Initial Type Assumption A_0

$$
\begin{array}{rcl}
A_0(x) & = & \forall a.\, a \;\; \text{for all } x \in \mathcal{V} \\
A_0(c) & = & \text{pre-defined type schema in HASKELL, for all } c \in \mathcal{C}_0 \\
A_0(\underline{\text{constr}}) & = & \forall\; (\underline{\text{type}}_1 \to \ldots \to \underline{\text{type}}_n \to (\underline{\text{tyconstr}}\, a_1 \ldots a_m)), \\
A_0(\texttt{bot}) & = & \forall a.\, a \\
A_0(\texttt{isa}_{()}) & = & (\,) \to \texttt{Bool} \\
A_0(\texttt{if}) & = & \forall a.\, \texttt{Bool} \to a \to a \to a \\
A_0(\texttt{fix}) & = & \forall a.\, (a \to a) \to a \\
A_0(\texttt{isa}_{\underline{\text{constr}}}) & = & \forall\; ((\underline{\text{tyconstr}}\, a_1 \ldots a_m)\; \texttt{->}\; \texttt{Bool}) \\
A_0(\texttt{argof}_{\underline{\text{constr}}}) & = & \forall\; ((\underline{\text{tyconstr}}\, a_1 \ldots a_m)\; \texttt{->}\; (\underline{\text{type}}_1, \ldots, \underline{\text{type}}_n)) \\
A_0(\texttt{sel}_{n,i}) & = & \forall a_1 \ldots a_n.\, (a_1, \ldots, a_n) \to a_i \\
A_0(\texttt{tuple}_n) & = & \forall a_1 \ldots a_n.\, a_1 \to \ldots \to a_n \to (a_1, \ldots, a_n)
\end{array}
$$

Here, constr is introduced by the declaration

$$
\texttt{data}\; \underline{\text{tyconstr}}\, a_1 \ldots a_m = \ldots \,|\, \underline{\text{constr}}\; \underline{\text{type}}_1 \ldots \underline{\text{type}}_n \,|\, \ldots
$$

Type Inference Algorithm $\mathcal{W}$

Let A be a type assumption, let $t \in \Lambda$.
$\mathcal{W}(A, t)$ is either a pair (θ, τ) or the computation fails because of a failing unification problem. Let $c \in \mathcal{C} \cup \mathcal{V}$.

- $\mathcal{W}(\, A + \{c :: \forall a_1, \ldots, a_n . \tau\},\ c\,) \;=\; (\, id,\ \tau[a_1/b_1, \ldots, a_n/b_n]\,)$,
  $b_1, \ldots, b_n$ are new variables

- $\mathcal{W}(\, A,\ \lambda x . t\,) \;=\; (\,\theta,\ b\theta \rightarrow \tau\,)$,
  where $\mathcal{W}(\, A + \{x :: b\},\ t\,) = (\theta, \tau)$, b is a new variable

- $\mathcal{W}(\, A,\ (t_1\, t_2)\,) \;=\; (\,\theta_1\, \theta_2\, \theta_3,\ b\theta_3\,)$,

  where
  $$\begin{aligned} \mathcal{W}(\, A,\ t_1\,) &= (\,\theta_1,\ \tau_1\,) \\ \mathcal{W}(\, A\,\theta_1,\ t_2\,) &= (\,\theta_2,\ \tau_2\,) \\ \theta_3 &= mgu(\tau_1\,\theta_2,\ \tau_2 \rightarrow b), \\ & \quad b \text{ is a new variable.} \end{aligned}$$

