Prof. Dr. Jürgen Giesl — Carsten Otto

Notes:

- The solutions must be handed in **directly before (very latest: at the beginning of)** the exercise course on Wednesday, 05.06.2013, in lecture hall **AH 2**. Alternatively you can drop your solutions into a box which is located right next to Prof. Giesl's office (this box is emptied **a few minutes before** the exercise course starts).
- Please write the **names** and **immatriculation numbers** of all (three) students on your solution. Also please staple the individual sheets!

Important: This sheet is only relevant for students attending the **V3M** version of the lecture.

Exercise 1 (Fixpoints): (3+2+2=7 points)

Consider the function $f : Pot(\mathbb{N}) \to Pot(\mathbb{N})$.

$$f(M) = \begin{cases} \{\sum\limits_{y \in X} 2y \mid X \subseteq M\}, & \text{if } M \text{ is finite} \\ \mathbb{N}, & \text{otherwise} \end{cases}$$

So for example, $f(\{2,5\}) = \{\sum\limits_{y \in \emptyset} 2y, \sum\limits_{y \in \{2\}} 2y, \sum\limits_{y \in \{5\}} 2y, \sum\limits_{y \in \{2,5\}} 2y\} = \{0, 4, 10, 14\}$.

a) Prove that f is monotonic.

b) Prove or disprove that f is continuous.

c) Please give all fixpoints of f and mark the least fixpoint.

Exercise 2 (Fixpoint Semantics): (2+1+1+1=5 points)

Consider the following logic program $\mathcal{P}$ over a signature (Σ, Δ) with `0`, `s` $\in \Sigma$ and `plus` $\in \Delta$.

```
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
plus(0,Z,Z).
```

1. For each $i \in \mathbb{N}$ explicitly give $\underline{\text{trans}}^i_{\mathcal{P}}(\emptyset)$.
2. Compute the set $\text{lfp}(\underline{\text{trans}}_{\mathcal{P}})$.
3. Give $F[\![\mathcal{P}, \{\neg$`plus(X, X, s(s(s(s(0)))))`$\}]\!]$.
4. Give $F[\![\mathcal{P}, \{\neg$`plus(X, s(0), Y)`$\}]\!]$.

Exercise 3 (Proofs): (2+2=4 points)

a) Please show that every continuous function $f : \mathcal{P}ot(M) \to \mathcal{P}ot(M)$ is monotonic.

b) Please show that for every *finite* chain

$$M_1 \subseteq M_2 \subseteq \cdots \subseteq M_n$$

and every monotonic function $f : \mathcal{P}ot(M) \to \mathcal{P}ot(M)$ we have:

$$f(\bigcup_{i=1}^{n} M_i) = \bigcup_{i=1}^{n} f(M_i)$$