$\left\{L_{1}, \ldots, L_{n}\right\}$ is unifiable iff there is a $\sigma$ with $\sigma\left(L_{1}\right)=\ldots=\sigma\left(L_{n}\right)$. $\sigma$ is $m g u$ iff for every unifier $\sigma^{\prime}$ there is a substitution $\delta$ with $\sigma^{\prime}=\delta \circ \sigma$.

## Unification Algorithm

1. Let $\sigma=\varnothing$ be the "identical" substitution.
2. If $|\sigma(K)|=1$, then stop and return $\sigma$.
3. Otherwise, check all $\sigma\left(L_{i}\right)$ in parallel from left to right, until there are different symbols in two literals.
4. If none of these symbols is a variable, then stop with clash failure.
5. Otherwise, let $X$ be the variable and $t$ be the subterm in the other literal. If $X$ occurs in $t$, then stop with occur failure.
6. Otherwise, let $\sigma=\{X / t\} \circ \sigma$ und go back to step 2 .

## Resolution for Predicate Logic

$R$ is a resolvent of $K_{1}$ and $K_{2}$ iff

- $\nu_{1}\left(K_{1}\right)$ and $\nu_{2}\left(K_{2}\right)$ are variable-disjoint
- $L_{1}, \ldots, L_{m} \in \nu_{1}\left(K_{1}\right), L_{1}^{\prime}, \ldots, L_{n}^{\prime} \in \nu_{2}\left(K_{2}\right)$ with $n, m \geq 1$ and $\left\{\overline{L_{1}}, \ldots, \overline{L_{m}}, L_{1}^{\prime}, \ldots, L_{n}^{\prime}\right\}$ has mgu $\sigma$
- $R=\sigma\left(\left(\nu_{1}\left(K_{1}\right) \backslash\left\{L_{1}, \ldots, L_{m}\right\}\right) \cup\left(\nu_{2}\left(K_{2}\right) \backslash\left\{L_{1}^{\prime}, \ldots, L_{n}^{\prime}\right\}\right)\right)$


## Example

$$
\begin{array}{lll}
\{\mathrm{p}(\mathrm{f}(X)), \neg \mathrm{q}(Z), \underline{\mathrm{p}(Z)\}} & \underline{f \mathrm{p}(X)}, \mathrm{r}(\mathrm{~g}(X))\} \\
& \{\neg \mathrm{q}(\mathrm{f}(X)), \mathrm{r}(\mathrm{~g}(\mathrm{f}(X)))\} & \begin{array}{l}
\nu_{1}=\varnothing \\
\\
\\
\\
\\
\\
\\
\sigma
\end{array}=\{X / U, U / X\} \\
& =\{Z / \mathrm{f}(X), U / \mathrm{f}(X)\}
\end{array}
$$

