$\cancel{o_1}, i_9 \mid 34 \mid \mathtt{t}: o_1, \mathtt{i}: i_{12} \mid \varepsilon$
$o_{11}: \mathtt{List}(\mathtt{n}{=}\mathtt{null}) \quad i_{12}: [\leq 0]$
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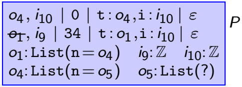
From intersection:

- o_5 is `null`
- i_6, i_{10} renamed to i_{12}
- i_9 renamed to i_{13}

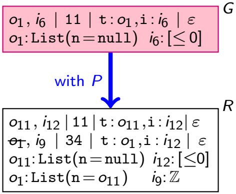
Converting call return edges



$$\mathsf{f}_{P,34}(\mathsf{f}_{G,11}(\mathsf{eos}, \mathsf{List}(\mathsf{null}), i_{12}, \mathsf{List}(\mathsf{null}), i_{12}), \mathsf{List}(\mathsf{List}(\mathsf{null})), i_{13}, \mathsf{List}(\mathsf{List}(\mathsf{null})), i_{12})$$

$$\longrightarrow$$

$$\mathsf{f}_{R,34}(\mathsf{f}_{R,11}(\mathsf{eos}, \mathsf{List}(\mathsf{null}), i_{12}, \mathsf{List}(\mathsf{null}), i_{12}), \mathsf{List}(\mathsf{List}(\mathsf{null})), i_9, \mathsf{List}(\mathsf{List}(\mathsf{null})), i_{12})$$



From intersection:

- o_5 is `null`
- i_6, i_{10} renamed to i_{12}
- i_9 renamed to i_{13}

The Example: TRS

TRS for appE

$$
\begin{array}{rcll}
f_A(L(n),\ i_6) & \to & f_G(L(n),\ i_6) & |\, i_6 \leq 0 \quad (1)\\
f_A(L(n),\ i_7) & \to & f_P(\ f_A(L(n), i_7 - 1),\ L(L(n)),\ i_7) & |\, i_7 > 0 \quad (2)\\
f_A(L(L(o_5)),\ i_3) & \to & f_P(\ f_A(L(o_5), i_3),\ L(L(o_5)),\ i_3) & \quad (3)\\
f_P(\ f_G(L(n), i_{12}),\ L(L(n)),\ i_9) & \to & f_V(L(L(n)),\ i_9) & \quad (4)\\
f_P(\ f_V(L(L(o_{20})), i_{19}),\ L(L(o_5)),\ i_9) & \to & f_V(L(L(L(o_{20}))),\ i_9) & \quad (5)
\end{array}
$$

TRS corresponds to source program:

```
public void appE(int i) {
  if (n == null) {
   if (i <= 0) return;
   n = new List();
   i--;
  }
  n.appE(i);
}
```

The Example: TRS

TRS for appE

$$
\begin{aligned}
f_A(L(n), i_6) &\to f_G(L(n), i_6) & |\, i_6 \le 0 \quad & (1)\\
f_A(L(n), i_7) &\to f_P(f_A(L(n), i_7 - 1), L(L(n)), i_7) & |\, i_7 > 0 \quad & (2)\\
f_A(L(L(o_5)), i_3) &\to f_P(f_A(L(o_5), i_3), L(L(o_5)), i_3) & & (3)\\
f_P(f_G(L(n), i_{12}), L(L(n)), i_9) &\to f_V(L(L(n)), i_9) & & (4)\\
f_P(f_V(L(L(o_{20})), i_{19}), L(L(o_5)), i_9) &\to f_V(L(L(L(o_{20}))), i_9) & & (5)
\end{aligned}
$$

TRS corresponds to source program:

(1) End of list and $\mathtt{i} \le 0$

```
public void appE(int i) {
  if (n == null) {
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  }
  n.appE(i);
}
```

The Example: TRS

TRS for appE

$$\begin{array}{rcll}
f_A(L(n),\ i_6) & \to & f_G(L(n),\ i_6) & | i_6 \leq 0 \quad (1) \\
f_A(L(n),\ i_7) & \to & f_P(\ f_A(L(n), i_7 - 1),\ L(L(n)),\ i_7) & | i_7 > 0 \quad (2) \\
f_A(L(L(o_5)),\ i_3) & \to & f_P(\ f_A(L(o_5), i_3),\ L(L(o_5)),\ i_3) & \quad (3) \\
f_P(\ f_G(L(n), i_{12}),\ L(L(n)),\ i_9) & \to & f_V(L(L(n)),\ i_9) & \quad (4) \\
f_P(\ f_V(L(L(o_{20})), i_{19}),\ L(L(o_5)),\ i_9) & \to & f_V(L(L(L(o_{20}))),\ i_9) & \quad (5)
\end{array}$$

TRS corresponds to source program:

(1) End of list and $\texttt{i} \leq 0$

(2) End of list and $\texttt{i} > 0$

```
public void appE(int i) {
  if (n == null) {
   if (i <= 0) return;
   n = new List();
   i--;
  }
  n.appE(i);
}
```

The Example: TRS

TRS for appE

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\begin{array}{rcll}
f_A(L(n),\ i_6) & \to & f_G(L(n),\ i_6) & \mid i_6 \leq 0 \quad (1)\\
f_A(L(n),\ i_7) & \to & f_P(\ f_A(L(n), i_7 - 1),\ L(L(n)),\ i_7) & \mid i_7 > 0 \quad (2)\\
f_A(L(L(o_5)),\ i_3) & \to & f_P(\ f_A(L(o_5), i_3),\ L(L(o_5)),\ i_3) & (3)\\
f_P(\ f_G(L(n), i_{12}),\ L(L(n)),\ i_9) & \to & f_V(L(L(n)),\ i_9) & (4)\\
f_P(\ f_V(L(L(o_{20})), i_{19}),\ L(L(o_5)),\ i_9) & \to & f_V(L(L(L(o_{20}))),\ i_9) & (5)
\end{array}
$$

