

**Signature  $\Sigma$ :**  $\Sigma_0 = \{\mathcal{O}\}$ ,  $\Sigma_1 = \{\text{succ}\}$ ,  $\Sigma_2 = \{\text{plus, times}\}$

**Terms  $\mathcal{T}(\Sigma, \mathcal{V})$ :** (1)  $\mathcal{V} \subseteq \mathcal{T}(\Sigma, \mathcal{V})$  and  
(2)  $f(t_1, \dots, t_n) \in \mathcal{T}(\Sigma, \mathcal{V})$ , if  $f \in \Sigma_n$ ,  $n \geq 0$ ,  $t_i \in \mathcal{T}(\Sigma, \mathcal{V})$

**Sets of Equations  $\mathcal{E}$ :**

$$\begin{array}{ll} \text{plus}(\mathcal{O}, y) \equiv y & f(f(x, y), z) \equiv f(x, f(y, z)) \\ \text{plus}(\text{succ}(x), y) \equiv \text{succ}(\text{plus}(x, y)) & f(x, e) \equiv x \\ & f(x, i(x)) \equiv e \end{array}$$

**Def. 2.2.1 (Interpretation, Algebra)**

$\Sigma$ -Interpretation  $I = (\mathcal{A}, \alpha, \beta)$  with

$$\begin{array}{ll} \mathcal{A} \neq \emptyset & \text{carrier of the interpretation} \\ \alpha = (\alpha_f)_{f \in \Sigma} & \alpha_f : \mathcal{A} \times \dots \times \mathcal{A} \rightarrow \mathcal{A} \text{ meaning of } f \text{ w.r.t. } I \\ \beta : \mathcal{V} \rightarrow \mathcal{A} & \text{variable assignment} \end{array}$$