

Initial Type Assumption A_0

$$A_0(x) = \forall a. a \text{ for all } x \in \mathcal{V}$$

$$A_0(c) = \text{pre-defined type schema in HASKELL, for all } c \in \mathcal{C}_0$$

$$A_0(\underline{\text{constr}}) = \forall (\underline{\text{type}}_1 \rightarrow \dots \rightarrow \underline{\text{type}}_n \rightarrow (\underline{\text{tyconstr}} a_1 \dots a_m)),$$

$$A_0(\text{bot}) = \forall a. a$$

$$A_0(\text{if}) = \forall a. \text{Bool} \rightarrow a \rightarrow a \rightarrow a$$

$$A_0(\text{fix}) = \forall a. (a \rightarrow a) \rightarrow a$$

$$A_0(\text{isa}_{\underline{\text{constr}}}) = \forall ((\underline{\text{tyconstr}} a_1 \dots a_m) \rightarrow \text{Bool})$$

$$A_0(\text{argof}_{\underline{\text{constr}}}) = \forall ((\underline{\text{tyconstr}} a_1 \dots a_m) \rightarrow (\underline{\text{type}}_1, \dots, \underline{\text{type}}_n))$$

$$A_0(\text{isa}_{n\text{-tuple}}) = \forall a_1 \dots a_n. (a_1, \dots, a_n) \rightarrow \text{Bool}$$

$$A_0(\text{sel}_{n,i}) = \forall a_1 \dots a_n. (a_1, \dots, a_n) \rightarrow a_i$$

$$A_0(\text{tuple}_n) = \forall a_1 \dots a_n. a_1 \rightarrow \dots \rightarrow a_n \rightarrow (a_1, \dots, a_n)$$

Here, constr is introduced by the declaration

$$\text{data } \underline{\text{tyconstr}} a_1 \dots a_m = \dots | \underline{\text{constr}} \underline{\text{type}}_1 \dots \underline{\text{type}}_n | \dots$$

Type Inference Algorithm \mathcal{W}

Let A be a type assumption, let $t \in \Lambda$.

$\mathcal{W}(A, t)$ is either a pair (θ, τ) or the computation fails because of a failing unification problem. Let $c \in \mathcal{C} \cup \mathcal{V}$.

- $\mathcal{W}(A + \{c :: \forall a_1, \dots, a_n. \tau\}, c) = (id, \tau[a_1/b_1, \dots, a_n/b_n])$,
 b_1, \dots, b_n are new variables
- $\mathcal{W}(A, \lambda x. t) = (\theta, b\theta \rightarrow \tau)$,
where $\mathcal{W}(A + \{x :: b\}, t) = (\theta, \tau)$, b is a new variable
- $\mathcal{W}(A, (t_1 t_2)) = (\theta_1 \theta_2 \theta_3, b\theta_3)$,

where

$$\mathcal{W}(A, t_1) = (\theta_1, \tau_1)$$

$$\mathcal{W}(A \theta_1, t_2) = (\theta_2, \tau_2)$$

$$\theta_3 = mgu(\tau_1 \theta_2, \tau_2 \rightarrow b),$$

b is a new variable.

