Hints:

- To solve the programming exercises you can use the Prolog interpreter SWI-Prolog, available for free at http://www.swi-prolog.org. You can use the command “pl” to start it and use “[exercise1].” to load the facts from file exercise1.pl in the current directory.

- Please solve these exercises in groups of two!

- The solutions must be handed in directly before (very latest: at the beginning of) the exercise course on Wednesday, October 27th, 2010, in lecture hall AH 3.

- Please write the names and immatriculation numbers of all (two) students on your solution. Also please staple the individual sheets!

- Please register at https://aprove.informatik.rwth-aachen.de/lp10/ (https, not http!).

- Exercises or exercise parts marked with a star are voluntary challenge exercises with advanced difficulty. They do not contribute to the points you need for taking part in the final exam.

Exercise 1 (Simple Prolog): (3.5 + 1.5 = 5 points)

Consider the following Prolog program.

```prolog
rightNeighbor(anna,boris).
rightNeighbor(erica,frank).
rightNeighbor(boris,chuck).
rightNeighbor(diana,erica).

leftNeighbor(X,Y) :- rightNeighbor(Y,X).
neighbor(X,Y) :- rightNeighbor(X,Y).
neighbor(X,Y) :- leftNeighbor(X,Y).
```

a) Implement a predicate neighborhood(X,Y) in Prolog which is true iff Y is in the neighborhood of X, i.e., neighborhood(X,Y) is true or there are \( N > 0 \) elements \( X_1, \ldots, X_N \) such that the following predicates are true:
   - \( \text{neighbor}(X,X_1) \)
   - \( \text{neighbor}(X,N,Y) \)
   - \( \text{neighbor}(X,Y,X_J) \) for all \( I,J \in \{1, \ldots, N\} \) with \( J = I + 1 \).

Make sure that the evaluation of all queries \(- \text{neighborhood}(\ldots, \ldots)\) terminates.

b) List all answers Prolog gives for the following queries in the order Prolog gives them:
   1. \(- \text{leftNeighbor}(X,\text{chuck}).\)
   2. \(- \text{neighbor}(X,Y).\)
   3. \(- \text{neighborhood}(\text{anna},Y).\)

c)* Please explain why there is no \textit{false} at the end of all answers for the second query from part b).
Exercise 2 (Syntax): (1 point)

Consider the set of formulas \( \Phi = \{ \)

\[
\text{state}(aachen, germany), \\
\text{state}(berlin, germany), \\
\text{state}(paris, france), \\
\text{state}(lyon, france), \\
\text{state}(kathmandu, nepal), \\
\text{continent}(germany, europe), \\
\text{continent}(france, europe), \\
\text{continent}(nepal, asia), \\
\text{connected}(europe, asia), \\
\text{connected}(asia, europe), \\
\forall A, B, X, Y, Z \ (\text{state}(X, A) \land \text{state}(Y, B) \land \text{continent}(A, Z) \land \text{continent}(B, Z) \rightarrow \text{samecontinent}(X, Y), \\
\forall X, Y \ (\text{samecontinent}(X, Y) \rightarrow \text{byfoot}(X, Y), \\
\forall X, Y, A, B, C, D \ (\text{state}(X, A) \land \text{state}(Y, B) \land \text{continent}(A, C) \land \text{continent}(B, D) \land \text{connected}(C, D) \rightarrow \text{byfoot}(X, Y)
\}

\} \text{ over } \Sigma = \Sigma_0 = \{ \text{aachen, berlin, paris, lyon, kathmandu, germany, france, nepal, europe, asia} \}, \\
\text{ and } \Delta = \Delta_2 = \{ \text{state, continent, connected, samecontinent, byfoot} \} \text{ and } V = \{ A, B, C, D, X, Y, Z \}. \\
\text{Construct the corresponding Prolog program based on } \Phi, \Sigma, \Delta \text{ and } V, \text{ where the order of clauses corresponds to the order of formulas given above.}

Exercise 3 (Induction): (3 points)

Let \( t \) be an arbitrary term. Then the size \(| t |\) of \( t \) is defined as follows. \(| X | = 1 \text{ if } X \text{ is a variable}. \text{ Otherwise we have for } n \geq 0 \text{ that } | f(t_1, \ldots, t_n) | = 1 + \sum_{i=1}^{n} | t_i |. \text{ Show by structural induction that for every term } t \text{ and every substitution } \sigma \text{ we have } | t | \leq | \sigma(t) |. \)

Exercise 4 (Semantics): (1.5 + 2.5 + 4 = 8 points)

Let \( (\Sigma, \Delta) \) be a signature with \( \Sigma = \Sigma_0 \cup \Sigma_1, \Delta = \Delta_3 = \{ \text{plus} \} \) and \( \Sigma_0 = \{ 0 \}, \Sigma_1 = \{ s \}. \text{ Moreover, let } \Phi = \{ \forall Y \ (\text{plus}(0, Y, Y)), \forall X, Y, Z \ (\text{plus}(X, Y, Z) \rightarrow \text{plus}(s(X), Y, s(Z))) \}. \text{ \( \varphi = \forall X, Y, Z \ (\text{plus}(X, Y, Z) \leftrightarrow \text{plus}(Y, X, Z)), \) \( S = \{ (\mathbb{N}, \alpha), \alpha_0 = 0, \alpha_s(x) = x + 1, \text{ and } \alpha_{\text{plus}} = \{ (x, y, z) \in \mathbb{N}^3 \mid x + y = z \}. \) \text{ Prove or disprove the following statements.} \text{ You may use that addition on natural numbers is commutative.} \)

a) \( S \models \varphi \)

b) \( \models \varphi \)

c) \( \Phi \models \varphi \)