Example

$$
\begin{array}{rcl}
\mathcal{W}(A_0, \texttt{fix}\,(\lambda\,\texttt{fact}\,x.\ \texttt{if}\ (x\ \texttt{<=0})\ \texttt{1}\ (\texttt{fact}\,(x-\texttt{1}) * x))) & = & ([\ldots], \texttt{Int} \rightarrow \texttt{Int}) \\
\mathcal{W}(A_0, \texttt{fix}) & = & (id, (a_1 \rightarrow a_1) \rightarrow a_1) \\
\mathcal{W}(A_0, \lambda\,\texttt{fact}\,x.\ \texttt{if}\ (x\ \texttt{<=0})\ \texttt{1}\ (\texttt{fact}\,(x-\texttt{1}) * x)) & = & ([\ldots], (\texttt{Int} \rightarrow \texttt{Int}) \rightarrow (\texttt{Int} \rightarrow \texttt{Int})) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1\}, \lambda\,x.\ \texttt{if}\ (x\ \texttt{<=0})\ \texttt{1}\ (\texttt{fact}\,(x-\texttt{1}) * x)) & = & ([b_1/\texttt{Int} \rightarrow \texttt{Int}, \ldots], \texttt{Int} \rightarrow \texttt{Int}) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: b_2\}, \texttt{if}\ (x\ \texttt{<=0})\ \texttt{1}\ (\texttt{fact}\,(x-\texttt{1}) * x)) & = & ([b_2/\texttt{Int}, b_1/\texttt{Int} \rightarrow \texttt{Int}, \ldots], \texttt{Int}) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: b_2\}, \texttt{if}\ (x\ \texttt{<=0})\ \texttt{1}) & = & ([b_2/\texttt{Int}, \ldots], \texttt{Int} \rightarrow \texttt{Int}) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: b_2\}, \texttt{if}\ (x\ \texttt{<=0})) & = & ([b_2/\texttt{Int}, \ldots], a_2 \rightarrow a_2 \rightarrow a_2) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: b_2\}, \texttt{if}) & = & (id, \texttt{Bool} \rightarrow a_2 \rightarrow a_2 \rightarrow a_2) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: b_2\}, (x\ \texttt{<=0})) & = & ([b_2/\texttt{Int}, \ldots], \texttt{Bool}) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: b_2\}, (x\ \texttt{<=})) & = & ([b_2/\texttt{Int}, \ldots], \texttt{Int} \rightarrow \texttt{Bool}) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: b_2\}, \texttt{<=}) & = & (id, \texttt{Int} \rightarrow \texttt{Int} \rightarrow \texttt{Bool}) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: b_2\}, x) & = & (id, b_2) \\
mgu(\texttt{Int} \rightarrow \texttt{Int} \rightarrow \texttt{Bool}, b_2 \rightarrow b_3) & = & [b_2/\texttt{Int}, b_3/\texttt{Int} \rightarrow \texttt{Bool}] \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: \texttt{Int}\}, \texttt{0}) & = & (id, \texttt{Int}) \\
mgu(\texttt{Int} \rightarrow \texttt{Bool}, \texttt{Int} \rightarrow b_4) & = & [b_4/\texttt{Bool}] \\
mgu(\texttt{Bool} \rightarrow a_2 \rightarrow a_2 \rightarrow a_2, \texttt{Bool} \rightarrow b_5) & = & [b_5/a_2 \rightarrow a_2 \rightarrow a_2] \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: \texttt{Int}\}, \texttt{1}) & = & (id, \texttt{Int}) \\
mgu(a_2 \rightarrow a_2 \rightarrow a_2, \texttt{Int} \rightarrow b_6) & = & [b_6/\texttt{Int} \rightarrow \texttt{Int}] \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: \texttt{Int}\}, \texttt{fact}\,(x-\texttt{1}) * x) & = & ([b_1/\texttt{Int} \rightarrow \texttt{Int}, \ldots], \texttt{Int}) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: \texttt{Int}\}, \texttt{fact}\,(x-\texttt{1}) *) & = & ([b_1/\texttt{Int} \rightarrow \texttt{Int}, \ldots], \texttt{Int} \rightarrow \texttt{Int}) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: \texttt{Int}\}, *)) & = & (id, \texttt{Int} \rightarrow \texttt{Int} \rightarrow \texttt{Int}) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: \texttt{Int}\}, \texttt{fact}\,(x-\texttt{1})) & = & ([b_1/\texttt{Int} \rightarrow b_9, \ldots], b_9)
\end{array}
$$

$$
\begin{aligned}
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: \texttt{Int}\}, \texttt{fact}) &= (id, b_1) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: \texttt{Int}\}, x - \texttt{1}) &= ([\ldots], \texttt{Int}) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: \texttt{Int}\}, x-) &= ([\ldots], \texttt{Int} \rightarrow \texttt{Int}) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: \texttt{Int}\}, -) &= (id, \texttt{Int} \rightarrow \texttt{Int} \rightarrow \texttt{Int}) \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: \texttt{Int}\}, x) &= (id, \texttt{Int}) \\
mgu(\texttt{Int} \rightarrow \texttt{Int} \rightarrow \texttt{Int}, \texttt{Int} \rightarrow b_7) &= [b_7/\texttt{Int} \rightarrow \texttt{Int}] \\
\mathcal{W}(A_0 + \{\texttt{fact} :: b_1, x :: \texttt{Int}\}, \texttt{1}) &= (id, \texttt{Int}) \\
mgu(\texttt{Int} \rightarrow \texttt{Int}, \texttt{Int} \rightarrow b_8) &= [b_8/\texttt{Int}] \\
mgu(b_1, \texttt{Int} \rightarrow b_9) &= [b_1/\texttt{Int} \rightarrow b_9] \\
mgu(\texttt{Int} \rightarrow \texttt{Int} \rightarrow \texttt{Int}, b_9 \rightarrow b_{10}) &= [b_9/\texttt{Int}, b_{10}/\texttt{Int} \rightarrow \texttt{Int}] \\
\mathcal{W}(A_0 + \{\texttt{fact} :: \texttt{Int} \rightarrow \texttt{Int}, x :: \texttt{Int}\}, x) &= (id, \texttt{Int}) \\
mgu(\texttt{Int} \rightarrow \texttt{Int}, \texttt{Int} \rightarrow b_{11}) &= [b_{11}/\texttt{Int}] \\
mgu(\texttt{Int} \rightarrow \texttt{Int}, \texttt{Int} \rightarrow b_{12}) &= [b_{12}/\texttt{Int}] \\
mgu((a_1 \rightarrow a_1) \rightarrow a_1, ((\texttt{Int} \rightarrow \texttt{Int}) \rightarrow (\texttt{Int} \rightarrow \texttt{Int})) \rightarrow b_{13}) &= [a_1/\texttt{Int} \rightarrow \texttt{Int}, b_{13}/\texttt{Int} \rightarrow \texttt{Int}]
\end{aligned}
$$