

$f : D_1 \rightarrow D_2$ is **monotonic** iff $f(d) \sqsubseteq_{D_2} f(d')$ for all $d \sqsubseteq_{D_1} d'$
 $\{d_1, d_2, \dots\}$ is a **chain** iff $d_1 \sqsubseteq d_2 \sqsubseteq d_3 \sqsubseteq \dots$

$\{\text{fact}_0, \text{fact}_1, \dots\}$ is a chain where

$$\begin{aligned} \text{fact}_0(x) &= \perp \text{ for all } x \in \mathbb{Z}_\perp \\ \text{fact}_1(x) &= \begin{cases} x!, & \text{for } 0 \leq x < 1 \\ 1, & \text{for } x < 0 \\ \perp, & \text{for } x = \perp \text{ or } 1 \leq x \end{cases} \\ \text{fact}_2(x) &= \begin{cases} x!, & \text{for } 0 \leq x < 2 \\ 1, & \text{for } x < 0 \\ \perp, & \text{for } x = \perp \text{ or } 2 \leq x \end{cases} \\ &\vdots \end{aligned}$$

Least upper bound: $\sqcup\{\text{fact}_0, \text{fact}_1, \text{fact}_2, \dots\} = \text{fact}$ with

$$\text{fact}(x) = \begin{cases} x!, & \text{for } 0 \leq x \\ 1, & \text{for } x < 0 \\ \perp, & \text{for } x = \perp \end{cases}$$