TRS corresponds to source program:

(1) End of list and $\mathtt{i} \leq 0$

(2) End of list and $\mathtt{i} > 0$

(3) Not end of list

```
public void appE(int i) {
  if (n == null) {
   if (i <= 0) return;
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   i--;
  }
  n.appE(i);
}
```

The Example: TRS

TRS for appE

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\begin{array}{rcll}
f_A(L(n),\ i_6) & \to & f_G(L(n),\ i_6) & |\, i_6 \le 0 \quad (1)\\
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f_A(L(L(o_5)),\ i_3) & \to & f_P(\ f_A(L(o_5), i_3),\ L(L(o_5)),\ i_3) & (3)\\
f_P(\ f_G(L(n), i_{12}),\ L(L(n)),\ i_9) & \to & f_V(L(L(n)),\ i_9) & (4)\\
f_P(\ f_V(L(L(o_{20})), i_{19}),\ L(L(o_5)),\ i_9) & \to & f_V(L(L(L(o_{20}))),\ i_9) & (5)
\end{array}
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(4), (5) Return from (1), (2)

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public void appE(int i) {
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The Example: TRS

TRS for appE

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f_A(L(n),\ i_7) & \to & f_P(\ f_A(L(n), i_7 - 1),\ L(L(n)),\ i_7) & \mid i_7 > 0 \quad (2) \\
f_A(L(L(o_5)),\ i_3) & \to & f_P(\ f_A(L(o_5), i_3),\ L(L(o_5)),\ i_3) & \quad (3) \\
f_P(\ f_G(L(n), i_{12}),\ L(L(n)),\ i_9) & \to & f_V(L(L(n)),\ i_9) & \quad (4) \\
f_P(\ f_V(L(L(o_{20})), i_{19}),\ L(L(o_5)),\ i_9) & \to & f_V(L(L(L(o_{20}))),\ i_9) & \quad (5)
\end{array}
$$

TRS corresponds to source program:

(1) End of list and $\texttt{i} \leq 0$

(2) End of list and $\texttt{i} > 0$

(3) Not end of list

(4), (5) Return from (1), (2)

Termination proof trivial.

```
public void appE(int i) {
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}
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The Example: TRS

TRS for appE

$$
\begin{array}{rcll}
f_A(L(n),\ i_6) & \to & f_G(L(n),\ i_6) & |\, i_6 \le 0 \quad (1)\\
f_A(L(n),\ i_7) & \to & f_P(\, f_A(L(n), i_7 - 1),\ L(L(n)),\ i_7) & |\, i_7 > 0 \quad (2)\\
f_A(L(L(o_5)),\ i_3) & \to & f_P(\, f_A(L(o_5), i_3),\ L(L(o_5)),\ i_3) & \quad (3)\\
f_P(\, f_G(L(n), i_{12}),\ L(L(n)),\ i_9) & \to & f_V(L(L(n)),\ i_9) & \quad (4)\\
f_P(\, f_V(L(L(o_{20})), i_{19}),\ L(L(o_5)),\ i_9) & \to & f_V(L(L(L(o_{20}))),\ i_9) & \quad (5)
\end{array}
$$

TRS corresponds to source program:

(1) End of list and `i` ≤ 0

(2) End of list and `i` > 0

(3) Not end of list

(4), (5) Return from (1), (2)

Termination proof trivial.

```
public void appE(int i) {
  if (n == null) {
   if (i <= 0) return;
   n = new List();
   i--;
  }
  n.appE(i);
}
```

TRS for cappE

$$
\begin{array}{rcll}
f_{A'}(\ldots) & \to & f_{B'}(f_A(\ldots), \ldots) & \quad (1')\\
f_{B'}(f_V(\ldots), \ldots) & \to & f_{D'}(f_{D'}(\ldots), \ldots) & \quad (2')
\end{array}
$$

Theoretical basis

Definition 1: Java and the Termination Graph

A *computation path* in the Termination Graph is the embedding of a standard JVM computation into the graph.

Theoretical basis

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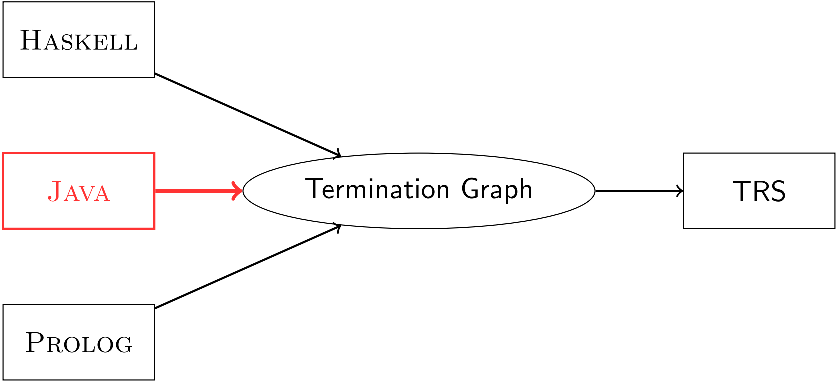
$\Rightarrow$ JBC program terminates for all states represented in Termination Graph.

Modular termination analysis of JBC via term rewriting



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