

Satz 3.2.12: $s \equiv_{\mathcal{E}} t$ gdw. $s \equiv t \in CC^S(\mathcal{E})$

- $S = \text{Subterms}(\mathcal{E}) \cup \text{Subterms}(s) \cup \text{Subterms}(t)$
- $\mathcal{E}_0^S = (\mathcal{E} \cup R) \cap S \times S$
- $\mathcal{E}_{i+1}^S = (\mathcal{E}_i^S \cup S(\mathcal{E}_i^S)) \cup T(\mathcal{E}_i^S) \cup C(\mathcal{E}_i^S) \cap S \times S$
- **Kongruenzabschluss bzgl. S :** $CC^S(\mathcal{E}) = \bigcup_{i \in \mathbb{N}} \mathcal{E}_i^S$

$$\mathcal{E} = \{i \equiv j, k \equiv l, f(i) \equiv g(k), j \equiv f(j), m \equiv g(l)\} \quad s = f(m) \quad t = g(k)$$

$$S = \{i, j, k, l, m, f(i), f(j), f(m), g(k), g(l)\}$$

$$\mathcal{E}_0^S = \mathcal{E} \cup \{i \equiv i, \dots, g(l) \equiv g(l)\}$$

$$\mathcal{E}_1^S = \mathcal{E}_0^S \cup \{j \equiv i, \dots, g(l) \equiv m\} \cup \{f(i) \equiv f(j)\} \cup \{f(i) \equiv f(j), g(k) \equiv g(l)\}$$

$$\mathcal{E} = \{i \equiv j, k \equiv l, f(i) \equiv g(k), j \equiv f(j), m \equiv g(l)\} \quad s = f(m) \quad t = g(k)$$

$$S = \{i, j, k, l, m, f(i), f(j), f(m), g(k), g(l)\}$$

Aufgrund \mathcal{E} : $\{i, j\}$, $\{k, l\}$, $\{f(i), g(k)\}$, $\{j, f(j)\}$, $\{m, g(l)\}$,

Aufgrund S : $\{i\}$, $\{j\}$, $\{k\}$, $\{l\}$, $\{m\}$, $\{f(i)\}$, $\{f(j)\}$, $\{f(m)\}$, $\{g(k)\}$, $\{g(l)\}$

1. Schritt: Äquivalenzrelation

$$S_0 : \{i, j, f(j)\}, \{k, l\}, \{f(i), g(k)\}, \{m, g(l)\}, \{f(m)\}$$

2. Schritt: Kongruenz

$$\{i, j, f(j)\}, \{k, l\}, \{f(i), g(k)\}, \{m, g(l)\}, \{f(m)\}, \\ \{f(i), f(j)\}, \{g(k), g(l)\}$$

3. Schritt: Äquivalenzrelation

$$S_1 : \{i, j, f(j), f(i), g(k), g(l), m\}, \{k, l\}, \{f(m)\}$$

4. Schritt: Kongruenz

$$\{i, j, f(j), f(i), g(k), g(l), m\}, \{k, l\}, \{f(m)\}, \\ \{f(i), f(j)\}, \{f(i), f(m)\}, \{f(j), f(m)\}, \{g(k), g(l)\}$$

5. Schritt: Äquivalenzrelation

$$S_2 : \{i, j, f(j), f(i), g(k), g(l), m, f(m)\}, \{k, l\}$$