Proving Termination of Heap-Manipulating Java Programs

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Automated Termination Analysis

Imperative Programs:

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- Termination Analysis for Java (via Path Length, CLP backend)
  Julia – (*Spoto, Mesnard, Payet*, since '08)
  COSTA – (*Albert, Arenas, Codish, Genaim, Puebla, Zanardini*, since '08)

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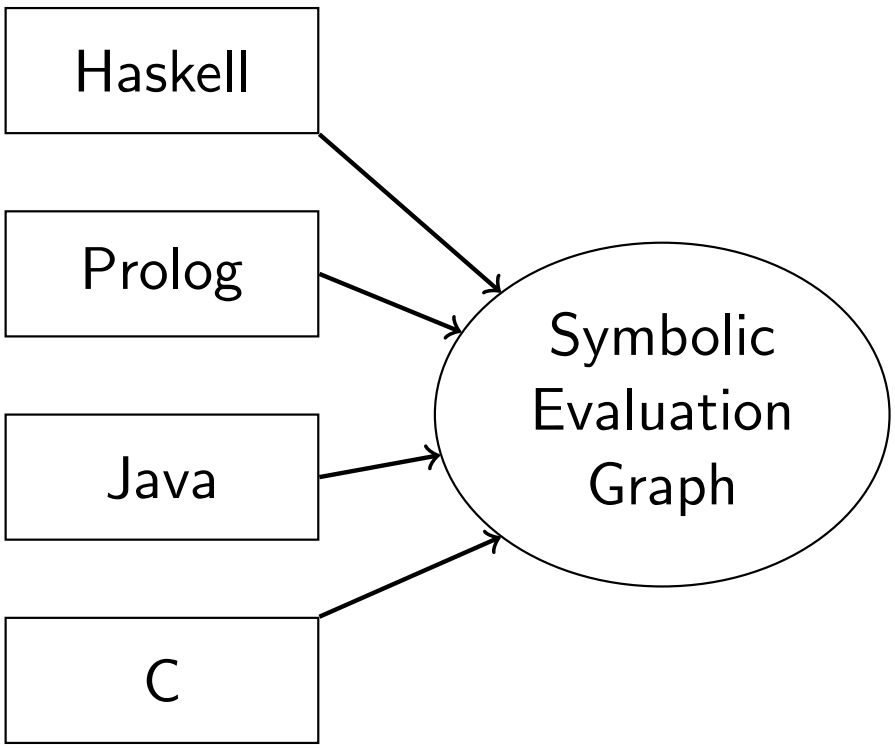

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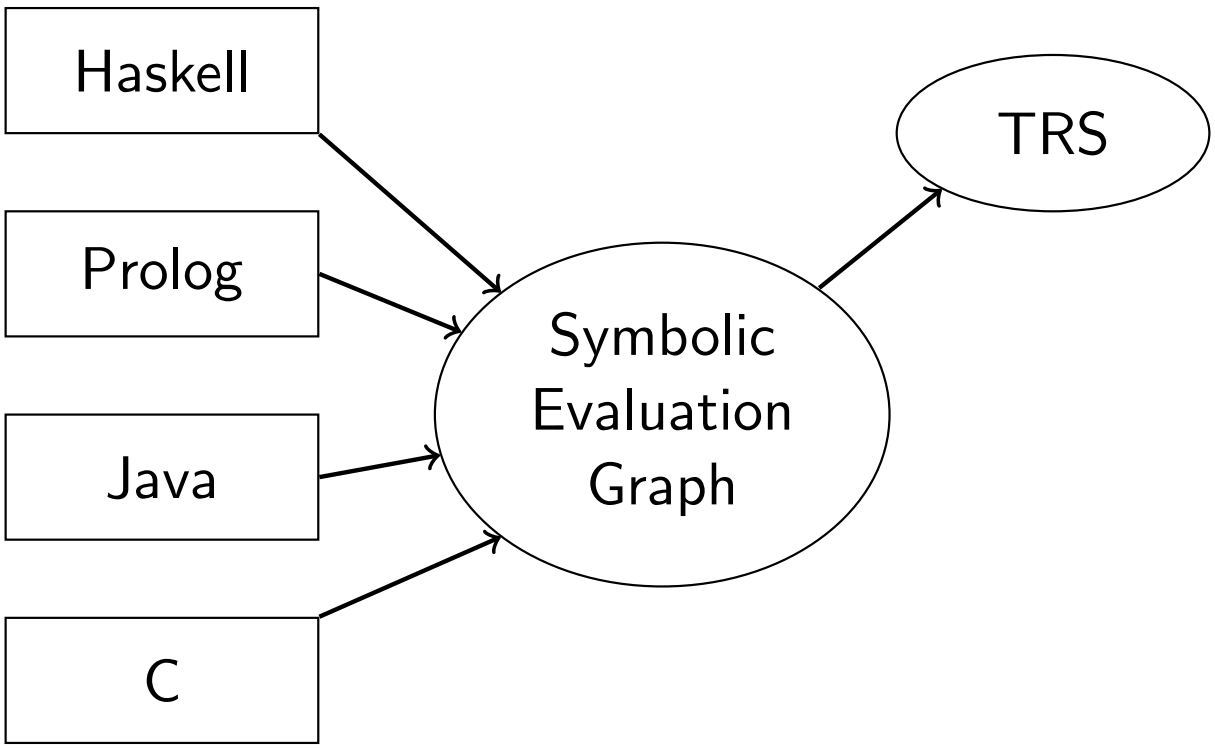

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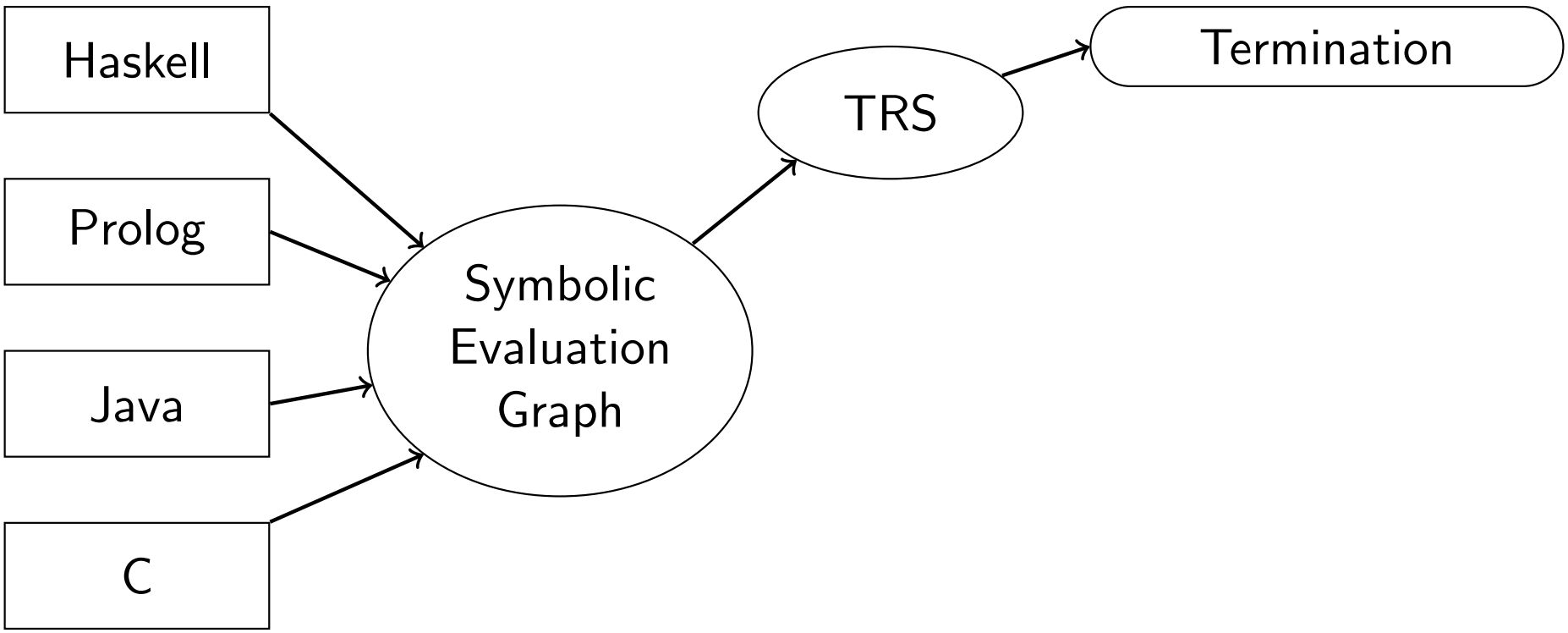

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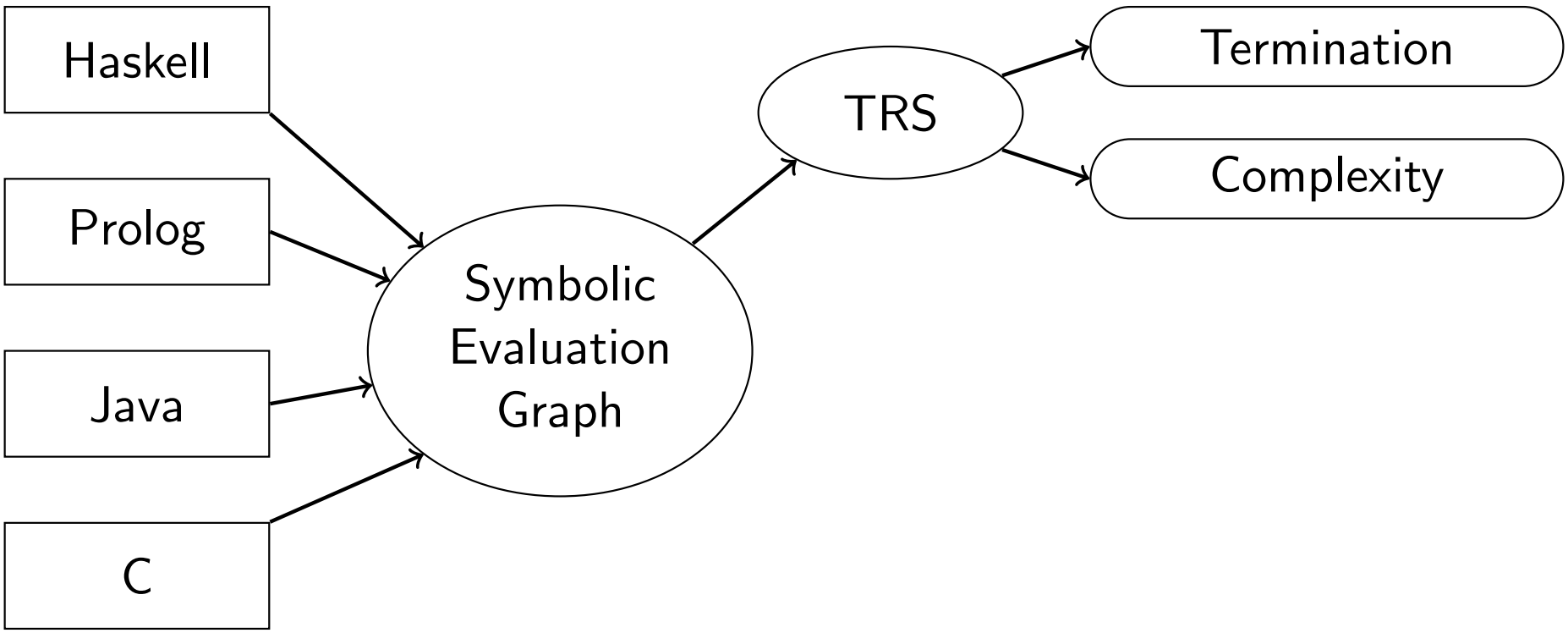

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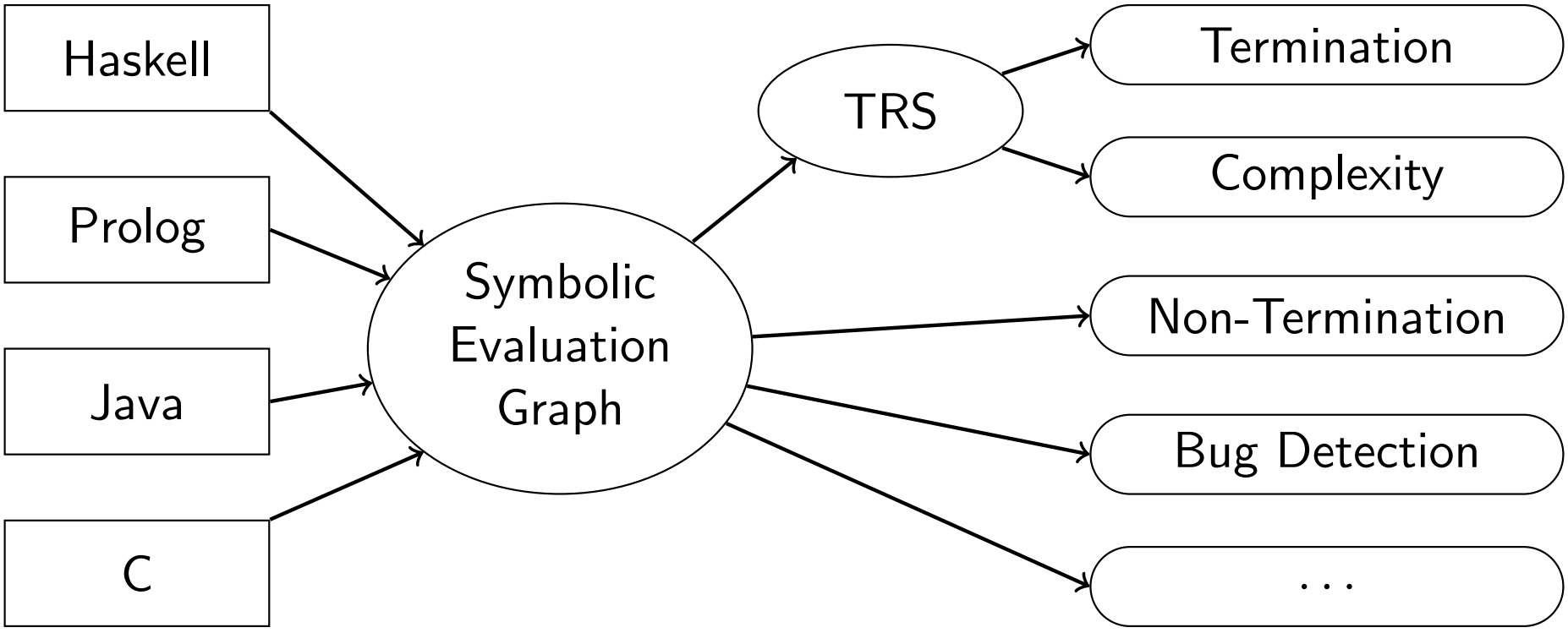

Rewriting-backed approach: Advantages

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public class List {
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- **TRS techniques** search for suitable orders automatically

$\Rightarrow$ Complex orders for user-defined data structures possible

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Overview

1. Introduction
2. Building Symbolic Evaluation Graphs
3. Generating TRSs from Symbolic Evaluation Graphs
4. Post-processing Symbolic Evaluation Graphs
5. Conclusion

length: the example

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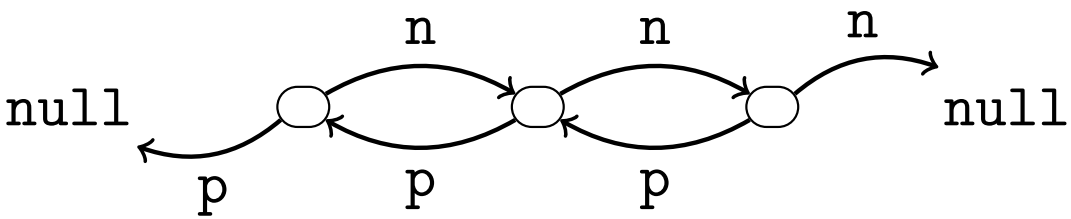

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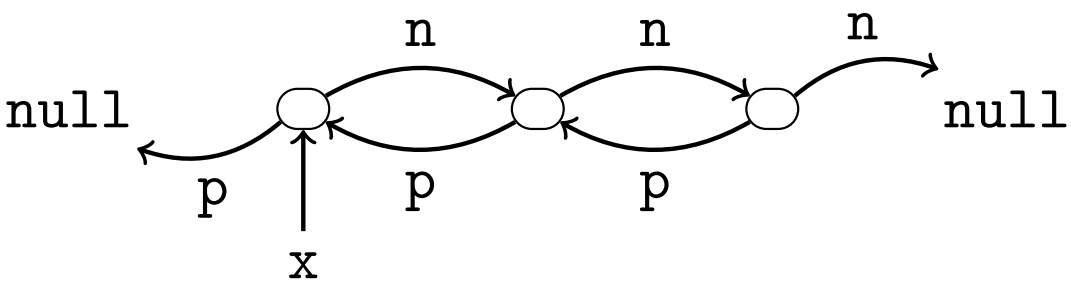

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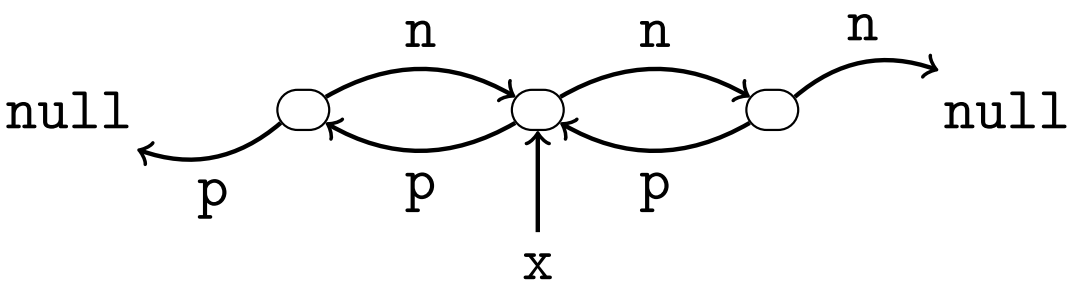

