Domain of a HASKELL-program

Least solution of

$$\text{Dom} = \text{Functions} \oplus \text{Tuples} \oplus \text{Constructions}_0 \oplus \text{Constructions}_1 \oplus \ \ldots$$

$$\begin{aligned} \text{where} \quad & \text{Functions} = \langle \text{Dom} \to \text{Dom} \rangle_\perp \\ & \text{Tuples} = \{\perp, (\,)\} \oplus (\text{Dom} \times \text{Dom}) \oplus (\text{Dom} \times \text{Dom} \times \text{Dom}) \oplus \ldots \\ & \text{Constructions}_n = (\text{Con}_n \times \text{Dom}^n)_\perp \end{aligned}$$

Environment

$\rho : \text{Var} \to \text{Dom}$ with $\rho = \{\underline{\text{var}}_1, \ldots, \underline{\text{var}}_n \,/\, d_1, \ldots, d_n\}$

$$(\rho_1 + \rho_2)(\underline{\text{var}}) = \begin{cases} \rho_2(\underline{\text{var}}), & \text{if } \rho_2(\underline{\text{var}}) \text{ is defined} \\ \rho_1(\underline{\text{var}}), & \text{otherwise.} \end{cases}$$

Initial environment ω

for all pre-defined variables in HASKELL

Simple HASKELL-Programs

- no type synonyms, no type classes, no pre-defined lists
- only one declaration of the form $\underline{\text{var}} = \underline{\text{exp}}$

$$
\begin{array}{rcl}
\underline{\text{exp}} & \rightarrow & \underline{\text{var}} \\
 & | & \underline{\text{constr}} \\
 & | & \underline{\text{integer}} \\
 & | & \underline{\text{float}} \\
 & | & \underline{\text{char}} \\
 & | & (\underline{\text{exp}}_1, \ldots, \underline{\text{exp}}_n), \quad n \geq 0 \\
 & | & (\underline{\text{exp}}_1\ \underline{\text{exp}}_2) \\
 & | & \texttt{if}\ \underline{\text{exp}}_1\ \texttt{then}\ \underline{\text{exp}}_2\ \texttt{else}\ \underline{\text{exp}}_3 \\
 & | & \texttt{let}\ \underline{\text{var}} = \underline{\text{exp}}\ \texttt{in}\ \underline{\text{exp}} \\
 & | & \backslash\ \underline{\text{var}} \rightarrow \underline{\text{exp}}
\end{array}
$$

Semantics of HASKELL-Programs

$$\mathcal{V}al\,[\![\underline{\mathsf{var}}]\!]\,\rho \quad = \quad \rho(\underline{\mathsf{var}})$$

$$\mathcal{V}al\,[\![\underline{\mathsf{constr}_0}]\!]\,\rho \quad = \quad \underline{\mathsf{constr}_0}\ \mathit{in}\ \mathsf{Constructions}_0\ \mathit{in}\ \mathsf{Dom}$$

$$\mathcal{V}al\,[\![\underline{\mathsf{constr}_n}]\!]\,\rho \quad = \quad f\ \mathit{in}\ \mathsf{Functions}\ \mathit{in}\ \mathsf{Dom},\ \text{where } f\,d_1\,d_2\,\ldots\,d_n = (\underline{\mathsf{constr}_n}, d_1, \ldots, d_n)\ \ \mathit{in}\ \mathsf{Constructions}_n\ \mathit{in}\ \mathsf{Dom}$$

$$\mathcal{V}al\,[\![(\underline{\mathsf{exp}_1},\ \ldots,\ \underline{\mathsf{exp}_n})]\!]\,\rho \quad = \quad (\,\mathcal{V}al\,[\![\underline{\mathsf{exp}_1}]\!]\,\rho, \ldots, \mathcal{V}al\,[\![\underline{\mathsf{exp}_n}]\!]\,\rho\,)\ \mathit{in}\ \mathsf{Tuples}\ \mathit{in}\ \mathsf{Dom},\ \text{where } n = 0 \text{ or } n \geq 2$$

$$\mathcal{V}al\,[\![(\underline{\mathsf{exp}})]\!]\,\rho \quad = \quad \mathcal{V}al\,[\![\underline{\mathsf{exp}}]\!]\,\rho$$

$$\mathcal{V}al\,[\![(\underline{\mathsf{exp}_1}\ \underline{\mathsf{exp}_2})]\!]\,\rho \quad = \quad f(\,\mathcal{V}al\,[\![\underline{\mathsf{exp}_2}]\!]\,\rho\,),\ \text{where } \mathcal{V}al\,[\![\underline{\mathsf{exp}_1}]\!]\,\rho = f\ \mathit{in}\ \mathsf{Functions}\ \mathit{in}\ \mathsf{Dom}$$

$$\mathcal{V}al \left[\!\!\left[\begin{array}{l} \texttt{if}\ \underline{\mathsf{exp}}_1\ \texttt{then}\ \underline{\mathsf{exp}}_2 \\ \qquad\quad\ \texttt{else}\ \underline{\mathsf{exp}}_3 \end{array} \right]\!\!\right] \rho \quad = \quad \begin{cases} \mathcal{V}al\, [\![\underline{\mathsf{exp}}_2]\!]\, \rho, & \text{if } \mathcal{V}al\, [\![\underline{\mathsf{exp}}_1]\!]\, \rho = \texttt{True} \\ & \qquad \mathit{in}\ \mathsf{Constructions}_0\ \mathit{in}\ \mathsf{Dom} \\ \mathcal{V}al\, [\![\underline{\mathsf{exp}}_3]\!]\, \rho, & \text{in } \mathcal{V}al\, [\![\underline{\mathsf{exp}}_1]\!]\, \rho = \texttt{False} \\ & \qquad \mathit{in}\ \mathsf{Constructions}_0\ \mathit{in}\ \mathsf{Dom} \\ \bot, & \text{otherwise} \end{cases}$$

$$\mathcal{V}al \left[\!\!\left[\begin{array}{l} \texttt{let}\ \underline{\mathsf{var}} = \underline{\mathsf{exp}} \\ \qquad \texttt{in}\ \underline{\mathsf{exp}'} \end{array} \right]\!\!\right] \rho \;=\; \mathcal{V}al\, [\![\underline{\mathsf{exp}'}]\!]\, (\rho + \{\underline{\mathsf{var}} \,/\, \mathrm{lfp}\ f\}),$$

where $f : \mathsf{Dom} \to \mathsf{Dom}$ and $f(d) = \mathcal{V}al\, [\![\underline{\mathsf{exp}}]\!]\, (\rho + \{\underline{\mathsf{var}}/d\})$

$$\mathcal{V}al\, [\![\backslash\ \underline{\mathsf{var}}\ \texttt{->}\ \underline{\mathsf{exp}}]\!]\, \rho \quad = \quad f\ \mathit{in}\ \mathsf{Functions}\ \mathit{in}\ \mathsf{Dom}$$

$$\qquad\qquad\qquad \text{where } f(d) = \mathcal{V}al\, [\![\underline{\mathsf{exp}}]\!]\, (\rho + \{\underline{\mathsf{var}} \,/\, d\})$$