Example

$$\begin{aligned} \mathcal{W}(A_0, \text{fix}(\lambda \text{fact } x. \text{if}(x \leq 0) 1 (\text{fact}(x-1) * x))) &= ([\dots], \text{Int} \rightarrow \text{Int}) \\ \mathcal{W}(A_0, \text{fix}) &= (id, (a_1 \rightarrow a_1) \rightarrow a_1) \\ \mathcal{W}(A_0, \lambda \text{fact } x. \text{if}(x \leq 0) 1 (\text{fact}(x-1) * x)) &= ([\dots], (\text{Int} \rightarrow \text{Int}) \rightarrow (\text{Int} \rightarrow \text{Int})) \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1\}, \lambda x. \text{if}(x \leq 0) 1 (\text{fact}(x-1) * x)) &= ([b_1/\text{Int} \rightarrow \text{Int}, \dots], \text{Int} \rightarrow \text{Int}) \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: b_2\}, \text{if}(x \leq 0) 1 (\text{fact}(x-1) * x)) &= ([b_2/\text{Int}, b_1/\text{Int} \rightarrow \text{Int}, \dots], \text{Int}) \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: b_2\}, \text{if}(x \leq 0) 1) &= ([b_2/\text{Int}, \dots], \text{Int} \rightarrow \text{Int}) \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: b_2\}, \text{if}(x \leq 0)) &= ([b_2/\text{Int}, \dots], a_2 \rightarrow a_2 \rightarrow a_2) \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: b_2\}, \text{if}) &= (id, \text{Bool} \rightarrow a_2 \rightarrow a_2 \rightarrow a_2) \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: b_2\}, (x \leq 0)) &= ([b_2/\text{Int}, \dots], \text{Bool}) \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: b_2\}, (x \leq)) &= ([b_2/\text{Int}, \dots], \text{Int} \rightarrow \text{Bool}) \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: b_2\}, \leq) &= (id, \text{Int} \rightarrow \text{Int} \rightarrow \text{Bool}) \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: b_2\}, x) &= (id, b_2) \\ \text{mgu}(\text{Int} \rightarrow \text{Int} \rightarrow \text{Bool}, b_2 \rightarrow b_3) &= [b_2/\text{Int}, b_3/\text{Int} \rightarrow \text{Bool}] \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: \text{Int}\}, 0) &= (id, \text{Int}) \\ \text{mgu}(\text{Int} \rightarrow \text{Bool}, \text{Int} \rightarrow b_4) &= [b_4/\text{Bool}] \\ \text{mgu}(\text{Bool} \rightarrow a_2 \rightarrow a_2 \rightarrow a_2, \text{Bool} \rightarrow b_5) &= [b_5/a_2 \rightarrow a_2 \rightarrow a_2] \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: \text{Int}\}, 1) &= (id, \text{Int}) \\ \text{mgu}(a_2 \rightarrow a_2 \rightarrow a_2, \text{Int} \rightarrow b_6) &= [b_6/\text{Int} \rightarrow \text{Int}] \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: \text{Int}\}, \text{fact}(x-1) * x) &= ([b_1/\text{Int} \rightarrow \text{Int}, \dots], \text{Int}) \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: \text{Int}\}, \text{fact}(x-1)*) &= ([b_1/\text{Int} \rightarrow \text{Int}, \dots], \text{Int} \rightarrow \text{Int}) \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: \text{Int}\}, *) &= (id, \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}) \\ \mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: \text{Int}\}, \text{fact}(x-1)) &= ([b_1/\text{Int} \rightarrow b_9, \dots], b_9) \end{aligned}$$

$$\begin{aligned}
\mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: \text{Int}\}, \text{fact}) &= (id, b_1) \\
\mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: \text{Int}\}, x - 1) &= ([\dots], \text{Int}) \\
\mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: \text{Int}\}, x -) &= ([\dots], \text{Int} \rightarrow \text{Int}) \\
\mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: \text{Int}\}, -) &= (id, \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}) \\
\mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: \text{Int}\}, x) &= (id, \text{Int}) \\
mgu(\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}, \text{Int} \rightarrow b_7) &= [b_7/\text{Int} \rightarrow \text{Int}] \\
\mathcal{W}(A_0 + \{\text{fact} :: b_1, x :: \text{Int}\}, 1) &= (id, \text{Int}) \\
mgu(\text{Int} \rightarrow \text{Int}, \text{Int} \rightarrow b_8) &= [b_8/\text{Int}] \\
mgu(b_1, \text{Int} \rightarrow b_9) &= [b_1/\text{Int} \rightarrow b_9] \\
mgu(\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}, b_9 \rightarrow b_{10}) &= [b_9/\text{Int}, b_{10}/\text{Int} \rightarrow \text{Int}] \\
\mathcal{W}(A_0 + \{\text{fact} :: \text{Int} \rightarrow \text{Int}, x :: \text{Int}\}, x) &= (id, \text{Int}) \\
mgu(\text{Int} \rightarrow \text{Int}, \text{Int} \rightarrow b_{11}) &= [b_{11}/\text{Int}] \\
mgu(\text{Int} \rightarrow \text{Int}, \text{Int} \rightarrow b_{12}) &= [b_{12}/\text{Int}] \\
mgu((a_1 \rightarrow a_1) \rightarrow a_1, ((\text{Int} \rightarrow \text{Int}) \rightarrow (\text{Int} \rightarrow \text{Int})) \rightarrow b_{13}) &= [a_1/\text{Int} \rightarrow \text{Int}, b_{13}/\text{Int} \rightarrow \text{Int}]
\end{aligned}$$