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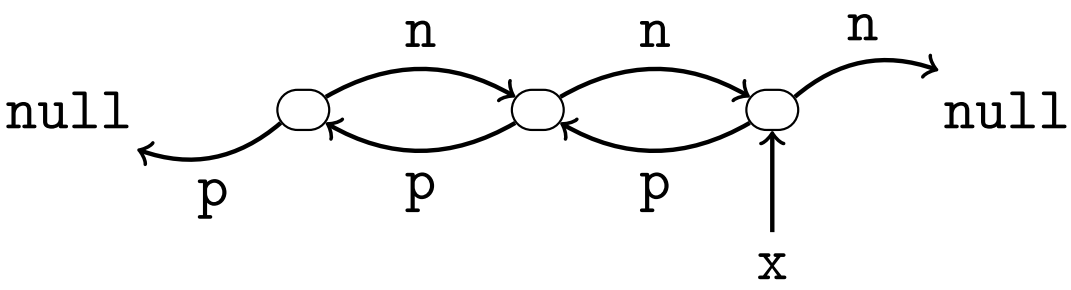

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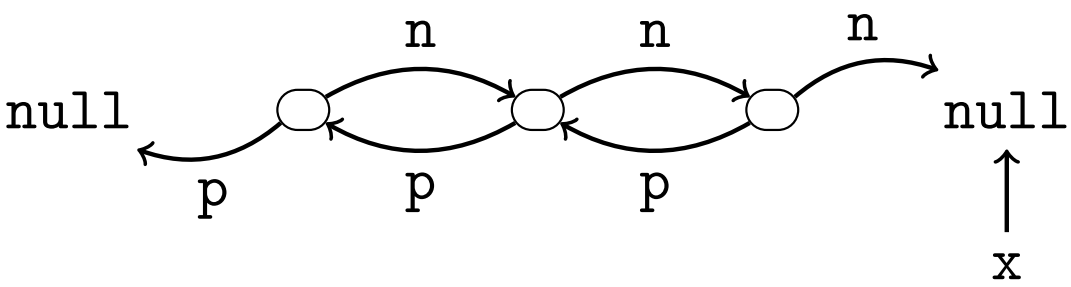

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- Reference might have cycles containing all fields F: $o_1 \circlearrowleft_F$

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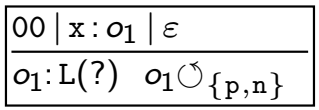

A

State A:

- x some list, might contain cycles using p and n

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int length(L x) {
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    x = x.n; r++;   }
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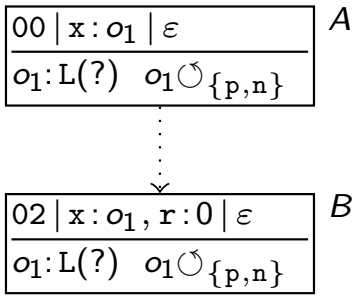

State *A*:

- `x` some list, might contain cycles using `p` and `n`

State *B*:

- Initialized variable `r` to 0

```
int length(L x) {
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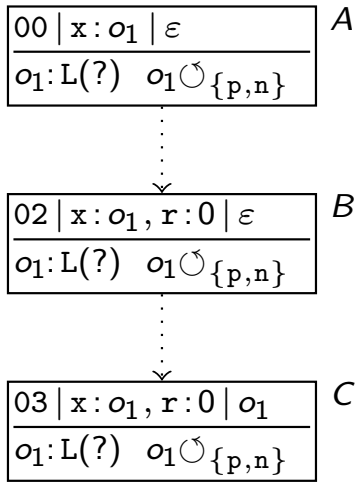

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- `x` (o_1) null? We do not know!

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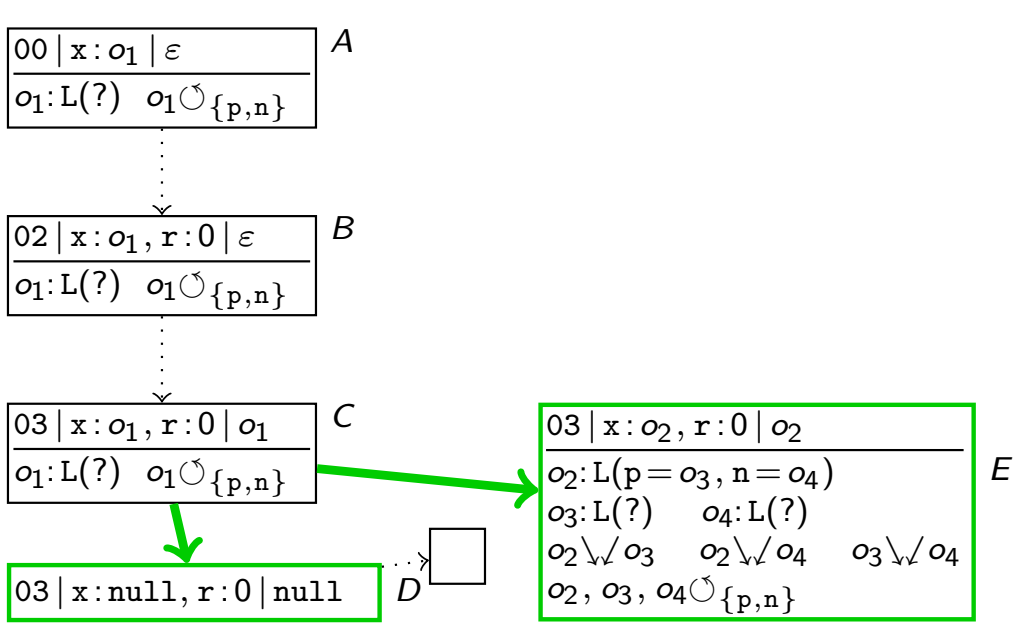

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⇒ Refinement

- In *D*: o_1 is null (↷ program ends)
- In *E*: o_1 replaced by o_2, which exists and has fields:
  - Field values can share (↷ add ↘↙)
  - Field values can be cyclic again (↷ add ↻)

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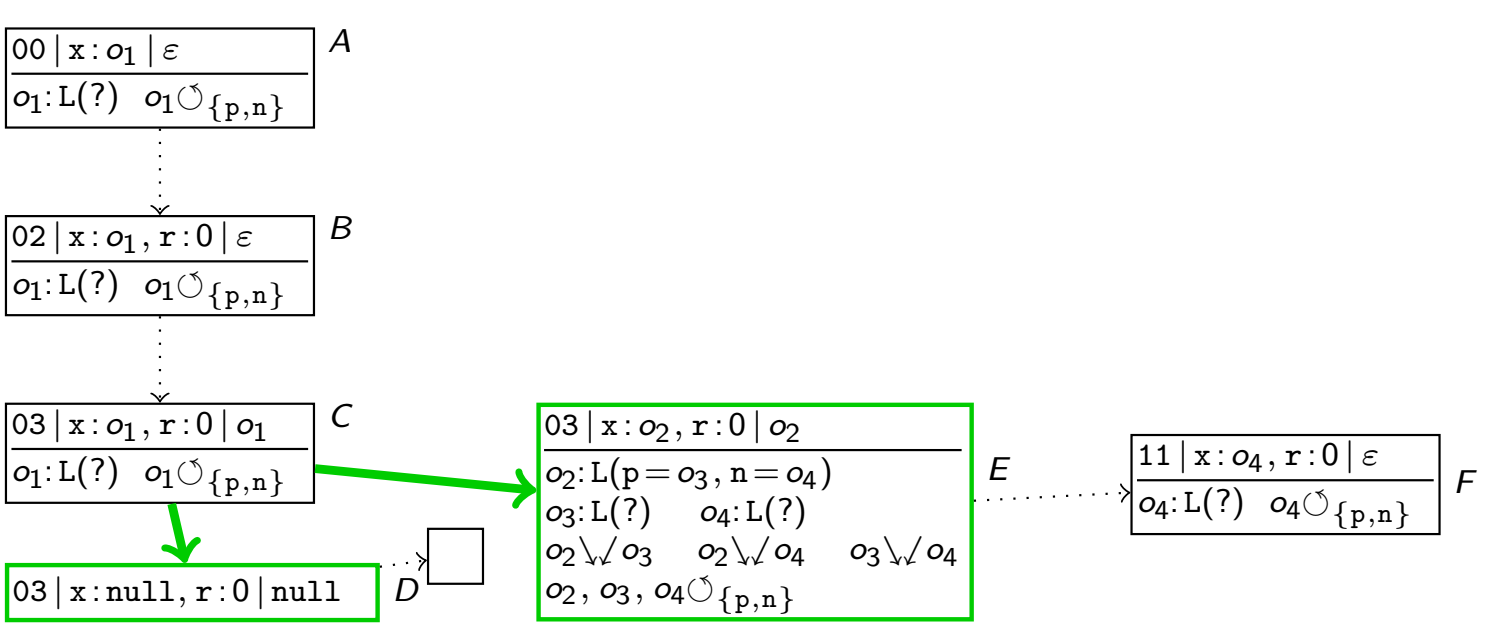

State *F*:

- Stored `x.n` to `x` (allowing for GC)

```
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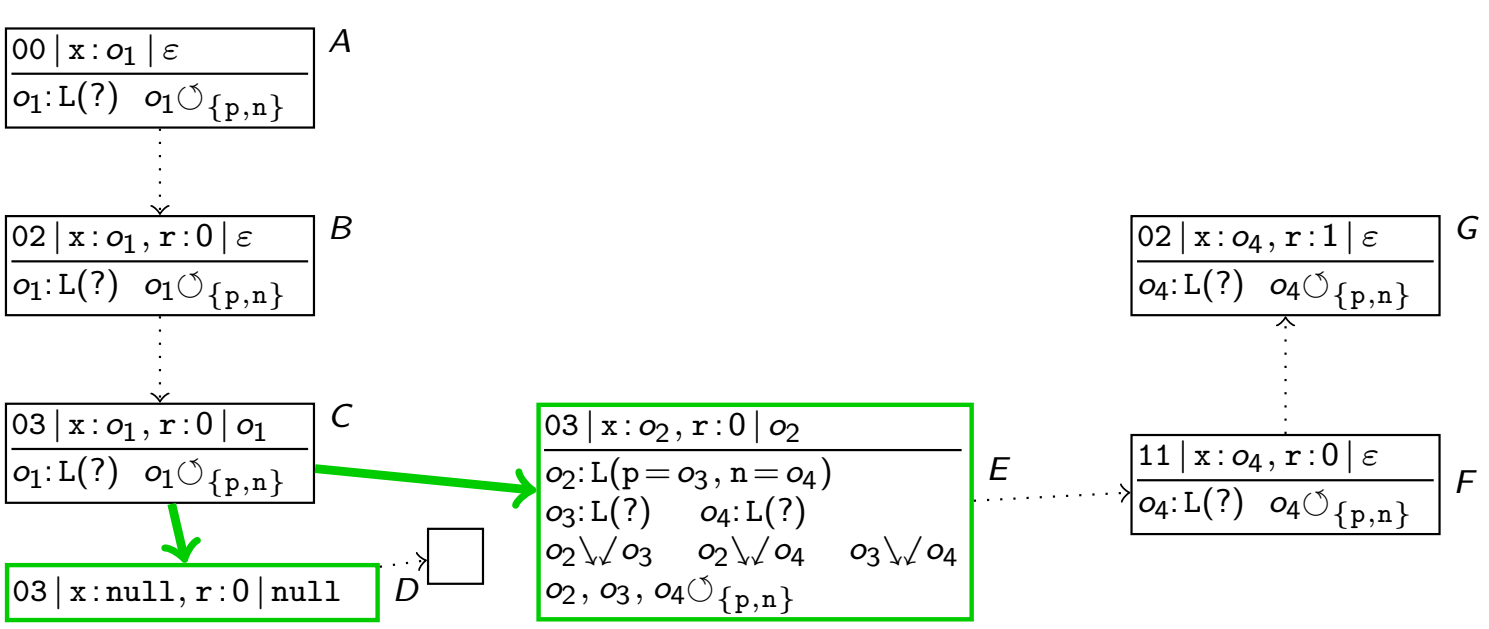

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State *G*:

- Incremented `r`, back to position 02 (as *B*)

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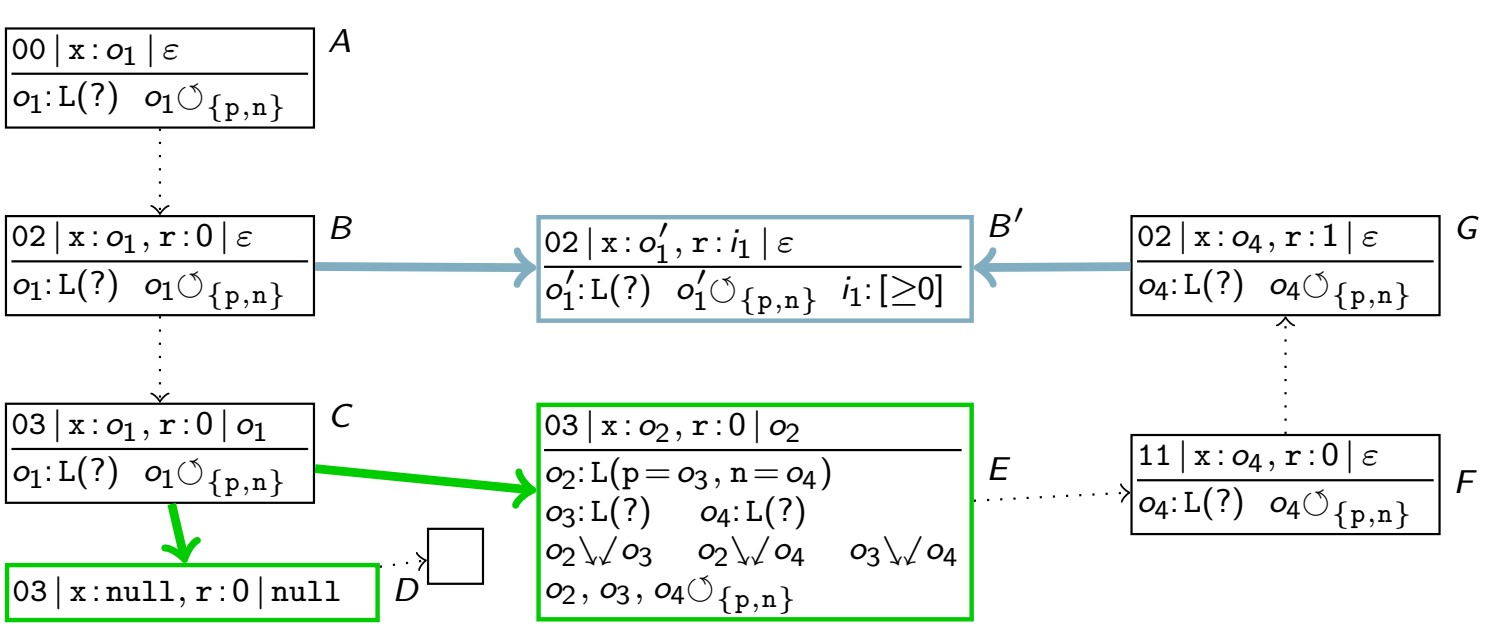

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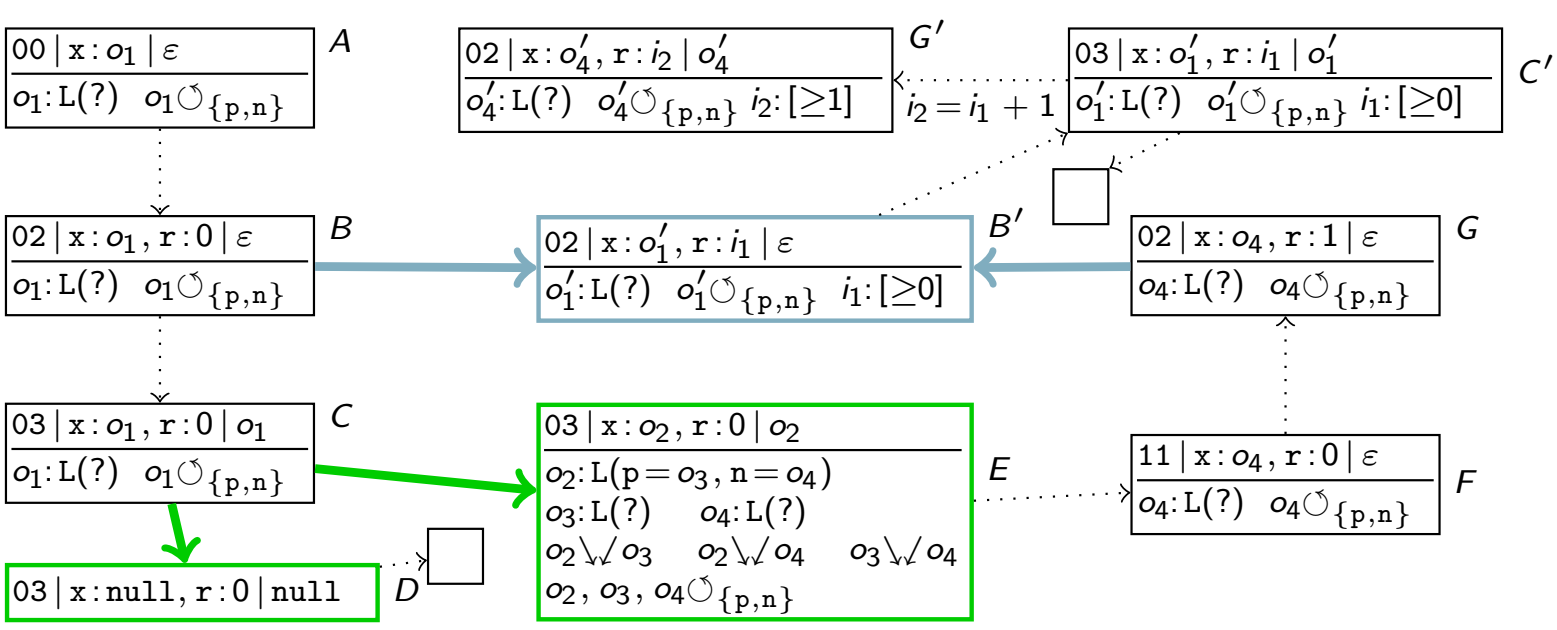

