RWTH RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

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INFORMATIK II
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Exercises *Functional Programming* – Sheet 6

Solutions will be collected until Friday, June 10, 2005 in the exercise course.
Exercises can be solved both in English and in German.

Exercise 1 (3 points)

Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ with $f(x, y) = x - y$.

Is there a function $f' : \mathbb{Z}_\perp \times \mathbb{Z}_\perp \to \mathbb{Z}_\perp$ such that f' is a monotonic extension of f with $f'(\perp_\mathbb{Z}, \perp_\mathbb{Z}) = 0$? Either give such a function f' or explain why such an f' does not exist.

Exercise 2 (4 points)

Let S be set of functions from D to D', i.e., $f : D \to D'$ for all $f \in S$. For all $i \in D$ we define $S_i = \{f(i) \mid f \in S\}$. Show that

- $\sqcup S$ exists if and only if $\sqcup S_i$ exists for all $i \in D$.
- If $\sqcup S$ exists, then $(\sqcup S)(i) = \sqcup S_i$.

Exercise 3 (2 + 1 + 2 points)

We consider $D = 2^\mathbb{Z}$ and the order $\subseteq$ on D, i.e., D contains all sets of integers and $\subseteq$ is the subset relation. For example, D contains $\{-1, 0, 1\}$ as well as $\{1, 2, 3, 4, \ldots\}$.

(a) What is the lub of a chain $\{M_1, M_2, \ldots\}$ with $M_i \in D$ for all i?
Show that $\subseteq$ is a complete order on D.

(b) Give an example of an infinite chain in $(D, \subseteq)$.

(c) Give an example of a monotonic, non-continuous function $f : D \to D$. Prove that your function is monotonic and not continuous. (Perhaps you can use the chain in part (b) to show that your function is not continuous.)

Exercise 4 (2 points)

Let D_1, D_2, and D_3 be domains with complete orders $\sqsubseteq_1, \sqsubseteq_2, \sqsubseteq_3$, respectively. Let $f : D_1 \to D_2$ and $g : D_2 \to D_3$ be continuous functions. Show that the composition $g \circ f : D_1 \to D_3$ is also continuous.

Exercise 5 (2 + 4 points)

Consider the function $\Phi_{\texttt{fact}} : \langle \mathbb{Z}_\perp \to \mathbb{Z}_\perp \rangle \to \langle \mathbb{Z}_\perp \to \mathbb{Z}_\perp \rangle$ defined as follows ($\Phi_{\texttt{fact}}$ corresponds to the function *ff* from the lecture):

$$(\Phi_{\texttt{fact}}(g))(x) = \begin{cases} 1, & \text{if } x \leq 0 \\ g(x-1) * x, & \text{otherwise} \end{cases}$$

Here, $-$ and $*$ are the strict extensions of the subtraction and the multiplication function.

Show that

(a) $\Phi_{\texttt{fact}}$ is monotonic.

(b) $\Phi_{\texttt{fact}}$ is continuous.

Hint: You may use the results of Exercise 2 (Lemma 2.1.11) and Theorem 2.1.15 in your proof.