

# Derivation of $f(m) \equiv g(k)$

Axioms:

$$\begin{aligned} i &\equiv j \\ k &\equiv l \\ f(i) &\equiv g(k) \\ j &\equiv f(j) \\ m &\equiv g(l) \end{aligned}$$

$$\begin{aligned} &f(m) \\ \leftrightarrow_{\mathcal{E}} &f(g(l)) \quad \text{using } m \equiv g(l) \\ \leftrightarrow_{\mathcal{E}} &f(g(k)) \quad \text{using } k \equiv l \\ \leftrightarrow_{\mathcal{E}} &f(f(i)) \quad \text{using } f(i) \equiv g(k) \\ \leftrightarrow_{\mathcal{E}} &f(f(j)) \quad \text{using } i \equiv j \\ \leftrightarrow_{\mathcal{E}} &f(j) \quad \text{using } j \equiv f(j) \\ \leftrightarrow_{\mathcal{E}} &f(i) \quad \text{using } i \equiv j \\ \leftrightarrow_{\mathcal{E}} &g(k) \quad \text{using } f(i) \equiv g(k) \end{aligned}$$

$$f(m) \equiv g(k) \in \mathcal{E}_g \subseteq CC(\mathcal{E})$$

