

# Term Rewriting Systems

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Apply the advanced completion algorithm to the following equations:

$$\begin{aligned}\text{plus}(\mathcal{O}, y) &\equiv y \\ \text{plus}(s(x), y) &\equiv s(\text{plus}(x, y)) \\ p(s(x)) &\equiv x \\ s(p(x)) &\equiv x\end{aligned}$$

Reduction order: LPO with precedence  $\text{plus} \sqsupset s \sqsupset p \sqsupset \mathcal{O}$

$$\{\text{plus}(\mathcal{O}, y) \equiv y, \text{plus}(s(x), y) \equiv s(\text{plus}(x, y)), p(s(x)) \equiv x, s(p(x)) \equiv x\},$$

$$\emptyset$$


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*(Orient  $s(p(x)) \equiv x$  and  $p(s(x)) \equiv x$ )*

$$\{\text{plus}(\mathcal{O}, y) \equiv y, \text{plus}(s(x), y) \equiv s(\text{plus}(x, y))\},$$

$$\{p(s(x)) \rightarrow x, s(p(x)) \rightarrow x\}$$

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*(Generate  $p(x) \equiv p(x)$  and  $s(x) \equiv s(x)$ )*

$$\text{plus}(\mathcal{O}, y) \equiv y, \text{plus}(s(x), y) \equiv s(\text{plus}(x, y)), p(x) \equiv p(x), s(x) \equiv s(x)\},$$

$$\{p(s(x)) \rightarrow x, s(p(x)) \rightarrow x\}$$

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*(Delete  $p(x) \equiv p(x)$  and  $s(x) \equiv s(x)$ )*

$$\{\text{plus}(\mathcal{O}, y) \equiv y, \text{plus}(s(x), y) \equiv s(\text{plus}(x, y))\},$$

$$\{p(s(x)) \rightarrow x, s(p(x)) \rightarrow x\}$$

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*(Orient  $\text{plus}(s(x), y) \equiv s(\text{plus}(x, y))$ )*

$$\{\text{plus}(\mathcal{O}, y) \equiv y\},$$

$$\{\text{plus}(s(x), y) \rightarrow s(\text{plus}(x, y)), p(s(x)) \rightarrow x, s(p(x)) \rightarrow x\}$$

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*(Generate  $s(\text{plus}(p(x), y)) \equiv \text{plus}(x, y)$ )*

$$\{\text{plus}(\mathcal{O}, y) \equiv y, s(\text{plus}(p(x), y)) \equiv \text{plus}(x, y)\},$$

$$\{\text{plus}(s(x), y) \rightarrow s(\text{plus}(x, y)), p(s(x)) \rightarrow x, s(p(x)) \rightarrow x\}$$

$$\{\text{plus}(\mathcal{O}, y) \equiv y, \text{s}(\text{plus}(\text{p}(x), y)) \equiv \text{plus}(x, y)\},$$

$$\{\text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$

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$$(\text{Orient } \text{s}(\text{plus}(\text{p}(x), y)) \equiv \text{plus}(x, y))$$

$$\{\text{plus}(\mathcal{O}, y) \equiv y\},$$

$$\{\text{s}(\text{plus}(\text{p}(x), y)) \rightarrow \text{plus}(x, y), \text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$

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$$(\text{Generate } \text{plus}(\text{p}(x), y) \equiv \text{p}(\text{plus}(x, y)))$$

$$\{\text{plus}(\mathcal{O}, y) \equiv y, \text{plus}(\text{p}(x), y) \equiv \text{p}(\text{plus}(x, y))\},$$

$$\{\text{s}(\text{plus}(\text{p}(x), y)) \rightarrow \text{plus}(x, y), \text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$

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$$(\text{Orient } \text{plus}(\text{p}(x), y) \equiv \text{p}(\text{plus}(x, y)))$$

$$\{\text{plus}(\mathcal{O}, y) \equiv y\},$$

$$\{\text{plus}(\text{p}(x), y) \rightarrow \text{p}(\text{plus}(x, y)), \text{s}(\text{plus}(\text{p}(x), y)) \rightarrow \text{plus}(x, y),$$

$$\text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$

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$$(\text{Reduce lhs of } \text{s}(\text{plus}(\text{p}(x), y)) \rightarrow \text{plus}(x, y))$$

$$\{\text{plus}(\mathcal{O}, y) \equiv y, \text{s}(\text{p}(\text{plus}(x, y))) \equiv \text{plus}(x, y)\},$$

$$\{\text{plus}(\text{p}(x), y) \rightarrow \text{p}(\text{plus}(x, y)), \text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$

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$$(\text{Reduce lhs of } \text{s}(\text{p}(\text{plus}(x, y))) \equiv \text{plus}(x, y))$$

$$\{\text{plus}(\mathcal{O}, y) \equiv y, \text{plus}(x, y) \equiv \text{plus}(x, y)\},$$

$$\{\text{plus}(\text{p}(x), y) \rightarrow \text{p}(\text{plus}(x, y)), \text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$

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$$(\text{Delete } \text{plus}(x, y) \equiv \text{plus}(x, y))$$

$$\{\text{plus}(\mathcal{O}, y) \equiv y\},$$

$$\{\text{plus}(\text{p}(x), y) \rightarrow \text{p}(\text{plus}(x, y)), \text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$

$$\{\text{plus}(\mathcal{O}, y) \equiv y\},$$

$$\{\text{plus}(\text{p}(x), y) \rightarrow \text{p}(\text{plus}(x, y)), \text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$

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*(Generate  $\text{p}(\text{plus}(\text{s}(x), y)) \equiv \text{plus}(x, y)$ )*

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$$\{\text{plus}(\mathcal{O}, y) \equiv y, \text{p}(\text{plus}(\text{s}(x), y)) \equiv \text{plus}(x, y)\},$$

$$\{\text{plus}(\text{p}(x), y) \rightarrow \text{p}(\text{plus}(x, y)), \text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$

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*(Reduce lhs of  $\text{p}(\text{plus}(\text{s}(x), y)) \equiv \text{plus}(x, y)$ )*

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$$\{\text{plus}(\mathcal{O}, y) \equiv y, \text{p}(\text{s}(\text{plus}(x, y))) \equiv \text{plus}(x, y)\},$$

$$\{\text{plus}(\text{p}(x), y) \rightarrow \text{p}(\text{plus}(x, y)), \text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$

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*(Reduce lhs of  $\text{p}(\text{s}(\text{plus}(x, y))) \equiv \text{plus}(x, y)$ )*

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$$\{\text{plus}(\mathcal{O}, y) \equiv y, \text{plus}(x, y) \equiv \text{plus}(x, y)\},$$

$$\{\text{plus}(\text{p}(x), y) \rightarrow \text{p}(\text{plus}(x, y)), \text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$

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*(Delete  $\text{plus}(x, y) \equiv \text{plus}(x, y)$ )*

---

$$\{\text{plus}(\mathcal{O}, y) \equiv y\},$$

$$\{\text{plus}(\text{p}(x), y) \rightarrow \text{p}(\text{plus}(x, y)), \text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$

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*(Orient  $\text{plus}(\mathcal{O}, y) \equiv y$ )*

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$$\emptyset,$$

$$\{\text{plus}(\mathcal{O}, y) \rightarrow y, \text{plus}(\text{p}(x), y) \rightarrow \text{p}(\text{plus}(x, y)), \text{plus}(\text{s}(x), y) \rightarrow \text{s}(\text{plus}(x, y)), \text{p}(\text{s}(x)) \rightarrow x, \text{s}(\text{p}(x)) \rightarrow x\}$$