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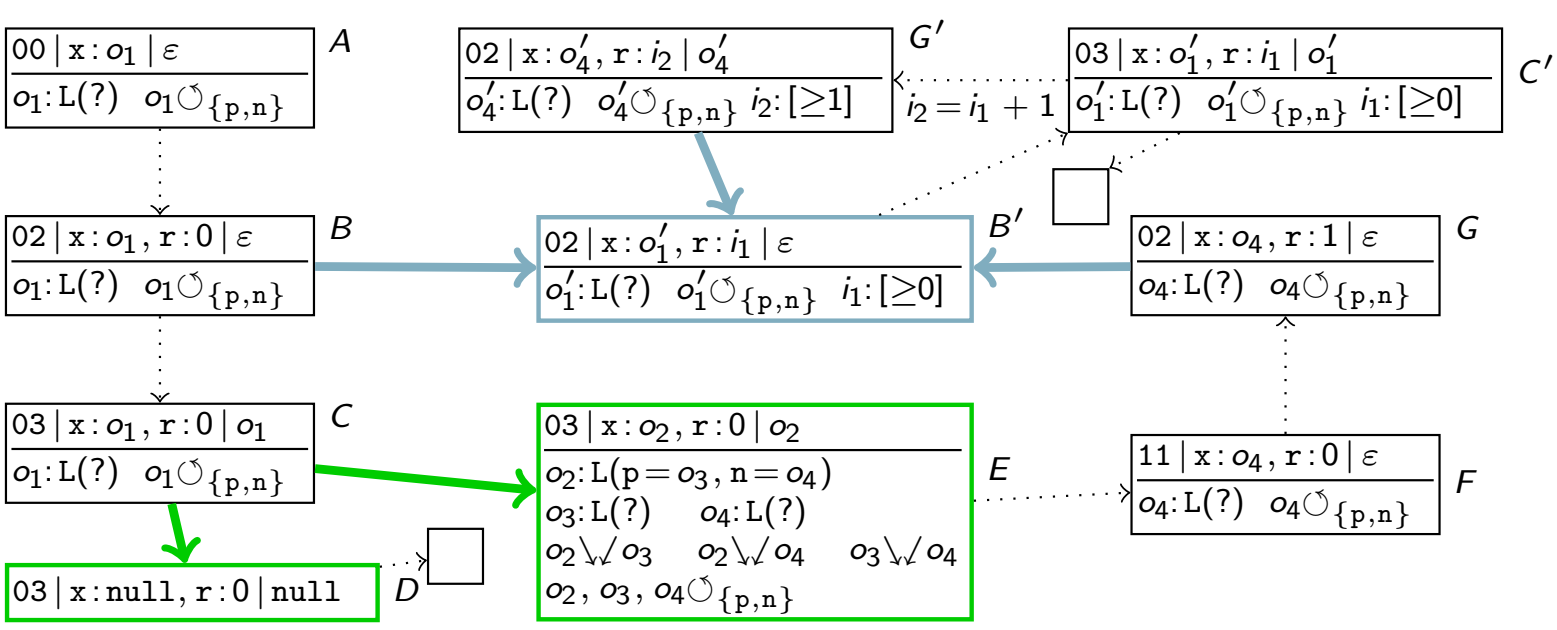

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$$\mathsf{app}(\mathsf{Nil}, ys) \to ys \quad (2)$$

- Rewriting of term t with rule $l \to r$:
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  2. Find variable instantiation σ with $\sigma(l) = s$
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$$\underline{\mathsf{app}(\mathsf{Cons}(1, \mathsf{Nil}), \mathsf{Cons}(2, \mathsf{Nil}))} \quad \text{with } (1), x = 1, xs = \mathsf{Nil}, ys = \mathsf{Cons}(2, \mathsf{Nil})$$

$$\to \mathsf{Cons}(1, \underline{\mathsf{app}(\mathsf{Nil}, \mathsf{Cons}(2, \mathsf{Nil}))}) \quad \text{with } (2), ys = \mathsf{Cons}(2, \mathsf{Nil})$$

$$\to \mathsf{Cons}(1, \mathsf{Cons}(2, \mathsf{Nil}))$$

Transforming values to terms

$$\frac{03 \mid \mathtt{x} : o_2, \mathtt{r} : 0 \mid o_2}{\begin{array}{l} o_2 : \mathrm{L}(\mathtt{p} = o_3, \mathtt{n} = o_4) \\ o_3 : \mathrm{L}(?) \quad o_4 : \mathrm{L}(?) \\ o_2 \between o_3 \quad o_2 \between o_4 \quad o_3 \between o_4 \\ o_2, o_3, o_4 \circlearrowleft_{\{\mathtt{p},\mathtt{n}\}} \end{array}} \; E$$

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- Known integers transformed to themselves
- Unknown values transformed to variables

$o_3, o_4 \quad 0$

Transforming values to terms

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- Data structures transformed to nested constructor terms:
  Class `Cl` with n fields $\curvearrowright$ symbol Cl of arity n

$$\overbrace{\mathsf{L}(o_3, o_4)}^{o_2}\ 0$$

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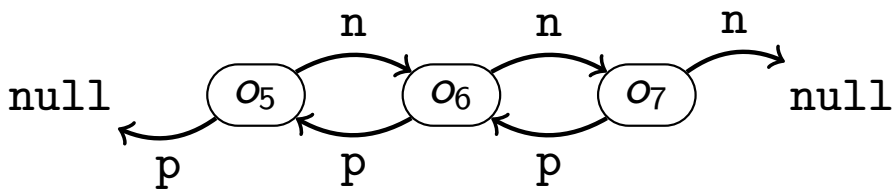
- Encoding cycles: Special symbol $\bigcirc$ for repetition

$o_5{:}\,\mathrm{L}(\mathrm{p}=\mathtt{null}, \mathrm{n}=o_6)$
$o_6{:}\,\mathrm{L}(\mathrm{p}=o_5, \mathrm{n}=o_7)$
$o_7{:}\,\mathrm{L}(\mathrm{p}=o_6, \mathrm{n}=\mathtt{null})$



Encoding of o_5: $\mathrm{L}(\mathtt{null}, \mathrm{L}(\bigcirc, \mathrm{L}(\bigcirc, \mathtt{null})))$
Encoding of o_6: $\mathrm{L}(\mathrm{L}(\mathtt{null}, \bigcirc), \mathrm{L}(\bigcirc, \mathtt{null}))$

Transforming states to terms

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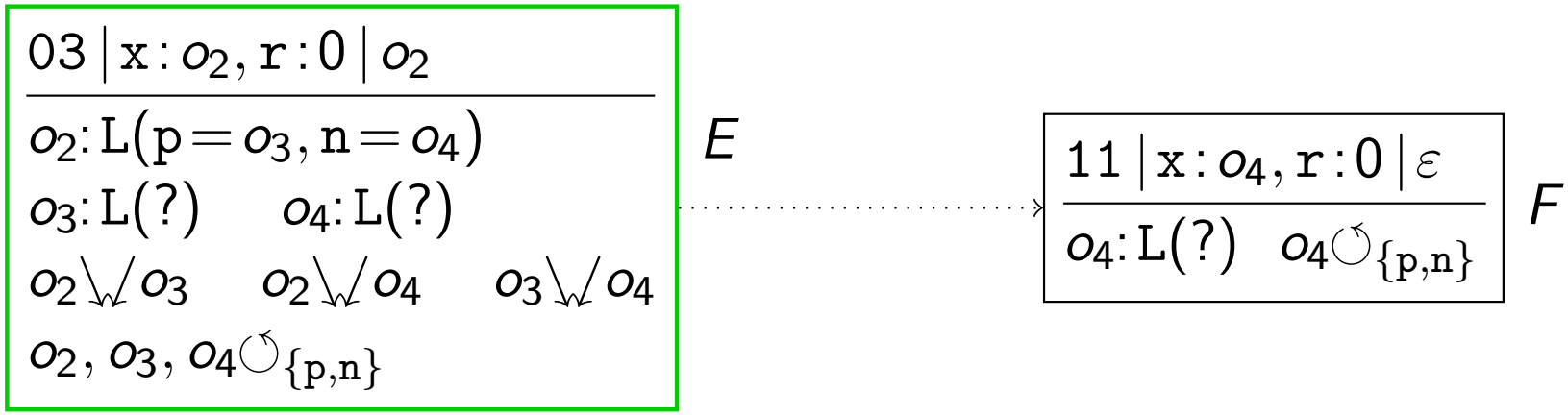
- State s term encoding:
  - Root symbol ($\equiv$ program position) f_s
  - All local variables, stack entries as arguments

$$\mathrm{f}_E(\overbrace{\mathrm{L}(o_3, o_4)}^{o_2}, 0, \overbrace{\mathrm{L}(o_3, o_4)}^{o_2})$$

Transforming edges to rules



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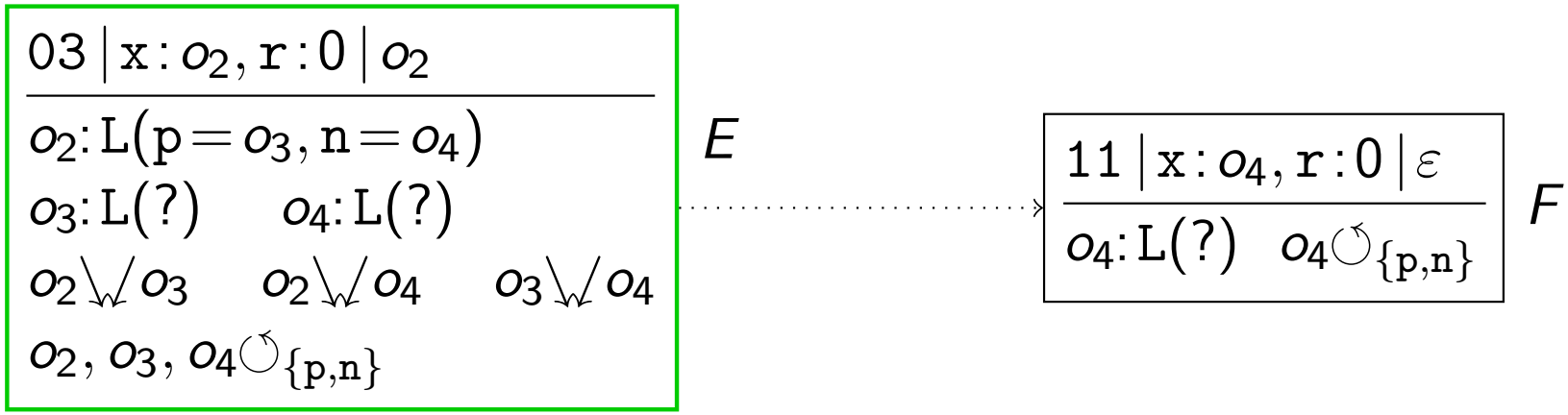

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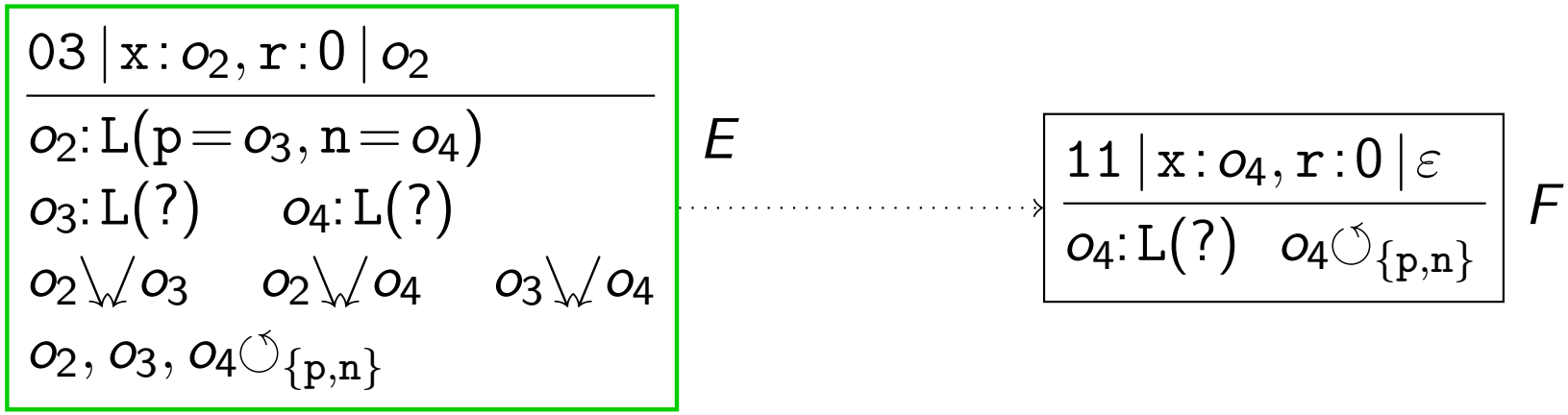

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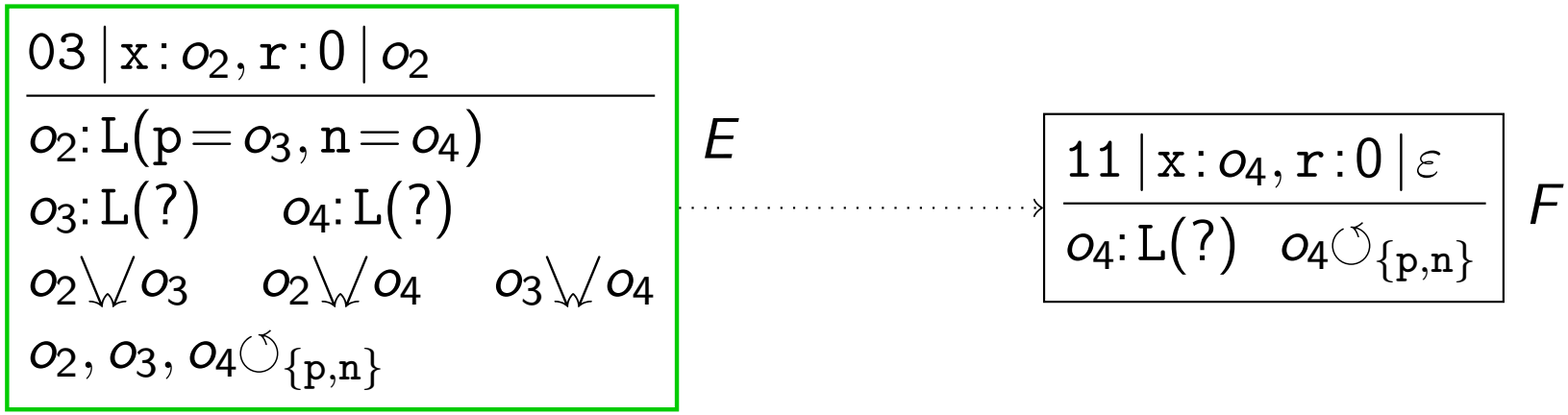

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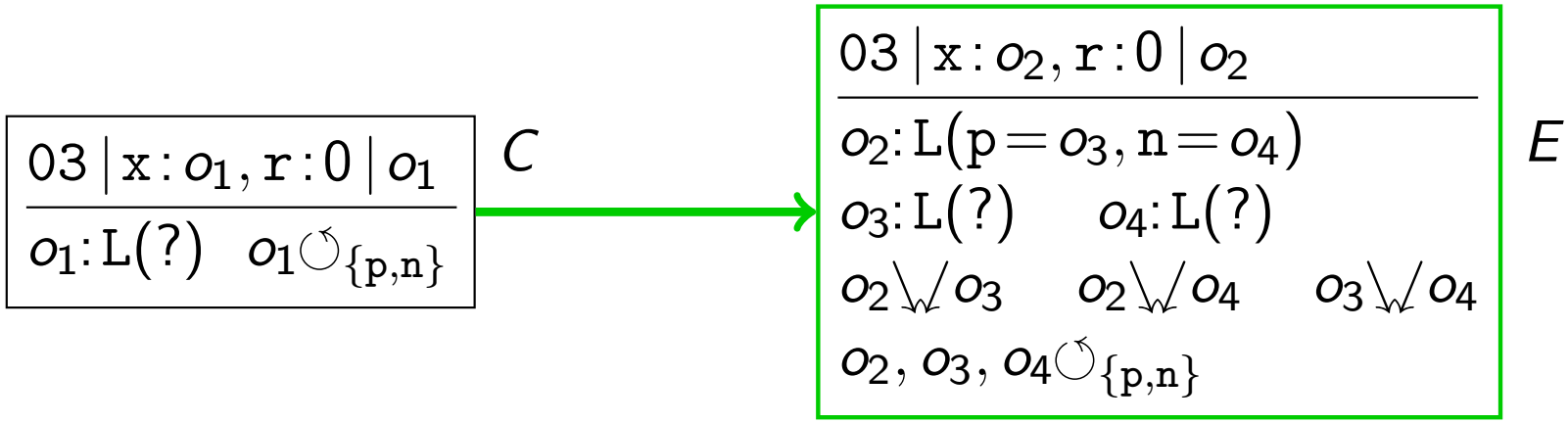

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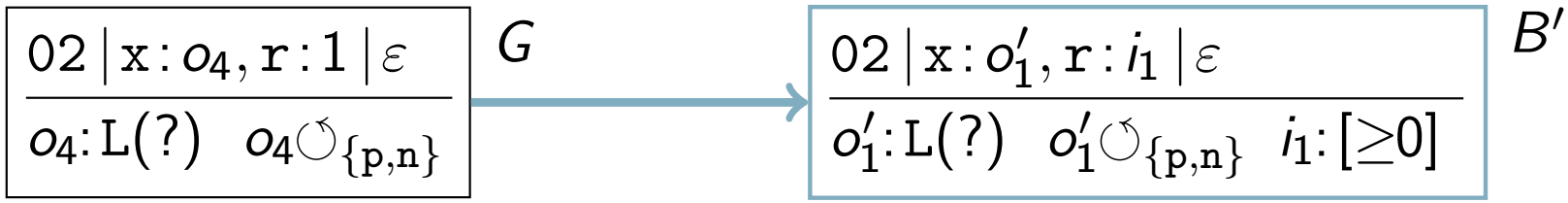
$$\mathrm{f}_C(\mathrm{L}(o_4), 0, \mathrm{L}(o_4)) \quad \rightarrow \quad \mathrm{f}_E(\mathrm{L}(o_4), 0, \mathrm{L}(o_4))$$

Transforming edges to rules



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$$\mathsf{f}_G(o_4, 2) \;\rightarrow\; \mathsf{f}_{B'}(o_4, 2)$$

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Rule: $\quad \mathsf{app}(\mathsf{Cons}(x, xs), ys) \to \mathsf{Cons}(x, \mathsf{app}(xs, ys))$

Choose $[\![\mathsf{app}]\!] = (x, y) \mapsto 1 + 2 \cdot x$, $[\![\mathsf{Cons}]\!] = (x, y) \mapsto 1 + y$,

