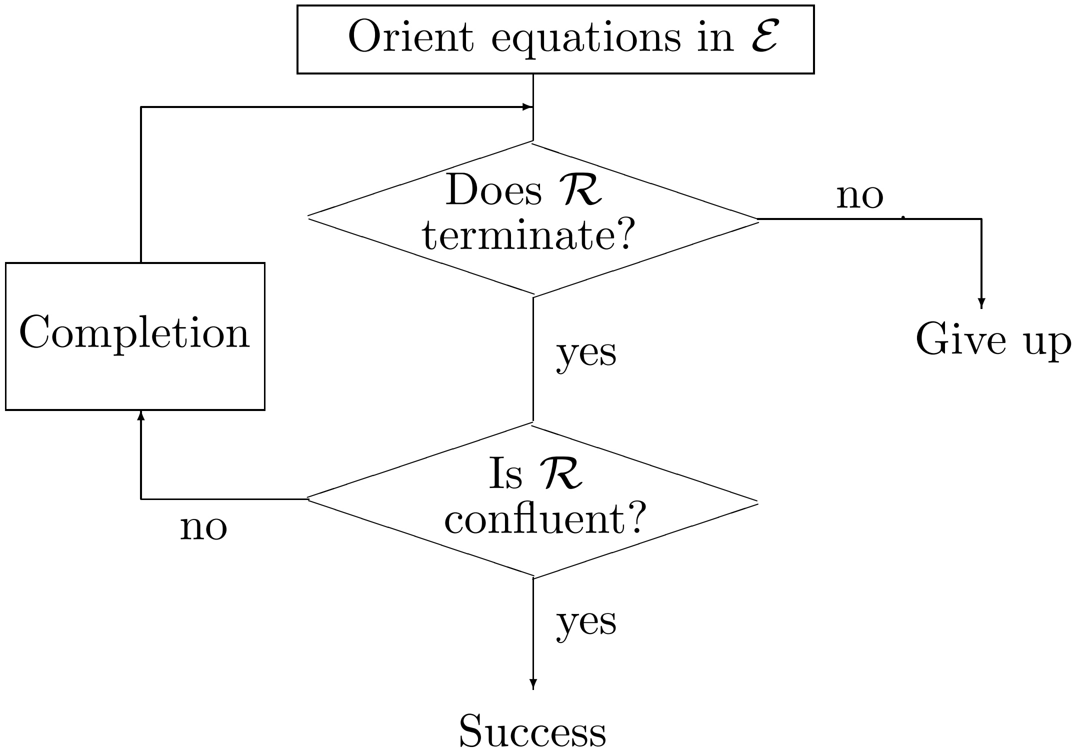
Decision Procedure for the Word Problem



Tasks

1. How can one prove *termination* of TRSs?
2. How can one prove *confluence* of TRSs?
3. How can one perform *completion* of TRSs?

Structural Induction:

$$\varphi(0) \wedge (\forall y \in \mathbb{N}.\quad \varphi(y) \Rightarrow \varphi(y+1)) \quad\Rightarrow\quad \forall x \in \mathbb{N}.\quad \varphi(x)$$

$$\begin{array}{l} (\forall x \in \mathcal{V}.\quad \varphi(x)) \quad \wedge \\ (\forall t_i \in \mathcal{T}(\Sigma, \mathcal{V}).\, \forall f \in \Sigma.\, \varphi(t_1) \wedge .. \wedge \varphi(t_n) \Rightarrow \varphi(f(t_1, .., t_n))) \end{array} \quad\Rightarrow\quad \forall t \in \mathcal{T}(\Sigma, \mathcal{V}).\, \varphi(t)$$

$$\varphi(\epsilon) \wedge (\forall \pi' \in \mathbb{N}^*.\, \forall i \in \mathbb{N}.\quad \varphi(\pi') \Rightarrow \varphi(i\pi')) \quad\Rightarrow\quad \forall \pi \in \mathbb{N}^*.\quad \varphi(\pi)$$

$$\varphi(\epsilon) \wedge (\forall \pi' \in \mathbb{N}^*.\, \forall i \in \mathbb{N}.\quad \varphi(\pi') \Rightarrow \varphi(\pi' i)) \quad\Rightarrow\quad \forall \pi \in \mathbb{N}^*.\quad \varphi(\pi)$$

Noetherian Induction:

$$(\ \forall m \in M.\ \ (\ \forall k \in M.\ \ m \succ k \ \Rightarrow\ \varphi(k)\ )\ \Rightarrow\ \varphi(m)\ ) \quad\Rightarrow\quad \forall n \in M.\ \varphi(n)$$

if $\succ$ is *well founded*