Recursive Path Order

Let $\sqsupset$ be well-founded order on Σ (*precedence*). We have $s \succ_{rpo} t$ iff

- $s = f(s_1, \ldots, s_n)$ and $s_i \succeq_{rpo} t$ for an i or
- $s = f(s_1, \ldots, s_n)$, $t = g(t_1, \ldots, t_m)$, $f \sqsupset g$, and $s \succ_{rpo} t_j$ for all j or
- $s = f(s_1, \ldots, s_n)$, $t = f(t_1, \ldots, t_n)$, $\{s_1, \ldots, s_n\} \, (\succ_{rpo})_{mul} \, \{t_1, \ldots, t_n\}$

RPO with Status $\succ_{rpos}$

Assign permutation of $1, \ldots, n$ or "multiset" to every n-ary symbol f, compare arguments lexicographically in this order or as multiset.

$$
\begin{array}{rcl}
\mathsf{sum}(\mathcal{O}, y) & \to & y \\
\mathsf{sum}(\mathsf{succ}(x), y) & \to & \mathsf{sum}(x, \mathsf{succ}(y))
\end{array}
\qquad
\begin{array}{rcl}
\mathsf{plus}(\mathcal{O}, y) & \to & y \\
\mathsf{plus}(\mathsf{succ}(x), y) & \to & \mathsf{succ}(\mathsf{plus}(y, x))
\end{array}
$$

- s and t are *unifiable* iff there exists a *unifier* σ with $s\sigma = t\sigma$
- *Unification Problem* $S = \{s_1 =^? t_1, \ldots, s_n =^? t_n\}$
- $\sigma \in U(S)$ iff $s_i\sigma = t_i\sigma$ for all $1 \leq i \leq n$
- σ is *more general* than σ' iff there is a substitution δ with $\sigma' = \sigma\delta$

Example $S = \{\mathsf{g}(\mathsf{f}(x), y) =^? \mathsf{g}(y, \mathsf{f}(z))\}$

$$U(S) \text{ contains} \quad \begin{aligned} \sigma \; &= \{x/z, y/\mathsf{f}(z)\} \\ \sigma_1 &= \{x/\mathsf{a}, y/\mathsf{f}(\mathsf{a}), z/\mathsf{a}\} \\ \sigma_2 &= \{x/\mathsf{f}(z), y/\mathsf{f}(\mathsf{f}(z)), z/\mathsf{f}(z)\} \\ \sigma_3 &= \{z/x, y/\mathsf{f}(x)\} \qquad \text{etc.} \end{aligned}$$

σ and σ_3 are *most general* unifiers, since

$$\begin{aligned} \sigma_1 &= \sigma\delta_1 \text{ with } \delta_1 = \{z/\mathsf{a}\} \\ \sigma_2 &= \sigma\delta_2 \text{ with } \delta_2 = \{z/\mathsf{f}(z)\} \\ \sigma_3 &= \sigma\delta_3 \text{ with } \delta_3 = \{z/x\} \end{aligned}$$