Ground Resolution Algorithm

Goal: Determine whether $\{\varphi_1, \ldots, \varphi_k\} \models \varphi$ holds

1. Let ξ be the formula $\varphi_1 \wedge \ldots \wedge \varphi_k \wedge \neg\varphi$.
2. Transform ξ into Skolem normal form $\forall X_1, \ldots, X_n\ \psi$.
3. Transform ψ into CNF resp. into clause set $\mathcal{K}(\psi)$.
4. Choose an enumeration $\{K_1, K_2, \ldots\}$ of all ground instances of the clauses from $\mathcal{K}(\psi)$.
5. Compute $Res^*(\{K_1, K_2\})$, $Res^*(\{K_1, K_2, K_3\})$, ... If one of these sets contains $\Box$, stop and return "**true**".

Resolution for Predicate Logic

$$\{\ \{\mathsf{p}(X), \neg\mathsf{q}(X)\},\ \{\neg\mathsf{p}(\mathsf{f}(Y))\},\ \{\mathsf{q}(\mathsf{f}(\mathsf{a}))\}\ \}$$

- use substitution $\{X/\mathsf{f}(Y)\}$ for resolution of the first two clauses
- $\mathsf{p}(X)[X/\mathsf{f}(Y)] = \mathsf{p}(\mathsf{f}(Y))$ and $\neg\mathsf{p}(\mathsf{f}(Y))[X/\mathsf{f}(Y)] = \neg\mathsf{p}(\mathsf{f}(Y))$
- $\{X/\mathsf{f}(Y)\}$ is *most general unifier* of $\{\mathsf{p}(X), \mathsf{p}(\mathsf{f}(Y))\}$
- resolvent is $\{\neg\mathsf{q}(X)[X/\mathsf{f}(Y)]\} = \{\neg\mathsf{q}(\mathsf{f}(Y))\}$