RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHR- UND FORSCHUNGSGEBIET INFORMATIK II
RWTH Aachen · D-52056 Aachen · GERMANY
http://www-i2.informatik.rwth-aachen.de/lufgi2

Prof. Dr. Jürgen Giesl
René Thiemann

Exercises *Functional Programming* – Sheet 8

Solutions will be collected until Thursday, Dec 12, 2002 in the exercise course.

Exercise 1 (4 points)

Let D_1, D_2 be domains, $D_1 \neq \emptyset$. Show that

$$\sqsubseteq_{D_2} \text{ is a cpo on } D_2 \;\Leftrightarrow\; \sqsubseteq_{D_1 \to D_2} \text{ is a cpo on } D_1 \to D_2.$$

Of course, you can use the results of the previous exercise sheets.

Exercise 2 (2 + 2 points)

(a) Let D be a domain. Show that if $\sqsubseteq_D$ is a cpo on D then $\sqsubseteq_{D_\perp}$ is a cpo on $D_\perp$. ($D_\perp$ denotes the lift of domain D.)

(b) Let $D_1, \dots, D_n$ be domains. Show that if $\sqsubseteq_{D_1}, \dots, \sqsubseteq_{D_n}$ are cpo's then $\sqsubseteq_{D_1 \oplus \dots \oplus D_n}$ is a cpo on $D_1 \oplus \dots \oplus D_n$.

Exercise 3 (1 + 5 points)

Let $D_1, \dots, D_n$ be domains. The domain constructors $\otimes$ and $+$ are defined as follows:

- $D_1 \otimes \dots \otimes D_n = \{(d_1, \dots, d_n) \mid d_1 \in D_1 - \{\perp_{D_1}\}, \dots, d_n \in D_n - \{\perp_{D_n}\}\} \cup \{(\perp_{D_1}, \dots, \perp_{D_n})\}$
  The relation $\sqsubseteq_{D_1 \otimes \dots \otimes D_n}$ is defined in the same way as $\sqsubseteq_{D_1 \times \dots \times D_n}$.
- $D_1 + \dots + D_n = \{d^{D_1} \mid d \in D_1\} \cup \dots \cup \{d^{D_n} \mid d \in D_n\} \cup \{\perp_{D_1 + \dots + D_n}\}$
  The relation $\sqsubseteq_{D_1 + \dots + D_n}$ is defined by $e \sqsubseteq_{D_1 + \dots + D_n} e'$ iff $e = \perp_{D_1 + \dots + D_n}$ or $e = d^{D_i}$, $e' = d'^{D_i}$ and $d \sqsubseteq_{D_i} d'$ for some $i \in \{1, \dots, n\}$.

(a) Represent the domain $D_1 + D_2$ graphically.

(b) Prove or disprove (with consideration of $\sqsubseteq$, but without consideration of the labels):

- $D_1 + D_2 = D_{1\perp} \oplus D_{2\perp}$
- $D_1 \times D_2 = D_{1\perp} \otimes D_{2\perp}$
- $(\{\underline{\mathsf{constr}}_1\} \times D)_\perp \oplus (\{\underline{\mathsf{constr}}_2\} \times D)_\perp = (\{\underline{\mathsf{constr}}_1, \underline{\mathsf{constr}}_2\} \times D)_\perp$

Exercise 4 (2 + 1 + 1 + 2 points)

Consider the following data type declarations

```
(i)   data BoolList = N | C Bool BoolList
(ii)  data Expr     = N | A Int | C Expr
```

(a) Give a graphical representation of the first four levels of the domain of (i). The fourth level contains the element `C True N`, for example.

(b) How many elements does the fifth level of the domain of (i) contain?

(c) Give Haskell expression that correspond to the following elements out of the domain from (i):

- `C` $\bot$ `(C` $\bot$ $\bot$`)`
- `C` $\bot$ `N`
- `C` $\bot$ `(C True` $\bot$`)`

(d) Give a graphical representation of the whole domain of (ii).