Prof. Dr. Jürgen Giesl

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Notes:

- Please solve these exercises in **groups of three or four**!
- The solutions must be handed in **directly before (very latest: at the beginning of)** the exercise course on **Tuesday, May 23rd, 2017 (8:30 am)**, in lecture hall **AH 2**. Alternatively you can drop your solutions into a box which is located right next to Prof. Giesl's office (this box is emptied **15 minutes before** the exercise course starts).
- Please write the **names** and **immatriculation numbers** of all students on your solution. Also please staple the individual sheets!

Exercise 1 (Herbrand model): (2 + 2 = 4 points)

Consider again the formula

$$\begin{aligned}\varphi = \; & \forall X \; \mathsf{p}(X, X) \\ & \wedge (\forall X, Y \; \mathsf{p}(X, Y) \rightarrow \mathsf{p}(X, \mathsf{f}(Y))) \\ & \wedge \neg \forall X \; \mathsf{p}(\mathsf{a}, X)\end{aligned}$$

from Exercise Sheet 2 over the signature (Σ, Δ) with $\Sigma = \Sigma_0 \cup \Sigma_1$, $\Sigma_0 = \{\mathsf{a}\}, \Sigma_1 = \{\mathsf{f}\}$, $\Delta = \Delta_2 = \{\mathsf{p}\}$, and its Skolem normal form

$$\psi = \forall X_1, X_2, Y \quad \mathsf{p}(X_1, X_1) \wedge (\neg \mathsf{p}(X_2, Y) \vee \mathsf{p}(X_2, \mathsf{f}(Y))) \wedge \neg \mathsf{p}(\mathsf{a}, \mathsf{g}(X_1, X_2, Y))$$

Hints:

- You can use $\mathsf{f}^i(\mathsf{a})$ as an abbreviation for $\underbrace{\mathsf{f}(\ldots(\mathsf{f}}_{i \text{ times}}(\mathsf{a}))\ldots)$.

a) Give a Herbrand model for φ or show why no such model exists.

b) Give a Herbrand model for ψ or show why no such model exists.

Exercise 2 (Gilmore's algorithm): (4 points)

Consider the following logic program

```
equals(0,0).
equals(s(X),s(Y)) :- equals(X,Y).
```

and the query

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? - equals(s(s(0)), s(s(0))).
```

Using Gilmore's algorithm show that the formulas φ_1 and φ_2 corresponding to the logic program entail the formula φ corresponding to the query (i.e., $\{\varphi_1, \varphi_2\} \models \varphi$).

Exercise 3 (Conjunctive Normal Form): (2 points)

Consider the following formula φ with $\mathsf{p}_1, \mathsf{p}_2, \mathsf{p}_3 \in \Delta_0$:

$$\varphi = (\neg(\mathsf{p}_1 \rightarrow \mathsf{p}_2) \vee (\neg \mathsf{p}_2 \rightarrow \mathsf{p}_3)) \vee \mathsf{p}_3$$

Use the algorithm presented in the proof of Theorem 3.3.2 to convert φ to an equivalent formula in *conjunctive normal form (CNF)*.

Exercise 4 (Resolution for Propositional Logic): (3 points)

Consider the following clause set $\mathcal{K}$ with $\mathsf{p}_1, \ldots, \mathsf{p}_4 \in \Delta_0$:

$$\mathcal{K} = \{\{\neg \mathsf{p}_1, \mathsf{p}_2\}, \{\mathsf{p}_1, \mathsf{p}_2\}, \{\neg \mathsf{p}_1, \neg \mathsf{p}_2\}, \{\mathsf{p}_1, \neg \mathsf{p}_3\}, \{\mathsf{p}_1, \neg \mathsf{p}_2, \mathsf{p}_3\}\}$$

Please show that $\mathcal{K}$ is unsatisfiable by using resolution for propositional logic (cf. Definition 3.3.4 and Example 3.3.5).

Hints:

- It suffices to perform four resolution steps.

Exercise 5 (Unification): (2 + 2 + 2 = 6 points)

Consider the signature (Σ, Δ) with $\Sigma_0 = \{\mathsf{a}, \mathsf{b}\}, \Sigma_1 = \{\mathsf{f}\}, \Sigma_2 = \{\mathsf{g}\}$ and $\Delta_3 = \{\mathsf{p}\}$. Use the algorithm from the lecture to decide whether the following clauses are unifiable. To document your application of the algorithm on some clause K, please write down the current substituted clause $\sigma(K)$ whenever the algorithm checks whether $|\sigma(K)| = 1$ and underline the position of the next symbols where the literals are not equal. Additionally, write down the resulting most general unifier (mgu) or the kind of failure (clash or occur) the algorithm returns.

a) $\{\mathsf{p}(X, \mathsf{f}(Y), \mathsf{g}(\mathsf{f}(\mathsf{a}), \mathsf{b})), \mathsf{p}(\mathsf{f}(Z), Z, \mathsf{g}(X, \mathsf{b}))\}$

b) $\{\mathsf{p}(X, \mathsf{g}(\mathsf{a}, \mathsf{f}(X)), Y), \mathsf{p}(Z, \mathsf{g}(\mathsf{a}, Y), Z)\}$

c) $\{\mathsf{p}(\mathsf{g}(X, Y), \mathsf{f}(Z), Z), \mathsf{p}(W, Y, \mathsf{f}(\mathsf{a}))\}$

Hint: To illustrate this exercise, we give a short example for the clause $\{\mathsf{p}(X, Y, Z), \mathsf{p}(Z, \mathsf{a}, \mathsf{b})\}$:

1. $\{\mathsf{p}(\underline{X}, \mathsf{f}(Y), Z), \mathsf{p}(\underline{Z}, \mathsf{f}(\mathsf{a}), \mathsf{b})\}$
2. $\{\mathsf{p}(Z, \mathsf{f}(\underline{Y}), Z), \mathsf{p}(Z, \mathsf{f}(\underline{\mathsf{a}}), \mathsf{b})\}$
3. $\{\mathsf{p}(Z, \mathsf{f}(\mathsf{a}), \underline{Z}), \mathsf{p}(Z, \mathsf{f}(\mathsf{a}), \underline{\mathsf{b}})\}$
4. $\{\mathsf{p}(\mathsf{b}, \mathsf{f}(\mathsf{a}), \mathsf{b})\}$
5. mgu: $\{X/\mathsf{b}, Y/\mathsf{a}, Z/\mathsf{b}\}$