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$$\text{Rule:} \quad \mathsf{app}(\mathsf{Cons}(x, xs), ys) \to \mathsf{Cons}(x, \mathsf{app}(xs, ys))$$

$$\text{Interpretation:} \quad 1 + 2 + 2 \cdot xs > 1 + 1 + 2 \cdot xs$$

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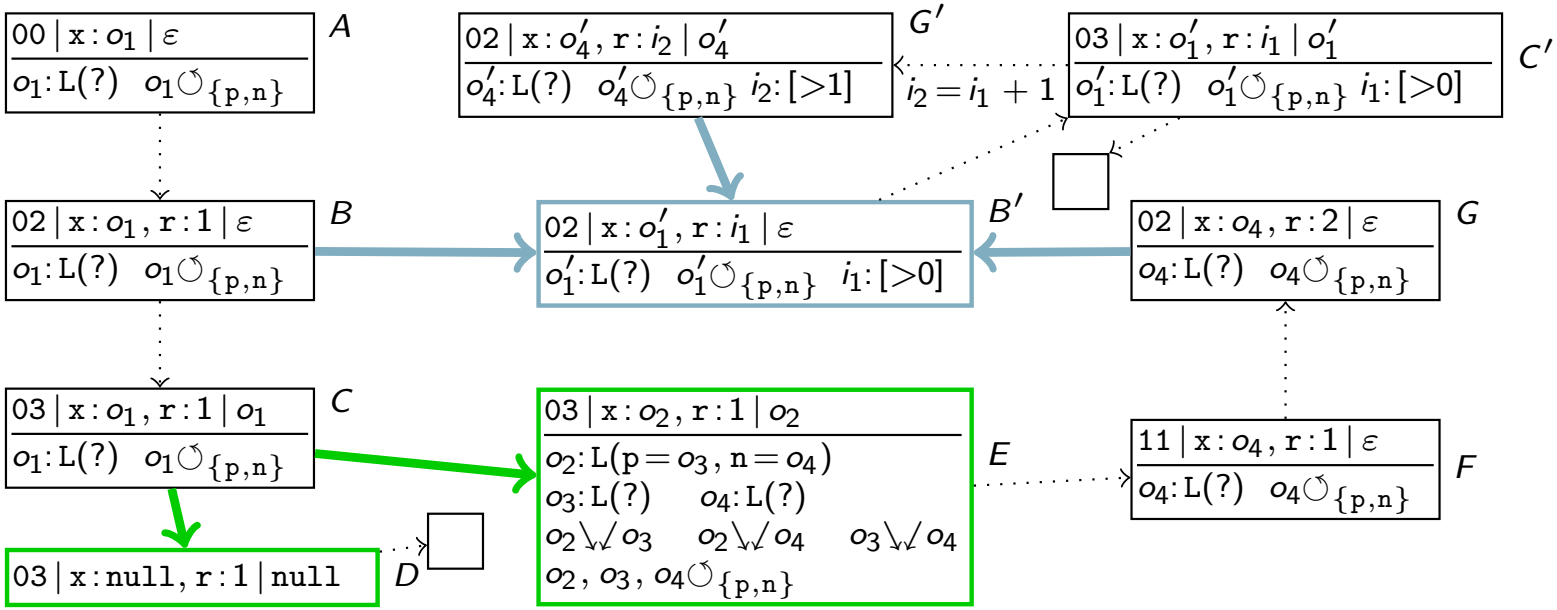

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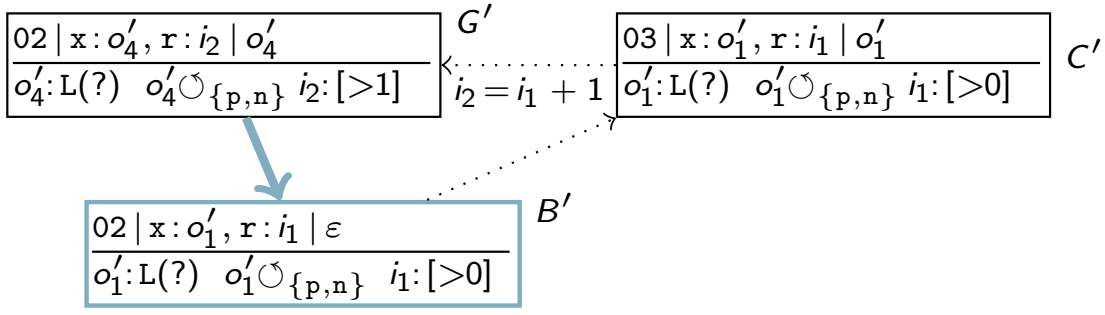

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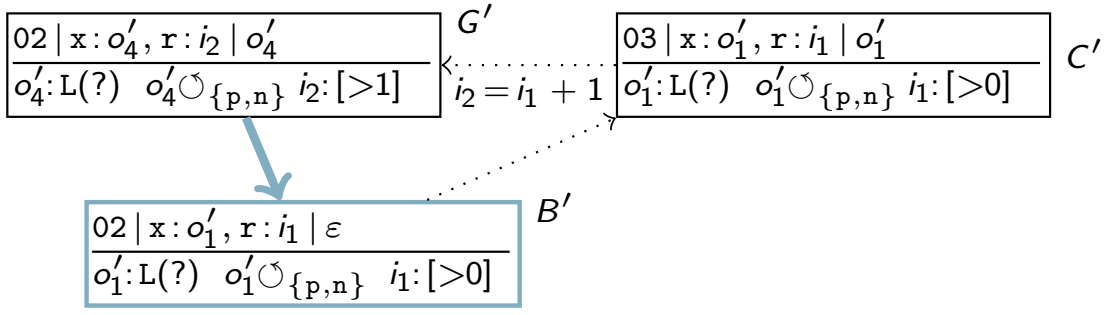

1. Only consider SCCs!
2. Transform all edges as before, simplify:

$$\mathsf{f}_{B'}(\mathrm{L}(o'_4), i_1) \quad \to \quad \mathsf{f}_{B'}(o'_4, i_1 + 1)$$

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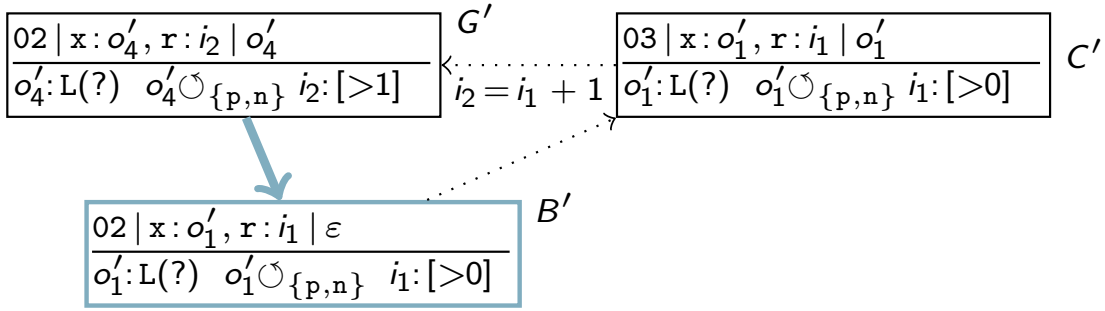


1. Only consider SCCs!
2. Transform all edges as before, simplify:

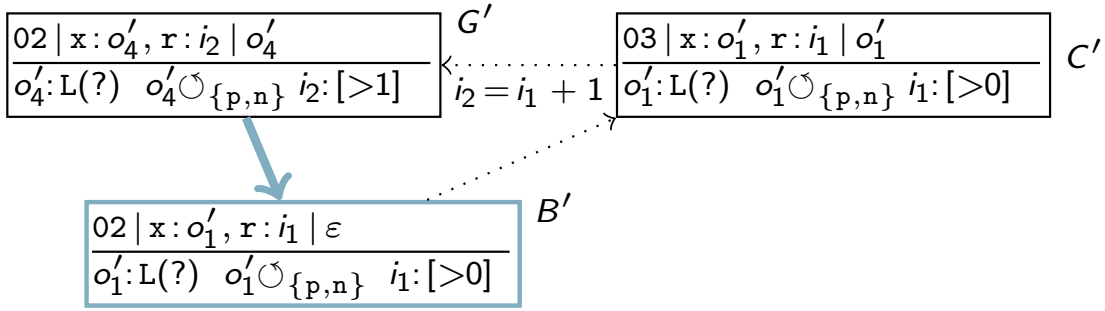
$$\mathsf{f}_{B'}(\mathsf{L}(o_4'), i_1) \quad \to \quad \mathsf{f}_{B'}(o_4', i_1 + 1)$$

3. Termination trivially proven with

$$[\![\mathsf{f}_{B'}]\!] = (x_1, x_2) \mapsto x_1$$
$$[\![\mathsf{L}]\!] = (x_1) \mapsto x_1 + 1$$

The example TRS

```
00: iconst_1
01: istore_1
02: aload_0
03: ifnull 17
06: aload_0
07: getfield n
10: astore_0
11: iinc 1, 1
14: goto 2
17: iload_1
18: ireturn
```



1. Only consider SCCs!
2. Transform all edges as before, simplify:

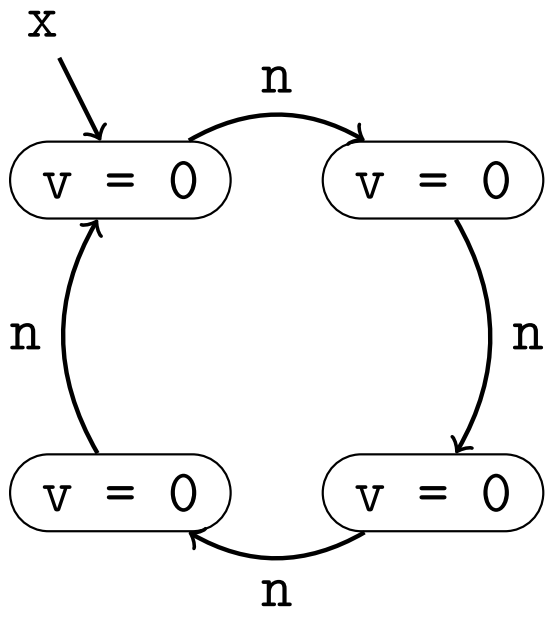
$$\begin{array}{rcl} \mathsf{f}_{B'}(\mathsf{L}(o_4'), i_1) & \to & \mathsf{f}_{B'}(o_4', i_1 + 1) \\ o_4' + 1 & > & o_4' \end{array}$$

3. Termination trivially proven with

$$[\![\mathsf{f}_{B'}]\!] = (x_1, x_2) \mapsto x_1$$
$$[\![\mathsf{L}]\!] = (x_1) \mapsto x_1 + 1$$

visit: the example

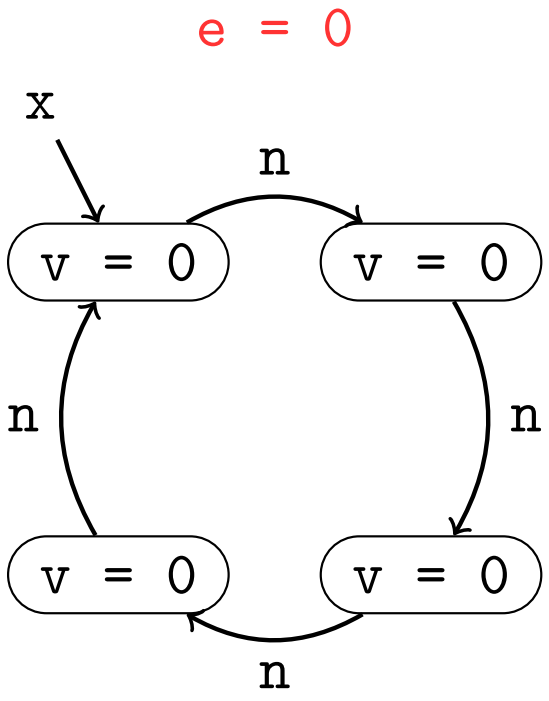
```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```



1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

visit: the example

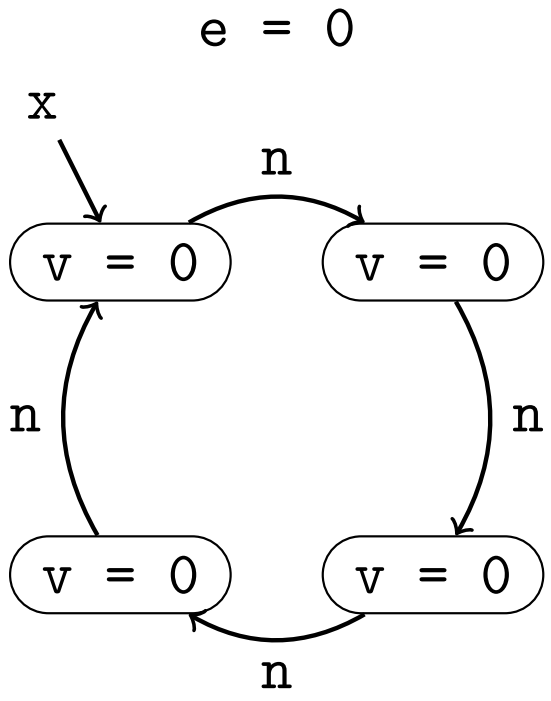
```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```



1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

visit: the example

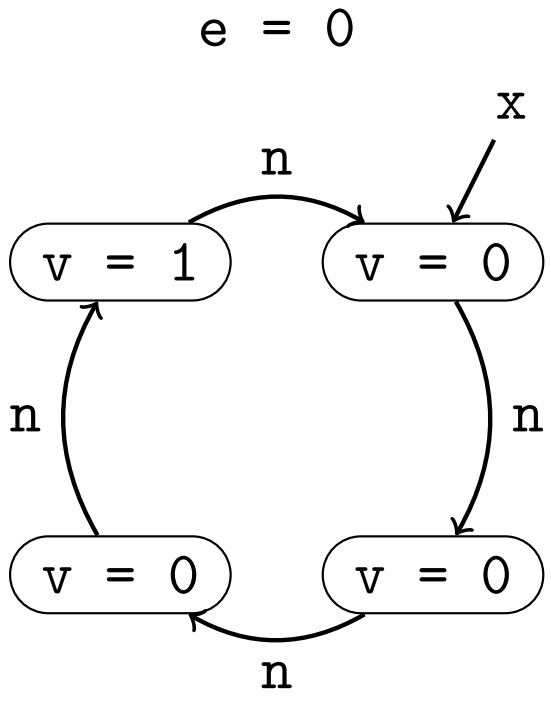
```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```



1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

visit: the example

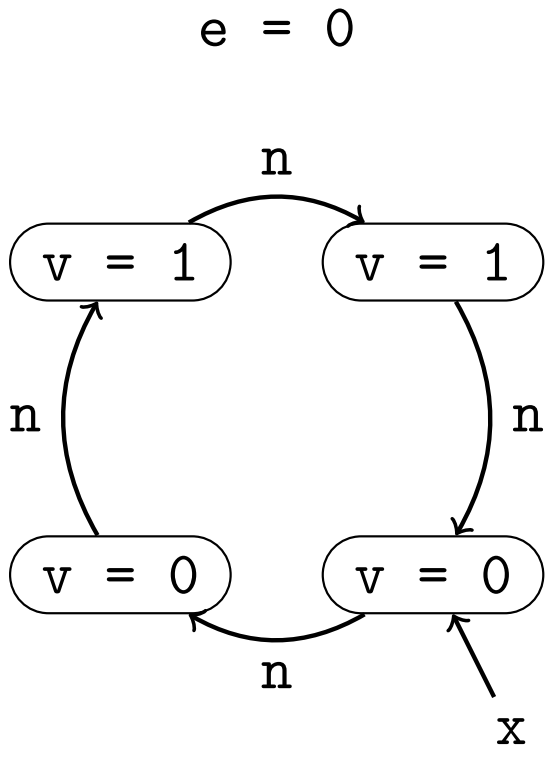
```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```



1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

visit: the example

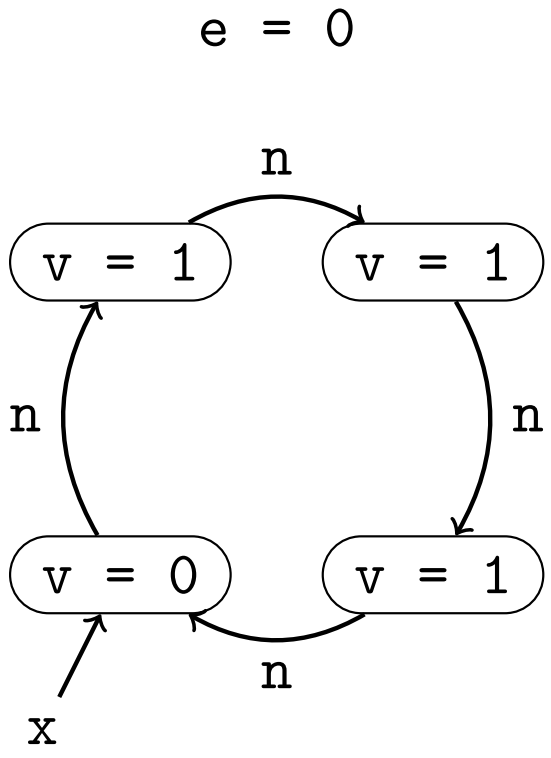
```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```



1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

visit: the example

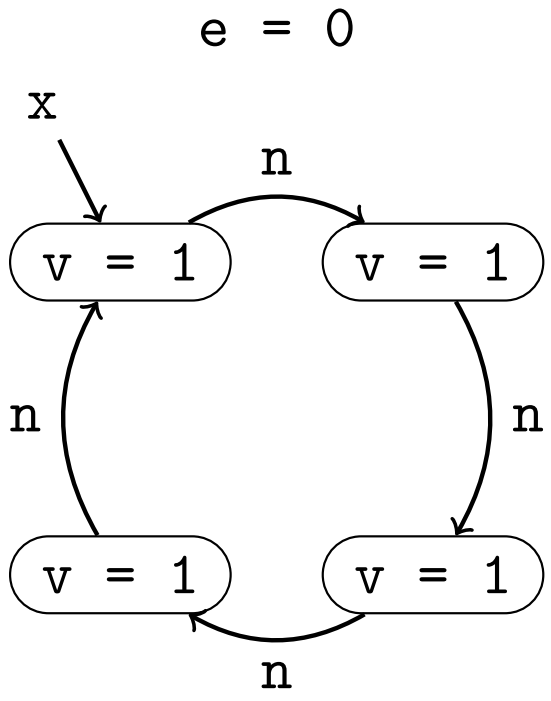
```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```



