$\succ$ *Reduction Order* iff $\succ$ well founded, stable, monotonic, and transitive.
$\succ$ *Simplification Order* iff $\succ$ reduction order and $\succ_{emb} \subseteq \succ$.

Thm. 4.4.2 (c) (follows from Kruskal's Theorem)
If $\succ$ is stable, monotonic, transitive, irreflexive,
and it satisfies the subterm property $f(x_1, \ldots, x_n) \succ x_i$,
then $\succ$ is a simplification order.

Lexicographic Combination

- $(s_1, \ldots, s_n) \succ_{1 \times \ldots \times n} (t_1, \ldots, t_n)$ iff there exists an i with $s_i \succ_i t_i$ and $s_j = t_j$ for $1 \leq j < i$.

- $\succ_1, \ldots, \succ_n$ are well founded iff $\succ_{1 \times \ldots \times n}$ is well founded.

- $\succ^n_{lex}$ is n-fold lexicographic combination of $\succ$ with itself

Lexicographic Path Order

Let $\sqsupset$ be well-founded order on Σ (*precedence*). We have $s \succ_{lpo} t$ iff

- $s = f(s_1, \ldots, s_n)$ and $s_i \succeq_{lpo} t$ for an i or
- $s = f(s_1, \ldots, s_n)$, $t = g(t_1, \ldots, t_m)$, $f \sqsupset g$, and $s \succ_{lpo} t_j$ for all j or
- $s = f(s_1, \ldots, s_{i-1}, s_i, s_{i+1}, \ldots, s_n)$, $t = f(s_1, \ldots, s_{i-1}, t_i, t_{i+1}, \ldots, t_n)$, $s_i \succ_{lpo} t_i$, and $s \succ_{lpo} t_j$ for all j.

$$
\begin{aligned}
\mathsf{plus}(\mathcal{O}, y) &\succ_{lpo} y \\
\mathsf{plus}(\mathsf{succ}(x), y) &\succ_{lpo} \mathsf{succ}(\mathsf{plus}(x, y)) \\
\mathsf{times}(\mathcal{O}, y) &\succ_{lpo} \mathcal{O} \\
\mathsf{times}(\mathsf{succ}(x), y) &\succ_{lpo} \mathsf{plus}(y, \mathsf{times}(x, y))
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{sum}(\mathcal{O}, y) &\succ_{lpo} y \\
\mathsf{sum}(\mathsf{succ}(x), y) &\succ_{lpo} \mathsf{sum}(x, \mathsf{succ}(y))
\end{aligned}
$$