$\equiv_{\mathcal{E}}$ stable: $s \equiv_{\mathcal{E}} t$ implies $s\sigma \equiv_{\mathcal{E}} t\sigma$

Lemma 3.1.4 ($\equiv_{\mathcal{E}}$ stable)

Let $I = (\mathcal{A}, \alpha, \beta)$, $I' = (\mathcal{A}, \alpha, \beta')$ with $\beta'(x) = I(x\sigma)$ for all $x \in \mathcal{V}$.

(a) $I(t\sigma) = I'(t)$

(b) $I \models s\sigma \equiv t\sigma$ iff $I' \models s \equiv t$

(c) If $A \models s \equiv t$, then $A \models s\sigma \equiv t\sigma$.

(d) If $s \equiv_{\mathcal{E}} t$, then $s\sigma \equiv_{\mathcal{E}} t\sigma$.

Structural Induction on Terms: Prove $\varphi(t)$ for all $t \in \mathcal{T}(\Sigma, \mathcal{V})$

- Induction Base: $\varphi(x)$ for all $x \in \mathcal{V}$
- Induction Step: $\underbrace{\varphi(t_1) \;\wedge\; \ldots \;\wedge\; \varphi(t_n)}_{Ind.Hyp.} \;\Rightarrow\; \varphi(f(t_1, \ldots, t_n))$