1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

visit: the example

```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```

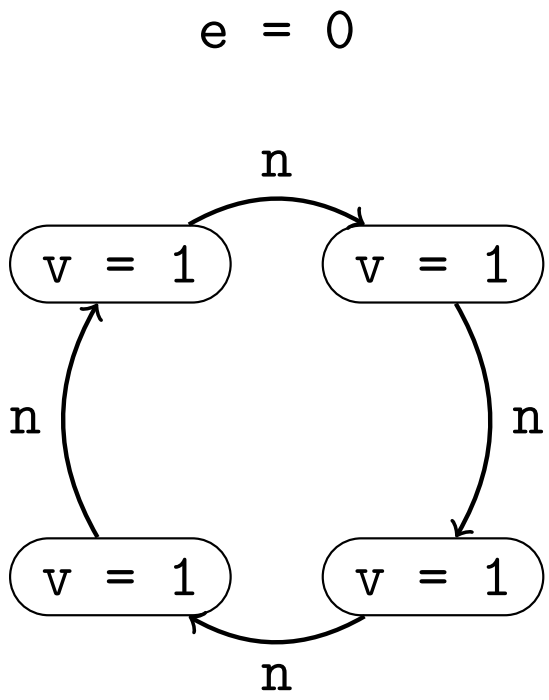


1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

visit: the example

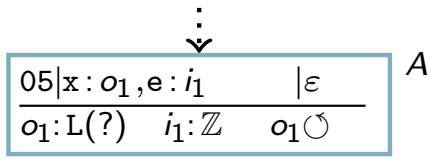
```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```

```
00: aload_0       #load x
01: getfield v    #get v from x
04: istore_1      #store to e
05: aload_0       #load x
06: getfield v    #get v from x
09: iload_1       #load e
10: if_icmpne 28  #jump if x.v != e
13: aload_0       #load x
14: iload_1       #load e
15: iconst_1      #load 1
16: iadd          #add e and 1
17: putfield v    #store to x.v
20: aload_0       #load x
21: getfield n    #get n from x
24: astore_0      #store to x
25: goto 5
28: return
```



1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



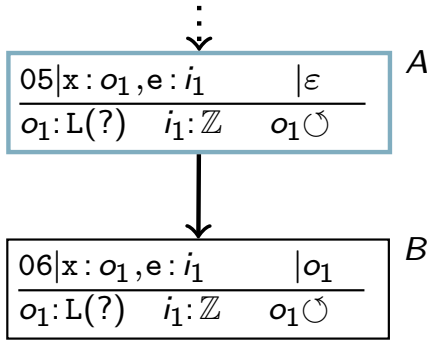
A

State *A*:

- x some (possibly cyclic) list
- e some integer

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

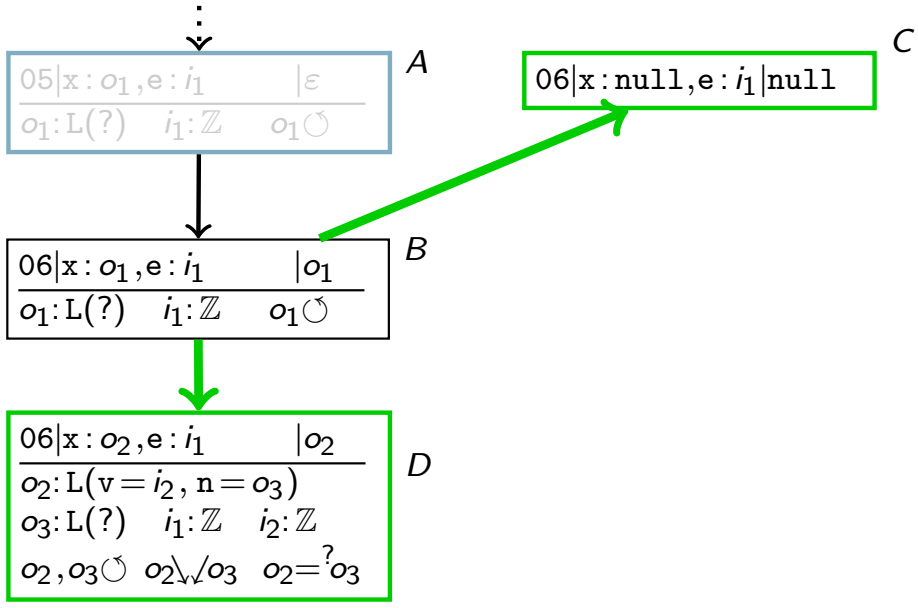


State *B*:

- *Evaluation* between *A* and *B*
- Need field of o_1

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

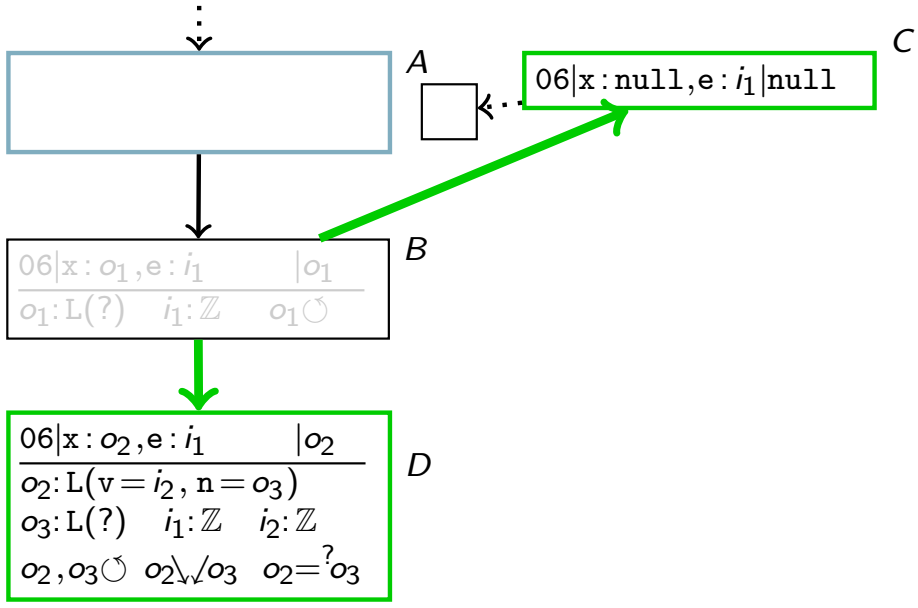



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States *B*, *C*, *D*:

- *Evaluation* between *A* and *B*
- Need field of o_1 ⇒ Refinement:
  - In *C*: o_1 is `null`
  - In *D*: o_1 renamed to o_2, pointing to L-object with successor o_3:
    - o_3 possibly cyclic
    - o_3 possibly equal to o_2 and may share with o_2

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

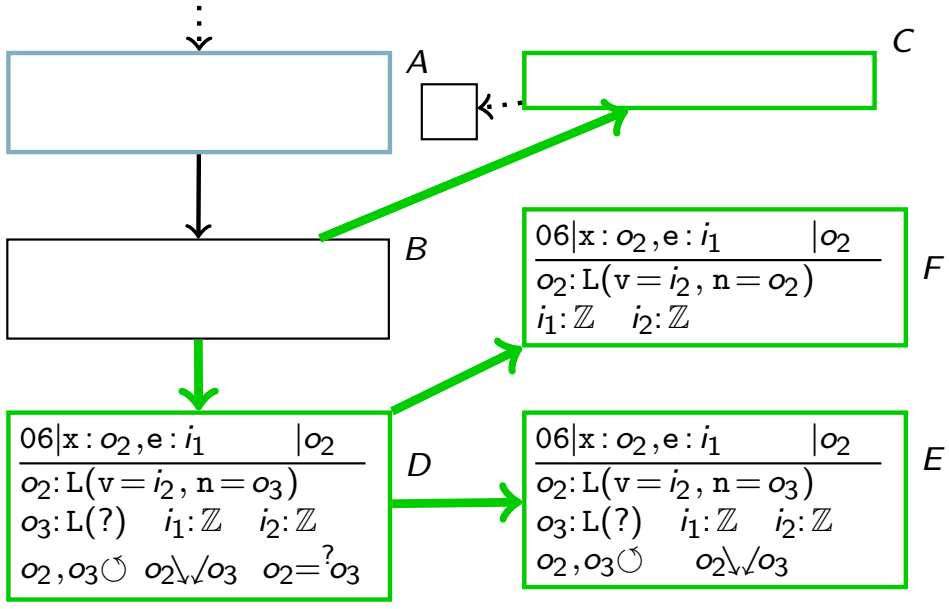



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States B, C, D:

- *Evaluation* between A and B
- Need field of o_1 $\Rightarrow$ Refinement:
  - In C: o_1 is `null` (program crashes)
  - In D: o_1 renamed to o_2, pointing to L-object with successor o_3:
    - o_3 possibly cyclic
    - o_3 possibly equal to o_2 and may share with o_2

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

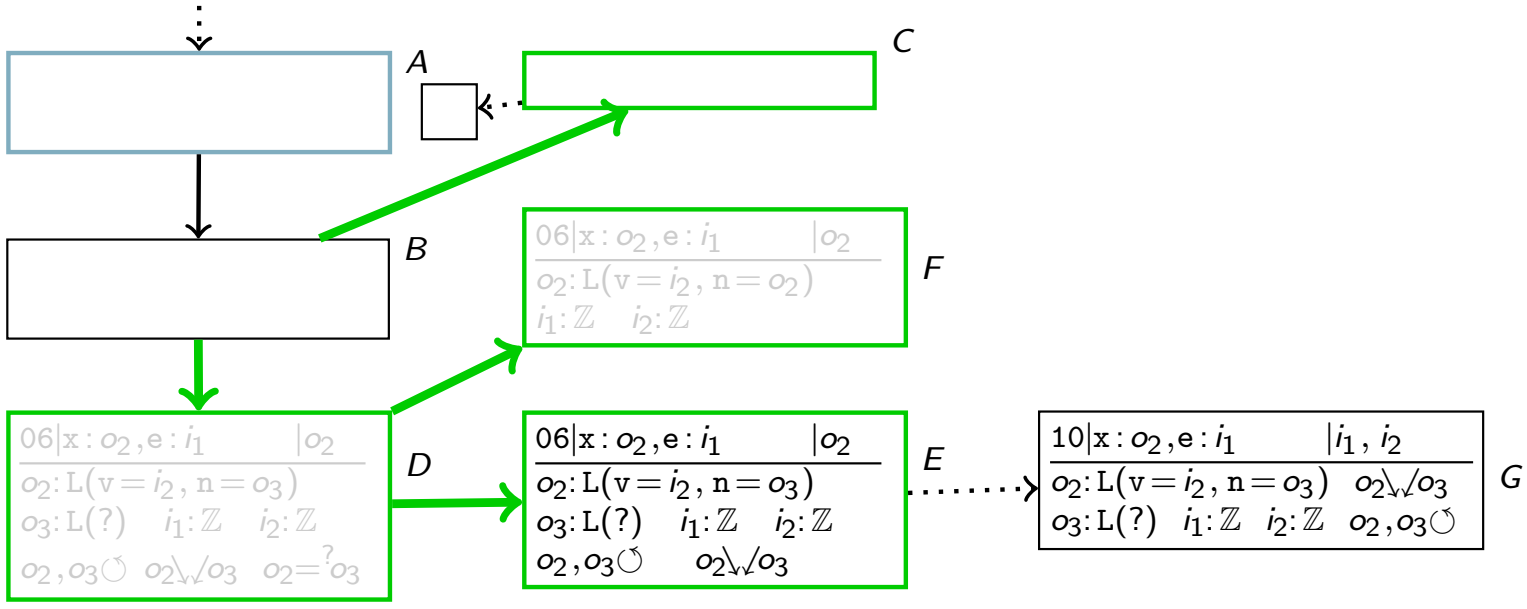


States E, F:

- Need to read field of o_2 $\Rightarrow$ Refinement
  - In E: $o_2 \neq o_3$
  - In F: $o_2 = o_3$

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

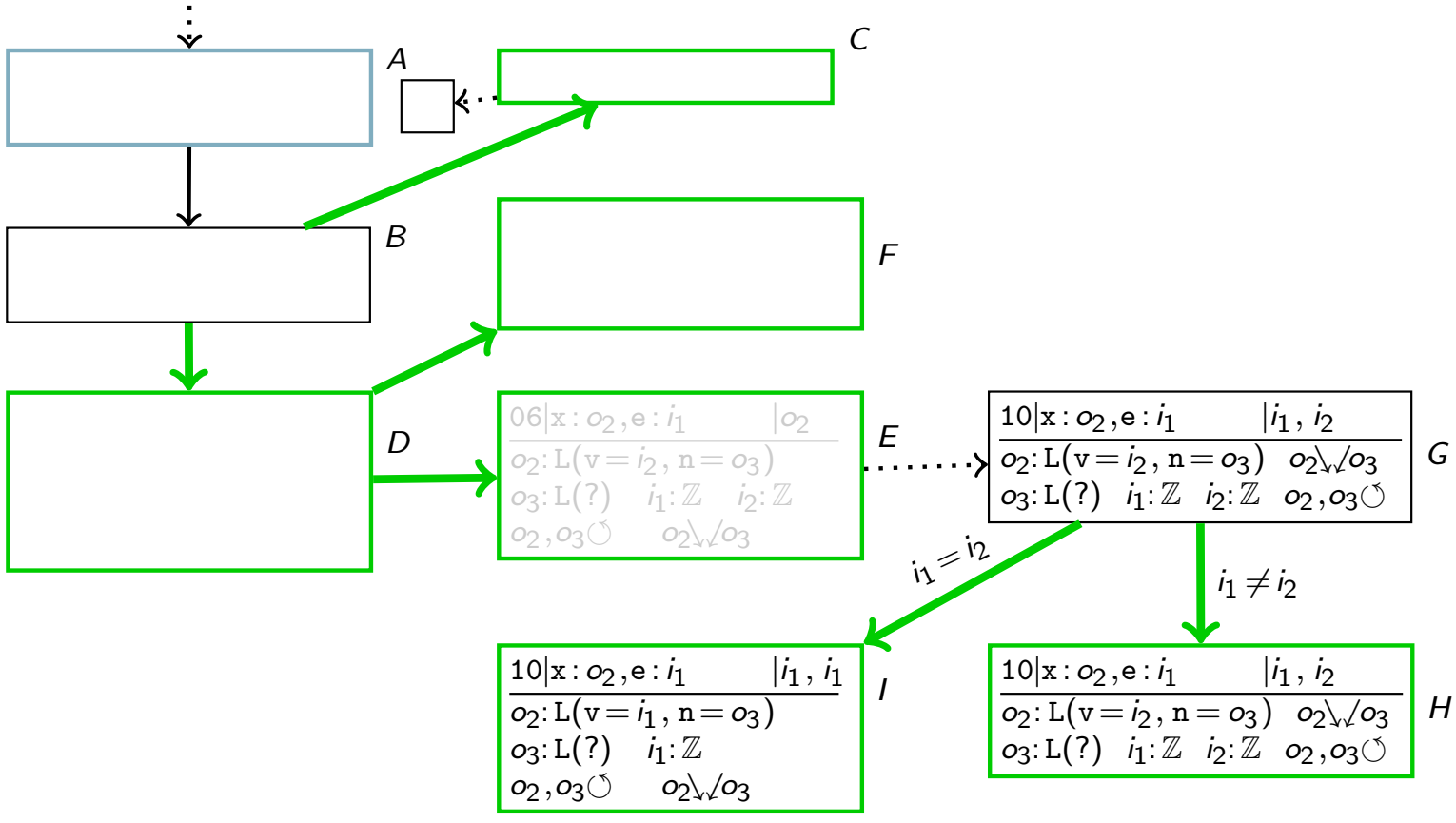


State G:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2$

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

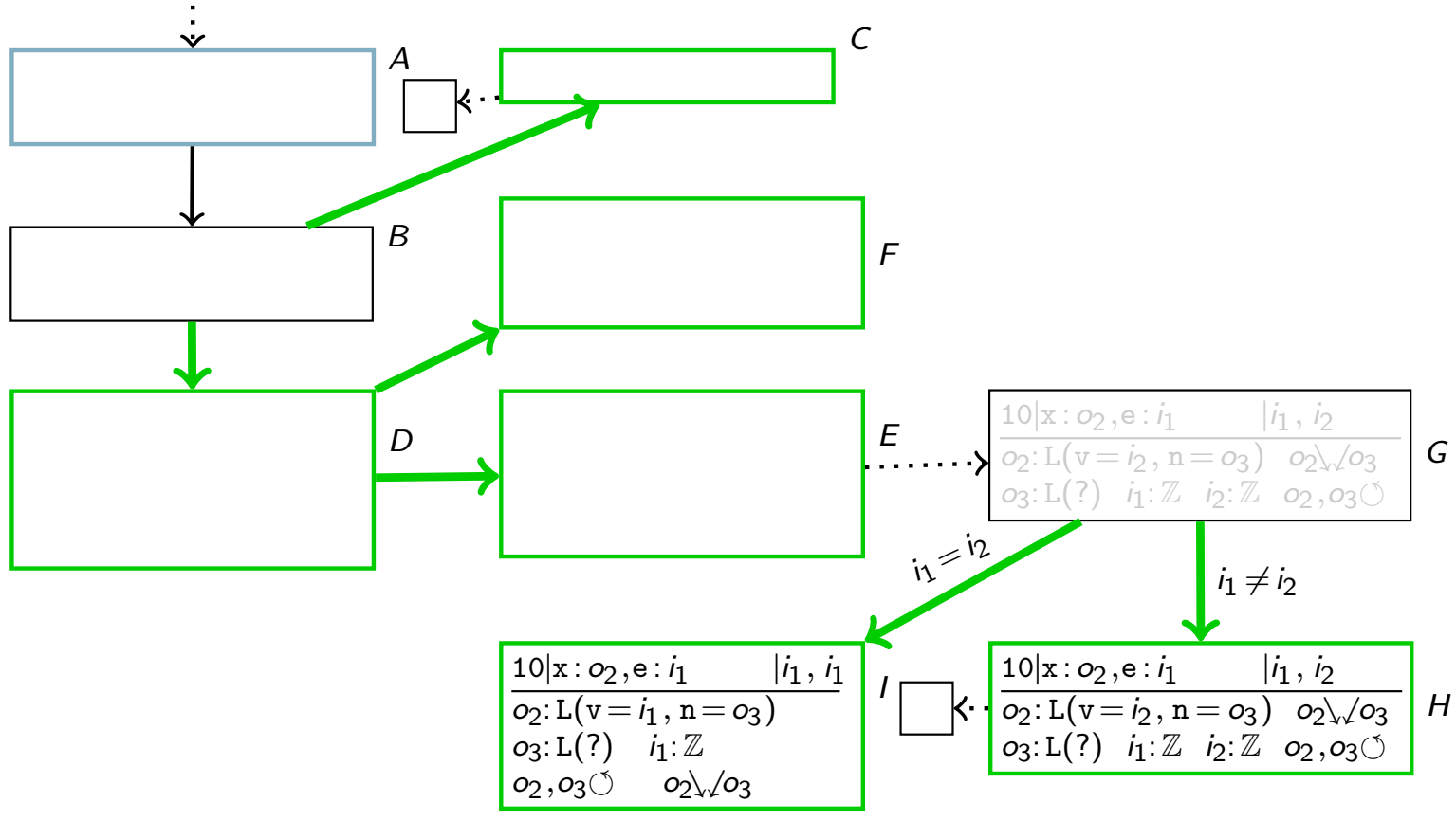



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States G, I, H:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2 \Rightarrow$ Refinement:
  - In I: $i_1 = i_2$
  - In H: $i_1 \neq i_2$

