Delete	$S \uplus \{t =^? t\}$	$\Longrightarrow$	S
Reduce Term	$S \uplus \{f(s_1, .., s_n) =^? f(t_1, .., t_n)\}$	$\Longrightarrow$	$S \cup \{s_1 =^? t_1, \ldots, s_n =^? t_n\}$
Exchange	$S \uplus \{t =^? x\}$	$\Longrightarrow$	$S \cup \{x =^? t\}$, if $t \notin \mathcal{V}$
Reduce Var.	$S \uplus \{x =^? t\}$	$\Longrightarrow$	$\{u\sigma =^? v\sigma \mid u =^? v \in S\} \cup \{x =^? t\}$, if $\sigma = \{x/t\}, x \notin \mathcal{V}(t), x \in \mathcal{V}(S)$

$$\{\mathsf{g}(\mathsf{f}(\mathsf{a}), \mathsf{g}(x, x)) =^? \mathsf{g}(x, \mathsf{g}(x, y))\} \Longrightarrow_{\text{ReduceTerm}}$$
$$\{\mathsf{f}(\mathsf{a}) =^? x, \mathsf{g}(x, x) =^? \mathsf{g}(x, y)\} \Longrightarrow_{\text{Exchange}}$$
$$\{x =^? \mathsf{f}(\mathsf{a}), \mathsf{g}(x, x) =^? \mathsf{g}(x, y)\} \Longrightarrow_{\text{ReduceVar.}}$$
$$\{x =^? \mathsf{f}(\mathsf{a}), \mathsf{g}(\mathsf{f}(\mathsf{a}), \mathsf{f}(\mathsf{a})) =^? \mathsf{g}(\mathsf{f}(\mathsf{a}), y)\} \Longrightarrow_{\text{ReduceTerm}}$$
$$\{x =^? \mathsf{f}(\mathsf{a}), \mathsf{f}(\mathsf{a}) =^? \mathsf{f}(\mathsf{a}), \mathsf{f}(\mathsf{a}) =^? y\} \Longrightarrow_{\text{Delete}}$$
$$\{x =^? \mathsf{f}(\mathsf{a}), \mathsf{f}(\mathsf{a}) =^? y\} \Longrightarrow_{\text{Exchange}}$$
$$\{x =^? \mathsf{f}(\mathsf{a}), y =^? \mathsf{f}(\mathsf{a})\}$$

Algorithm UNIFY(S)

1. While there exists an S' with $S \Longrightarrow S'$, let $S := S'$ und goto 1.
2. If S is in solved form, return σ_S. Otherwise, return "*False*".

Delete	$S \uplus \{t =^? t\}$	$\Longrightarrow$	S
Reduce Term	$S \uplus \{f(s_1, .., s_n) =^? f(t_1, .., t_n)\}$	$\Longrightarrow$	$S \cup \{s_1 =^? t_1, \ldots, s_n =^? t_n\}$
Exchange	$S \uplus \{t =^? x\}$	$\Longrightarrow$	$S \cup \{x =^? t\}$, if $t \notin \mathcal{V}$
Reduce Var.	$S \uplus \{x =^? t\}$	$\Longrightarrow$	$\{u\sigma =^? v\sigma \mid u =^? v \in S\} \cup \{x =^? t\}$, if $\sigma = \{x/t\}, x \notin \mathcal{V}(t), x \in \mathcal{V}(S)$

Algorithm UNIFY(S)

1. While there exists an S' with $S \Longrightarrow S'$, let $S := S'$ und goto 1.
2. If S is in solved form, return σ_S. Otherwise, return "*False*".

Thm. 5.1.9 (Soundness of the Unification Algorithm)

(a) The relation $\Longrightarrow$ is well founded.
(b) If $S \Longrightarrow S'$, then we have $U(S) = U(S')$.
(c) If S is solvable and in normal form w.r.t. $\Longrightarrow$, then S is in solved form.
(d) The algorithm UNIFY terminates and is correct.