Algorithm BASIC_COMPLETION($\mathcal{E}, \succ$)

Input: A set of equations $\mathcal{E}$ and a reduction order $\succ$.

Output: A convergent TRS $\mathcal{R}$ that is equivalent to $\mathcal{E}$ or "*Fail*" or non-termination.

1. If $s \equiv t \in \mathcal{E}$ with $s \neq t$, $s \not\succ t$, $t \not\succ s$, then return "*Fail*" and stop.

2. Let $i = 0$ and $\mathcal{R}_0 = \{l \to r \mid l \succ r$ and $(l \equiv r \in \mathcal{E}$ or $r \equiv l \in \mathcal{E})\}$.

3. Let $\mathcal{R}_{i+1} = \mathcal{R}_i$.

4. For all $\langle s, t \rangle \in CP(\mathcal{R}_i)$:

   4.1. Compute $\mathcal{R}_i$-normal forms s' and t' of s and t.
   4.2. If $s' \neq t'$, $s' \not\succ t'$, and $t' \not\succ s'$, then return "*Fail*" and stop.
   4.3. If $s' \succ t'$, then let $\mathcal{R}_{i+1} = \mathcal{R}_{i+1} \cup \{s' \to t'\}$.
   4.4. If $t' \succ s'$, then let $\mathcal{R}_{i+1} = \mathcal{R}_{i+1} \cup \{t' \to s'\}$.

5. If $\mathcal{R}_{i+1} = \mathcal{R}_i$, then return $\mathcal{R}_i$ and stop.

6. Let $i = i + 1$ and go back to Step 3.

Central Gruppoids

$$\mathcal{E} = \{\mathsf{f}(\mathsf{f}(x, y), \mathsf{f}(y, z)) \equiv y\}$$

$$\mathcal{R}_0 = \{\mathsf{f}(\mathsf{f}(x, y), \mathsf{f}(y, z)) \to y\}$$

$$\mathsf{f}(\underline{\mathsf{f}(\mathsf{f}(x, y), \mathsf{f}(y, z))}, \mathsf{f}(\mathsf{f}(y, z), z'))$$

$$\mathsf{f}(y, z) \leftarrow \quad \rightarrow \mathsf{f}(y, \mathsf{f}(\mathsf{f}(y, z), z'))$$

$$\mathsf{f}(\mathsf{f}(x', \mathsf{f}(x, y)), \underline{\mathsf{f}(\mathsf{f}(x, y), \mathsf{f}(y, z))})$$

$$\mathsf{f}(x, y) \leftarrow \quad \rightarrow \mathsf{f}(\mathsf{f}(x', \mathsf{f}(x, y)), y)$$

$$\begin{aligned} \mathcal{R}_1 = \quad \{\mathsf{f}(\mathsf{f}(x, y), \mathsf{f}(y, z)) &\to y, \\ \mathsf{f}(y, \mathsf{f}(\mathsf{f}(y, z), z')) &\to \mathsf{f}(y, z), \\ \mathsf{f}(\mathsf{f}(x', \mathsf{f}(x, y)), y) &\to \mathsf{f}(x, y)\} \end{aligned}$$