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

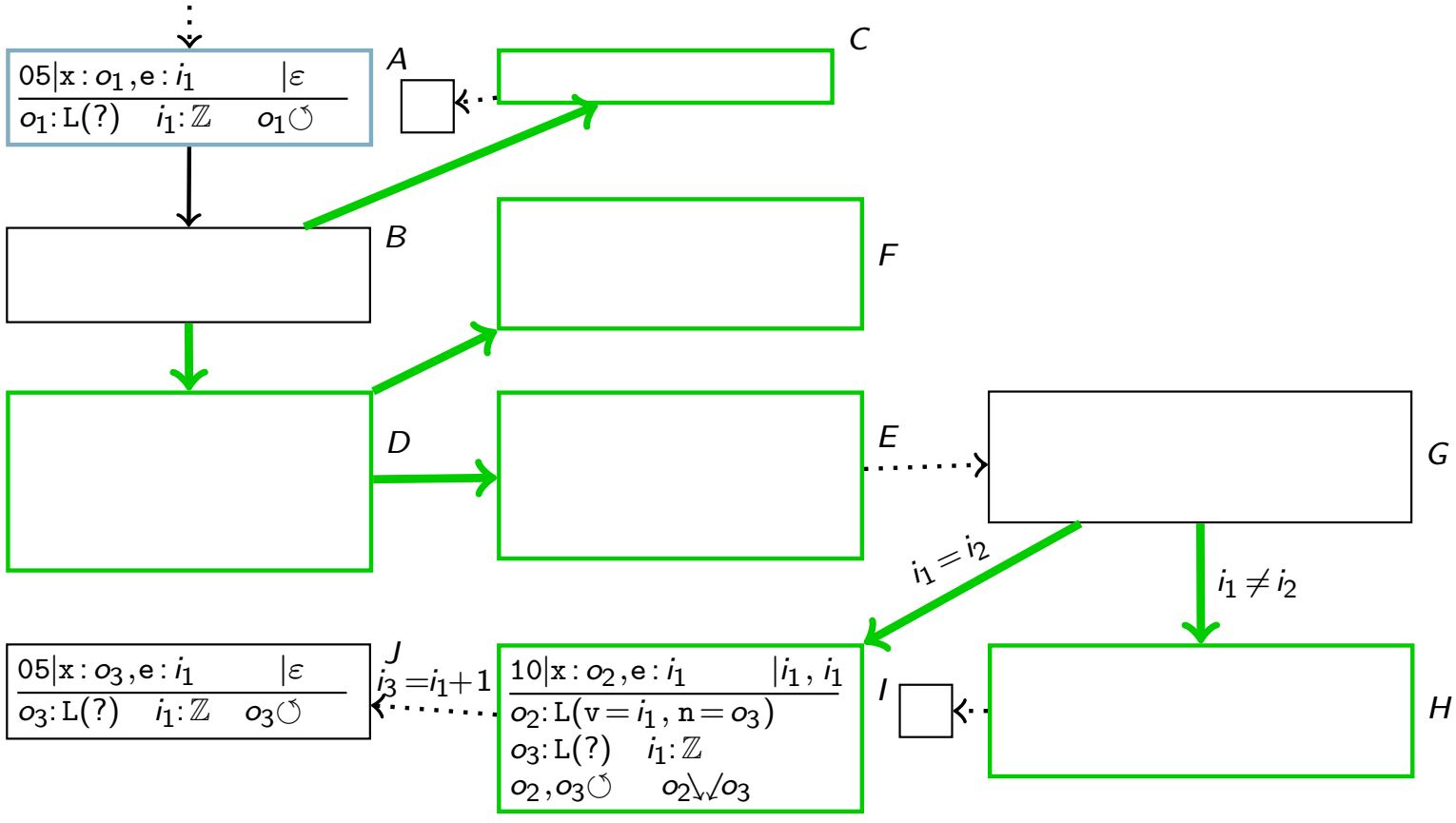



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States G, I, H:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2 \Rightarrow$ Refinement:
  - In I: $i_1 = i_2$ (program ends)
  - In H: $i_1 \neq i_2$

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

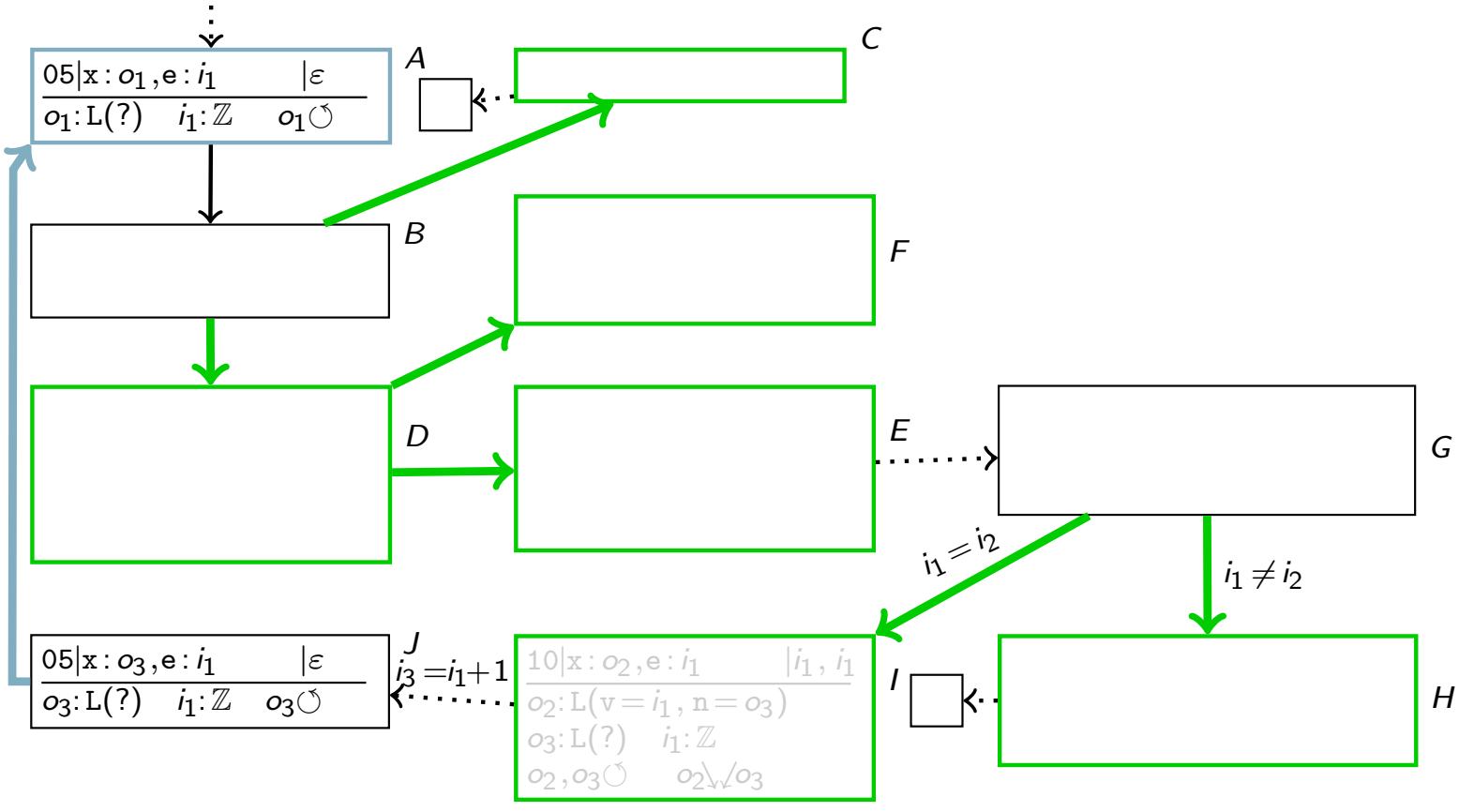



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States G, I, H:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2 \Rightarrow$ Refinement:
  - In I: $i_1 = i_2$ (program ends)
  - In H: $i_1 \neq i_2$
- State J reached by evaluation

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

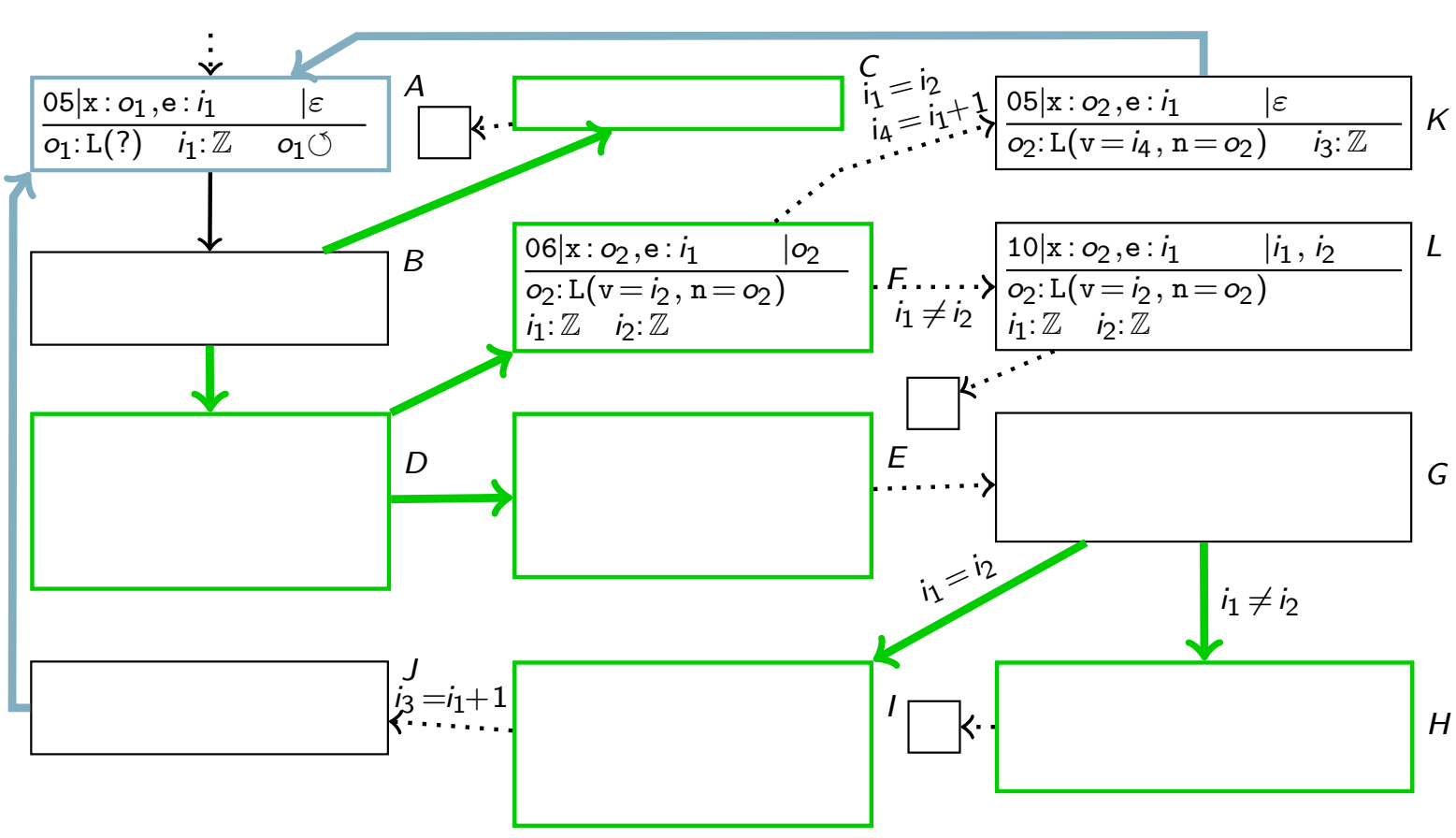



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States G, I, H:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2 \Rightarrow$ Refinement:
  - In I: $i_1 = i_2$ (program ends)
  - In H: $i_1 \neq i_2$
- State J reached by evaluation, represented by (instance of) A

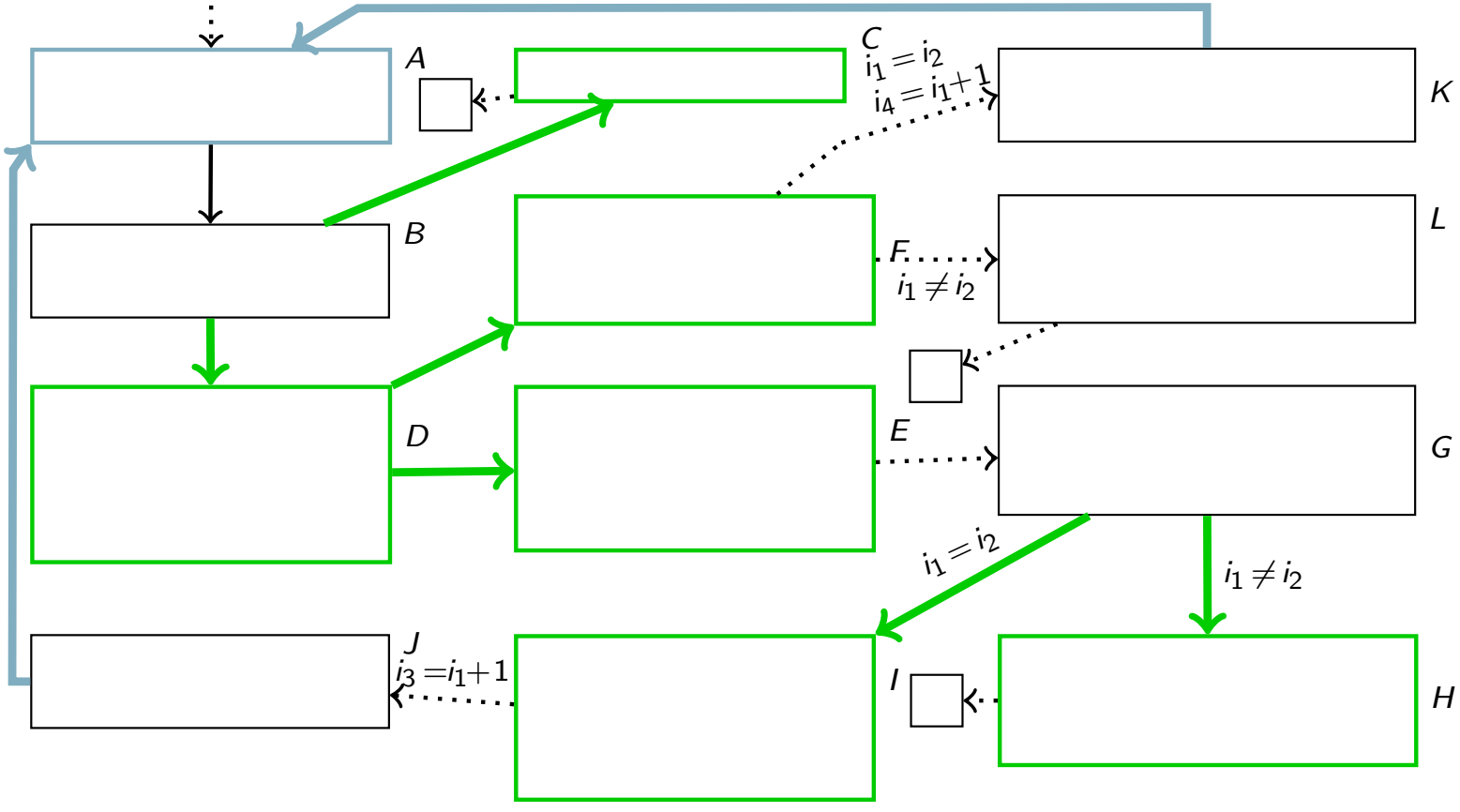
```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



States K, L: Analogous for one-element list

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

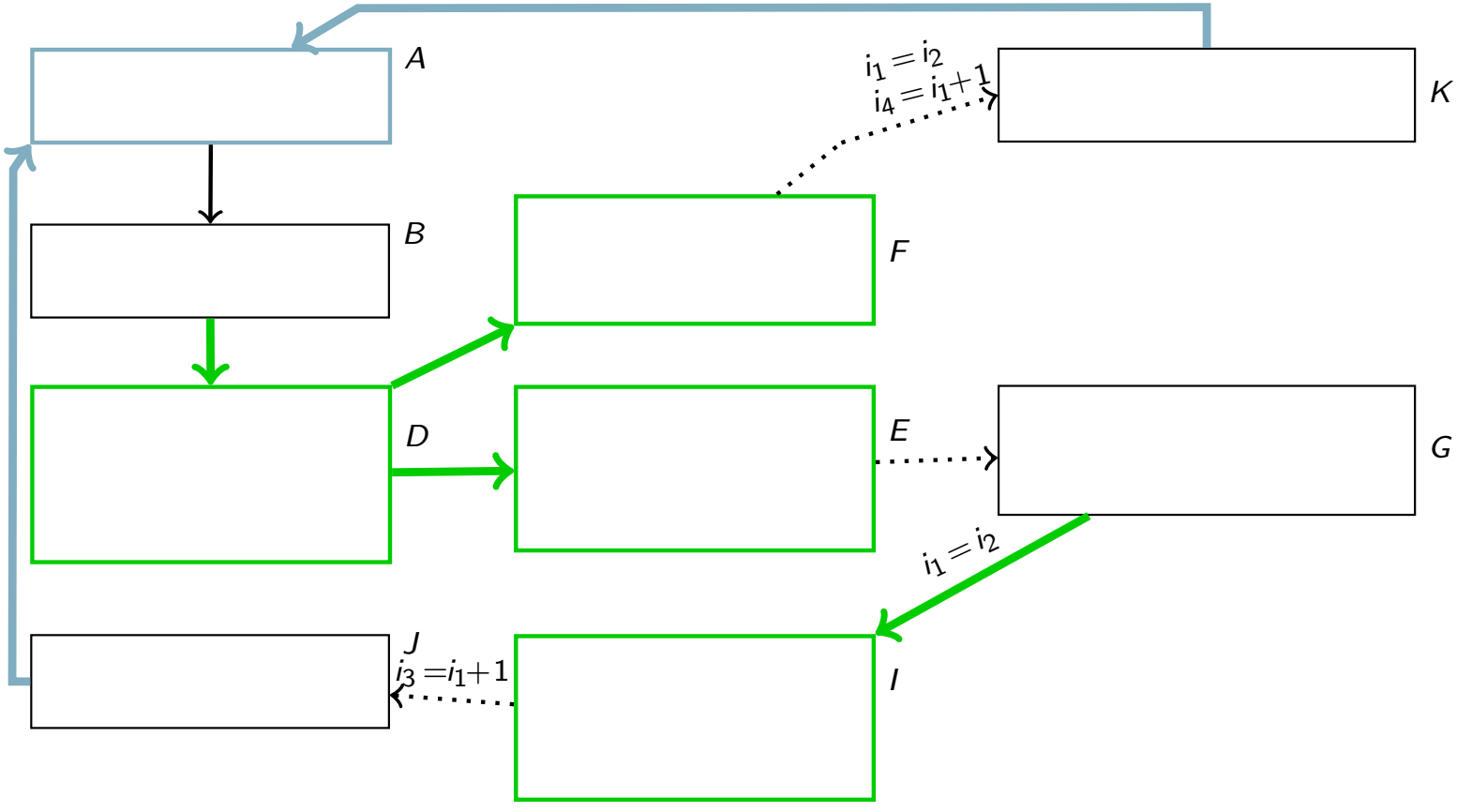
```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



- All leaves program ends ⇒ Graph finished
- How can we prove termination?

