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LEHR- UND FORSCHUNGSGEBIET INFORMATIK II
RWTH Aachen · D-52056 Aachen · GERMANY
http://www-i2.informatik.rwth-aachen.de/lufgi2

Prof. Dr. Jürgen Giesl
René Thiemann

Exercises *Functional Programming* – Sheet 6

Solutions will be collected until Thursday, Nov 28, 2002 in the exercise course.

Exercise 1 (2 points)

Prove that if all $\sqsubseteq_{D_i}$ are reflexive orderings, then $\sqsubseteq_{D_1 \times \ldots \times D_n}$ is also a reflexive ordering.

Exercise 2 (3 points)

Find all monotonic extensions of the following total functions (given their usual interpretation):

(a) $\neg : \mathbb{B} \to \mathbb{B}$

(b) $\wedge, \vee : \mathbb{B}^2 \to \mathbb{B}$

(c) $-, + : \mathbb{N}^2 \to \mathbb{N}$ where $x - y = 0$ if $x < y$

Exercise 3 (3 points)

Let $(D_1, \sqsubseteq), \ldots, (D_n, \sqsubseteq), (E, \sqsubseteq)$ be partial orderings, $n \geq 1$. A function $g : D_1 \times \ldots \times D_n \to E$ is called *monotonic in the i-th argument* if whenever $d_i, d'_i \in D_i$ such that $d_i \sqsubseteq d'_i$, then for all

$$(d_1, \ldots, d_{i-1}, d_{i+1}, \ldots, d_n) \in D_1 \times \ldots \times D_{i-1} \times D_{i+1} \times \ldots \times D_n$$

one has:

$$g(d_1, \ldots, d_{i-1}, d_i, d_{i+1}, \ldots, d_n) \sqsubseteq g(d_1, \ldots, d_{i-1}, d'_i, d_{i+1}, \ldots, d_n)$$

Show that g is monotonic (in the usual sense) iff it is monotonic in every argument.

Exercise 4 (3 points)

(a) Present the partial ordering $(\mathbb{B}_\perp \to \mathbb{B}_\perp, \sqsubseteq)$ graphically. How many elements does it have? How many elements are maximal?

(b) How many elements does the partial ordering $(\mathbb{B}_\perp^2 \to \mathbb{B}_\perp, \sqsubseteq)$ have?

Exercise 5 (1 point)

Show that each chain of a partial ordering has at most one least upper bound.

Exercise 6 (1 point)

Consider the non strict function `cond` defined as follows:

```
cond :: (Bool, Int, Int) -> Int
cond (True,  x, y) = x
cond (False, x, y) = y
```

As interpretation of `cond` we have the function $f : \mathbb{B}_\perp \times \mathbb{Z}_\perp \times \mathbb{Z}_\perp \to \mathbb{Z}_\perp$ with

$$f(b, x, y) = \begin{cases} x, & \text{if } b = \texttt{True} \\ y, & \text{if } b = \texttt{False} \\ \perp_{\mathbb{Z}}, & \text{if } b = \perp_{\mathbb{B}} \end{cases}$$

Because $f(\texttt{True}, 2, \perp_{\mathbb{Z}}) = 2$, f is non strict. Show that f is monotonic.

Exercise 7 (5 points)

Let D, D' be domains, S be a set of functions from D to D'. For all $i \in D$ define $S_i = \{f(i) | f \in S\}$. Show that

- $\sqcup S$ exists iff $\sqcup S_i$ exists for all $i \in D$.
- If $\sqcup S$ exists, then $(\sqcup S)(i) = \sqcup S_i$.