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

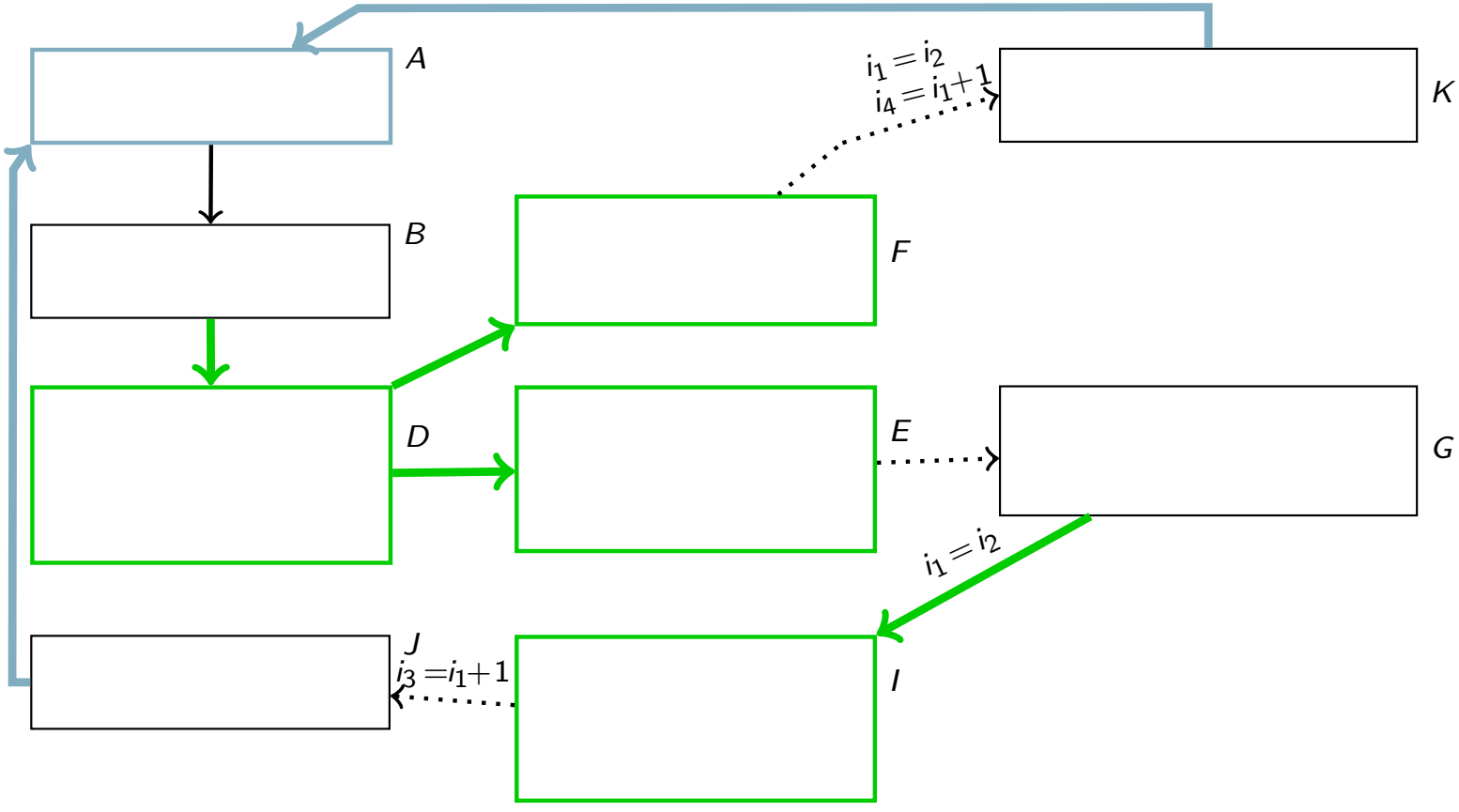
```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



- All leaves program ends ⇒ Graph finished
- How can we prove termination?
- Only consider SCCs

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

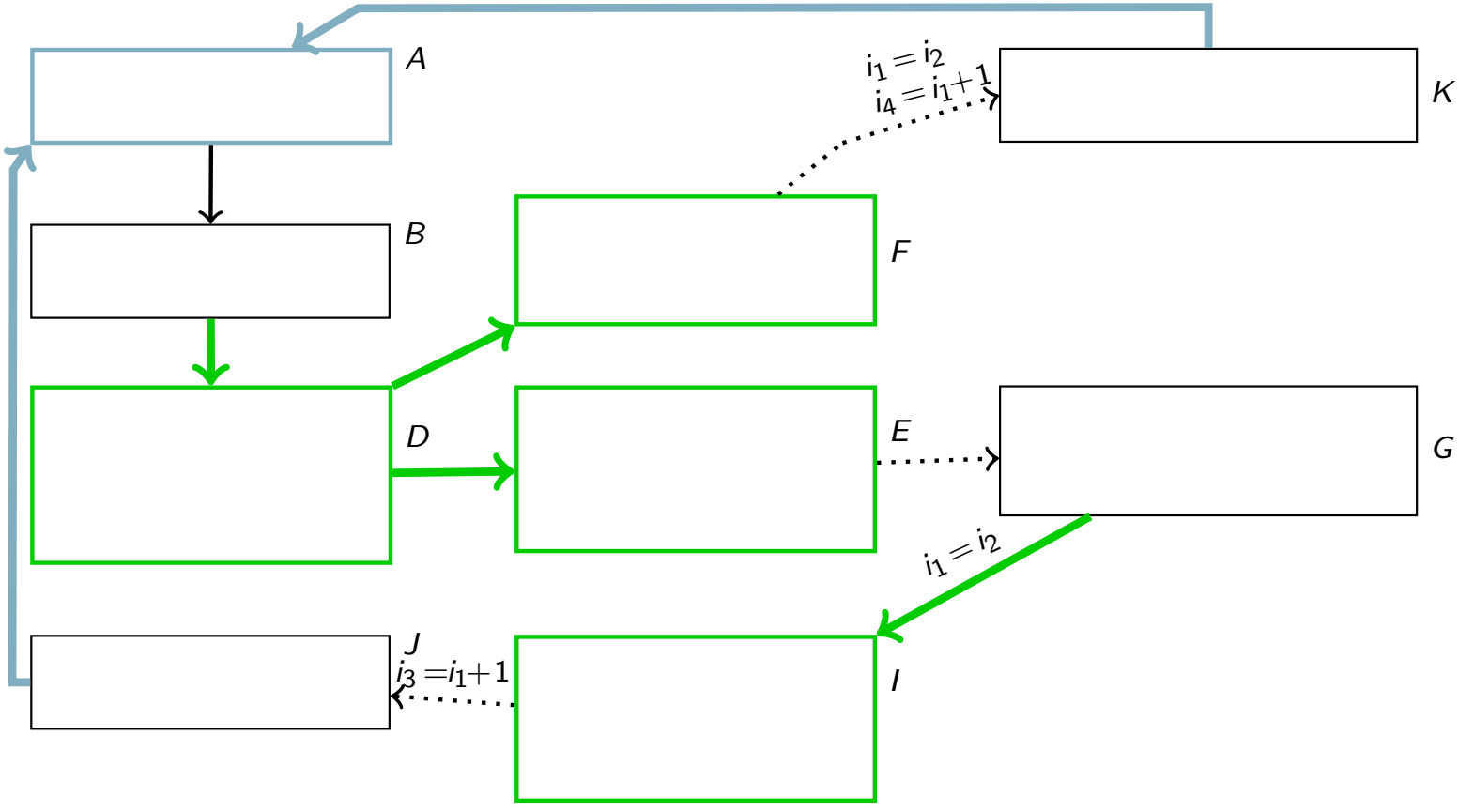



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

- All leaves program ends $\Rightarrow$ Graph finished
- How can we prove termination?
- Only consider SCCs

  High-level argument: Number of unvisited elements strictly decreasing

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



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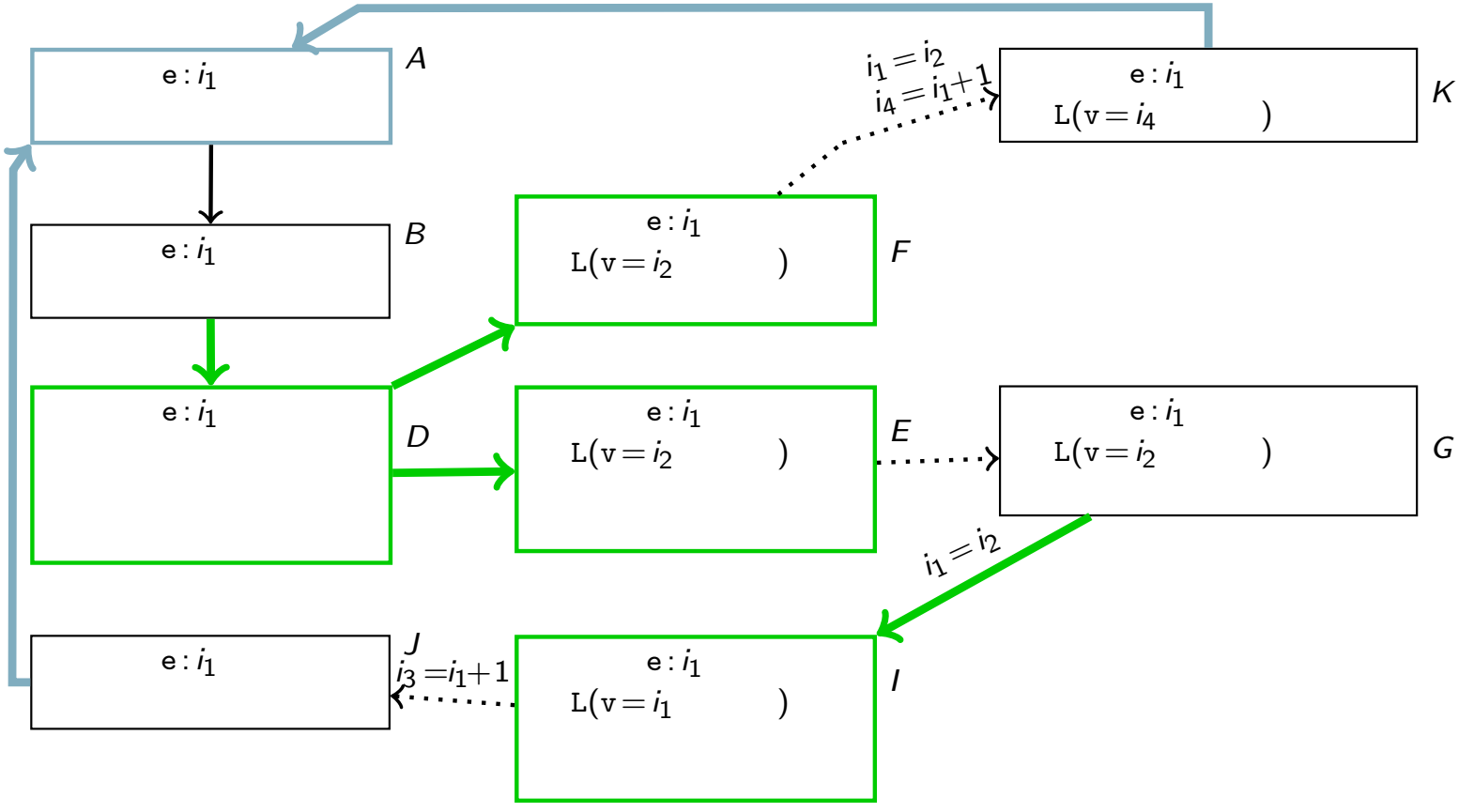

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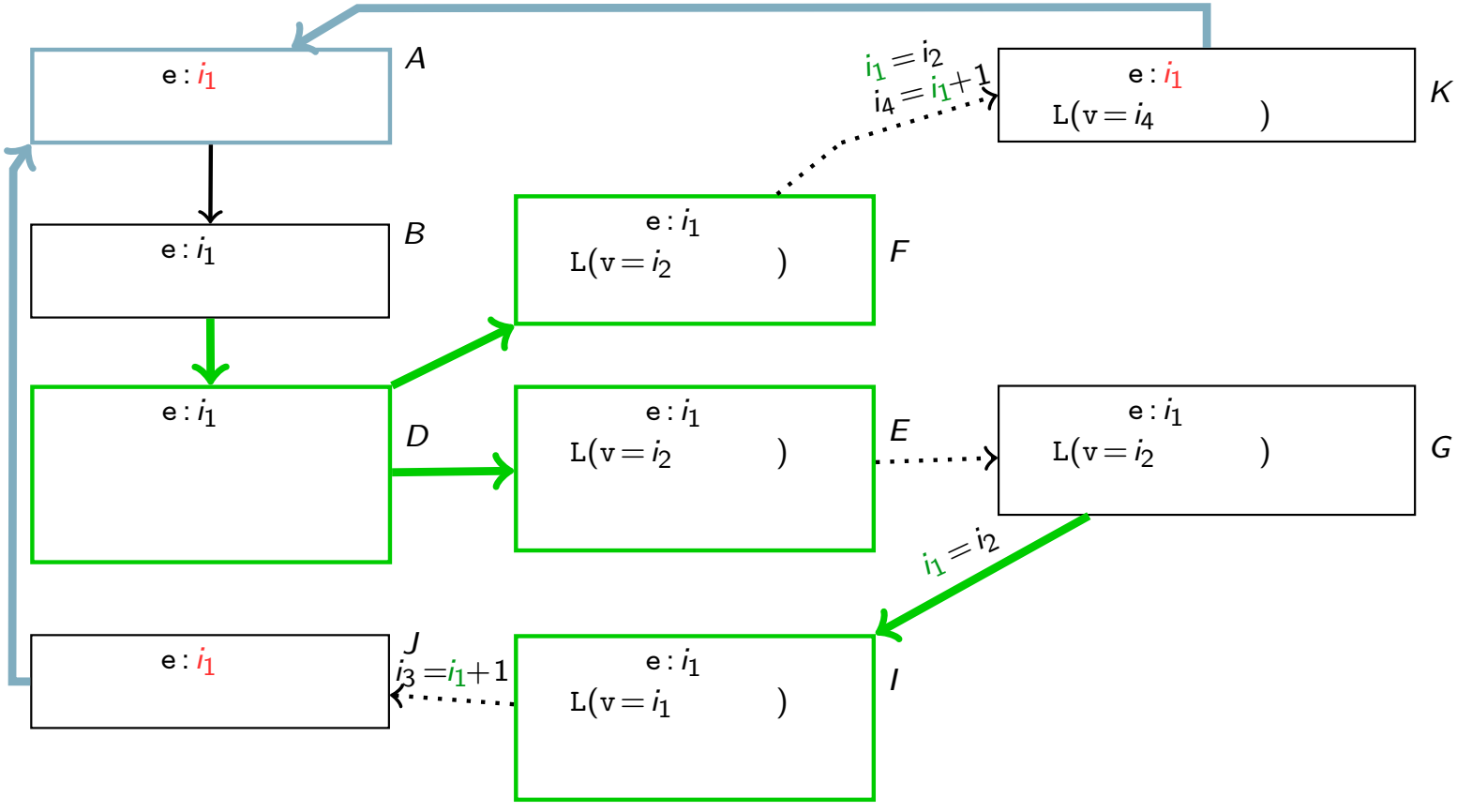

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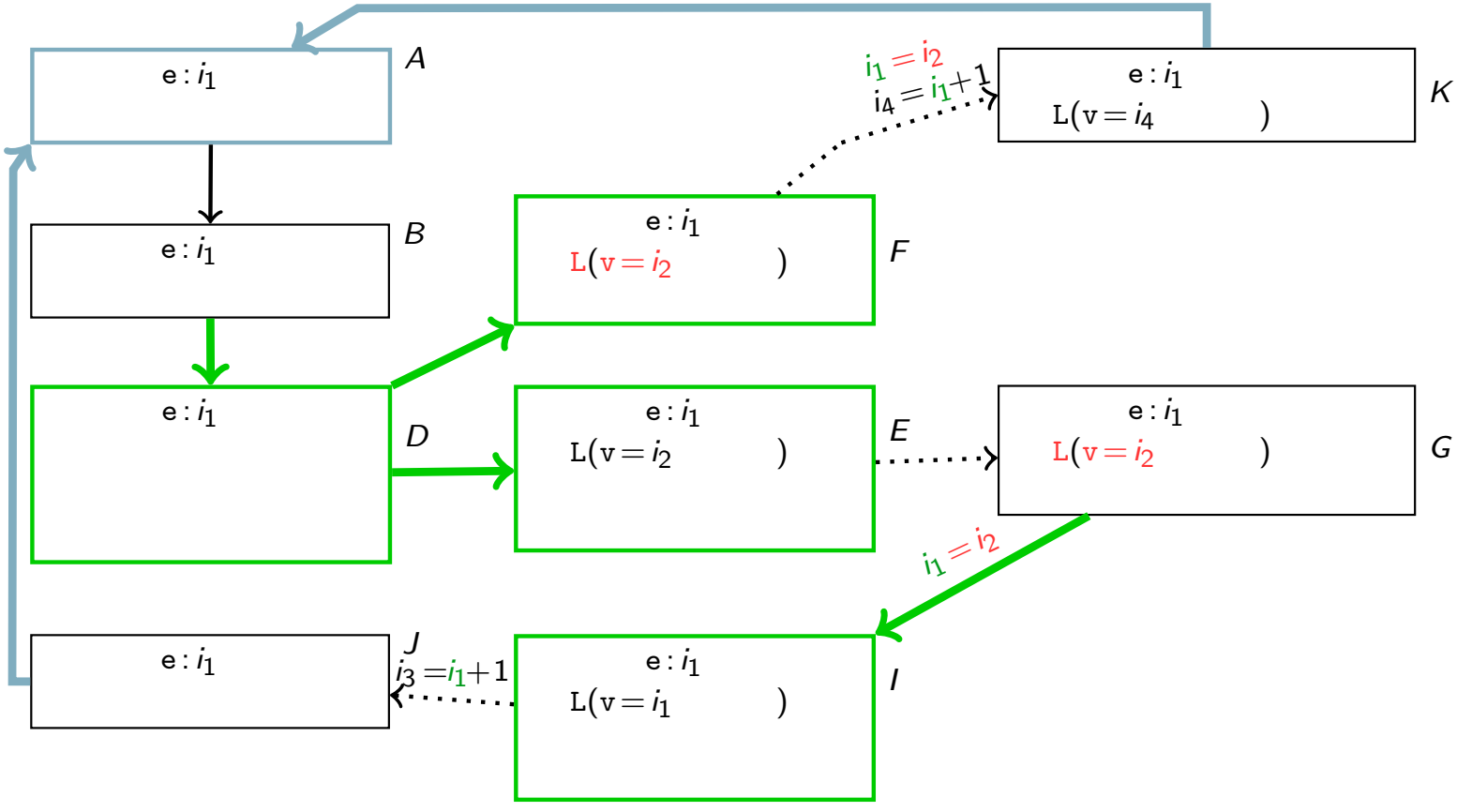


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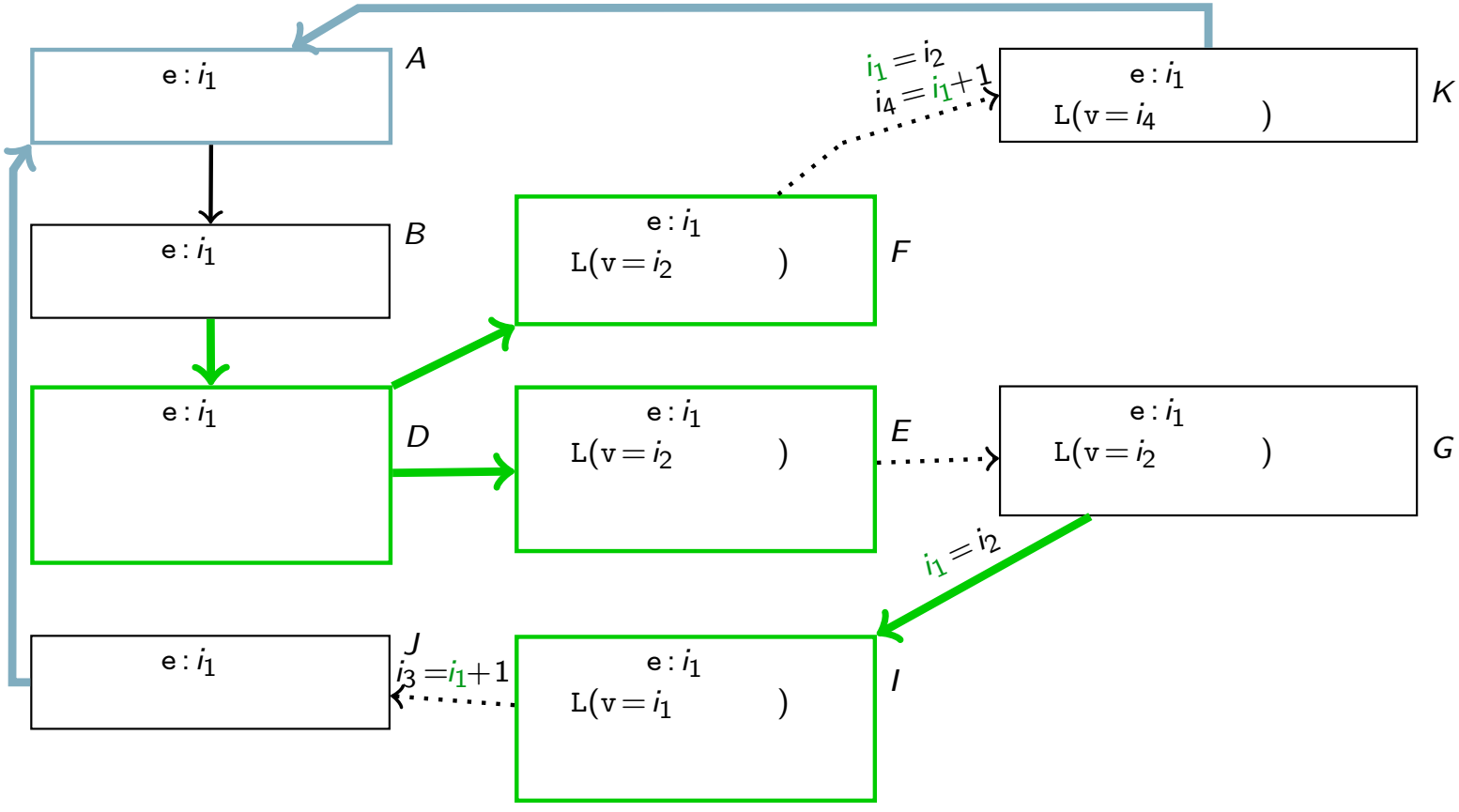


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- Track number of objects where C.f $\bowtie$ c holds ($\#_M$)

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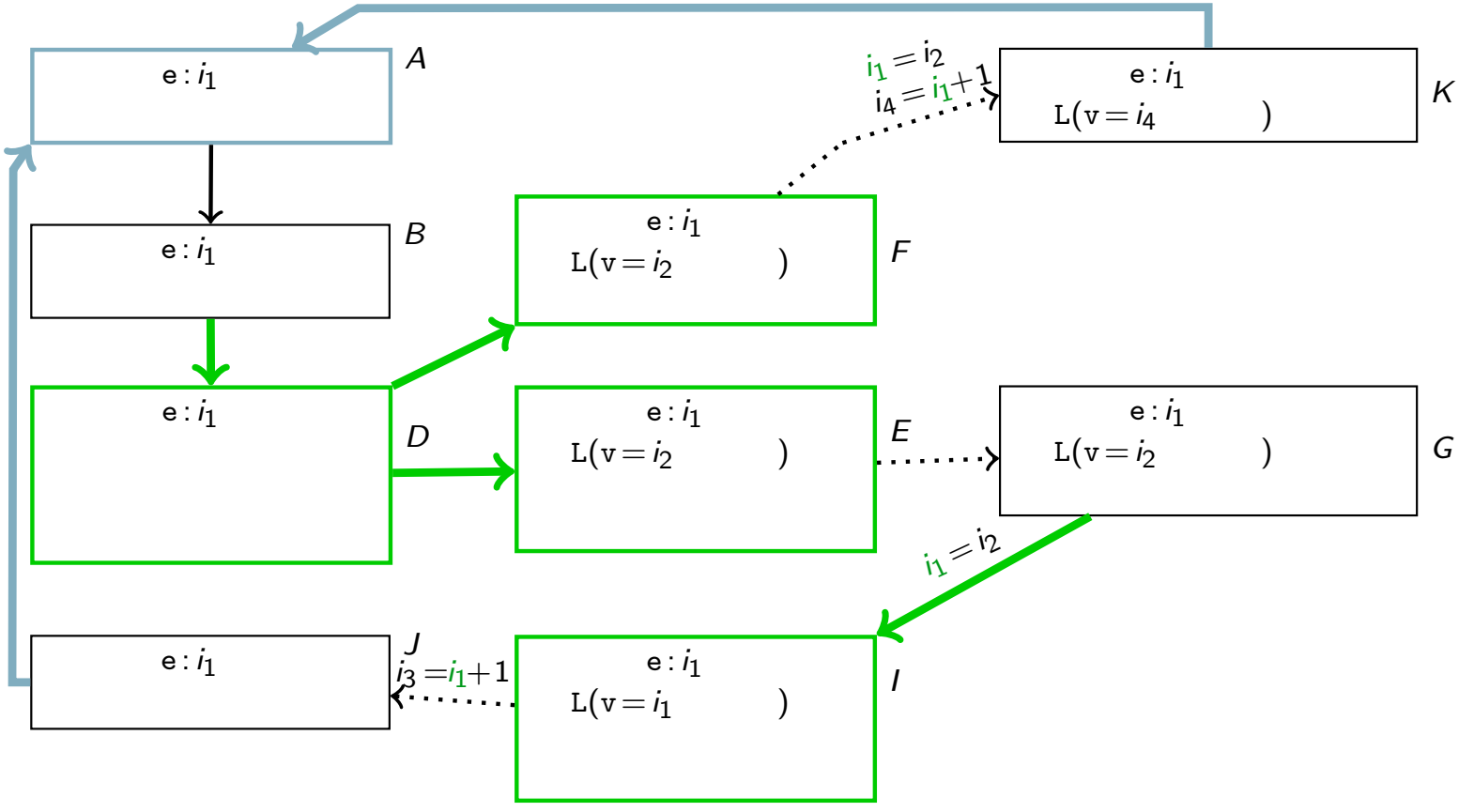

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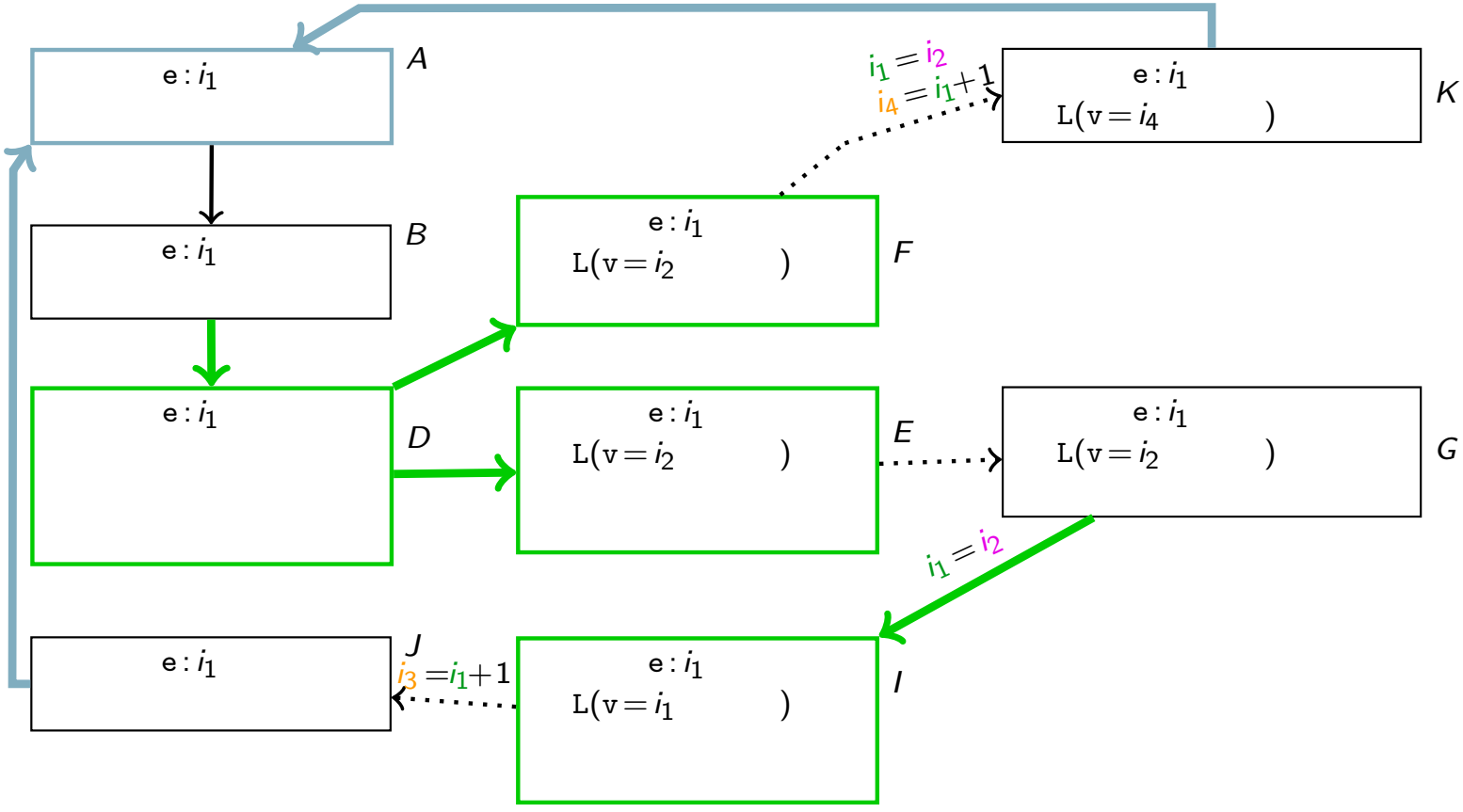

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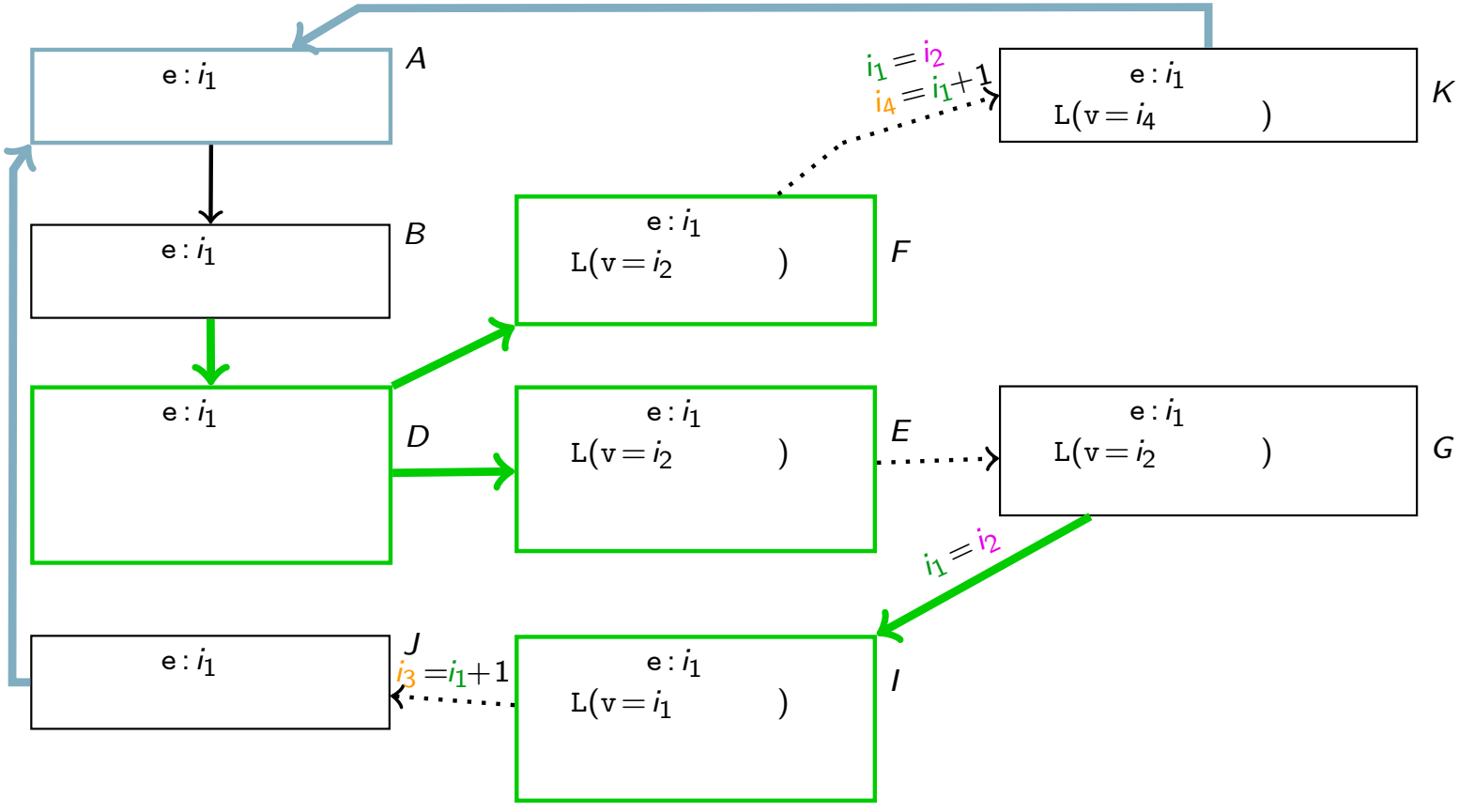

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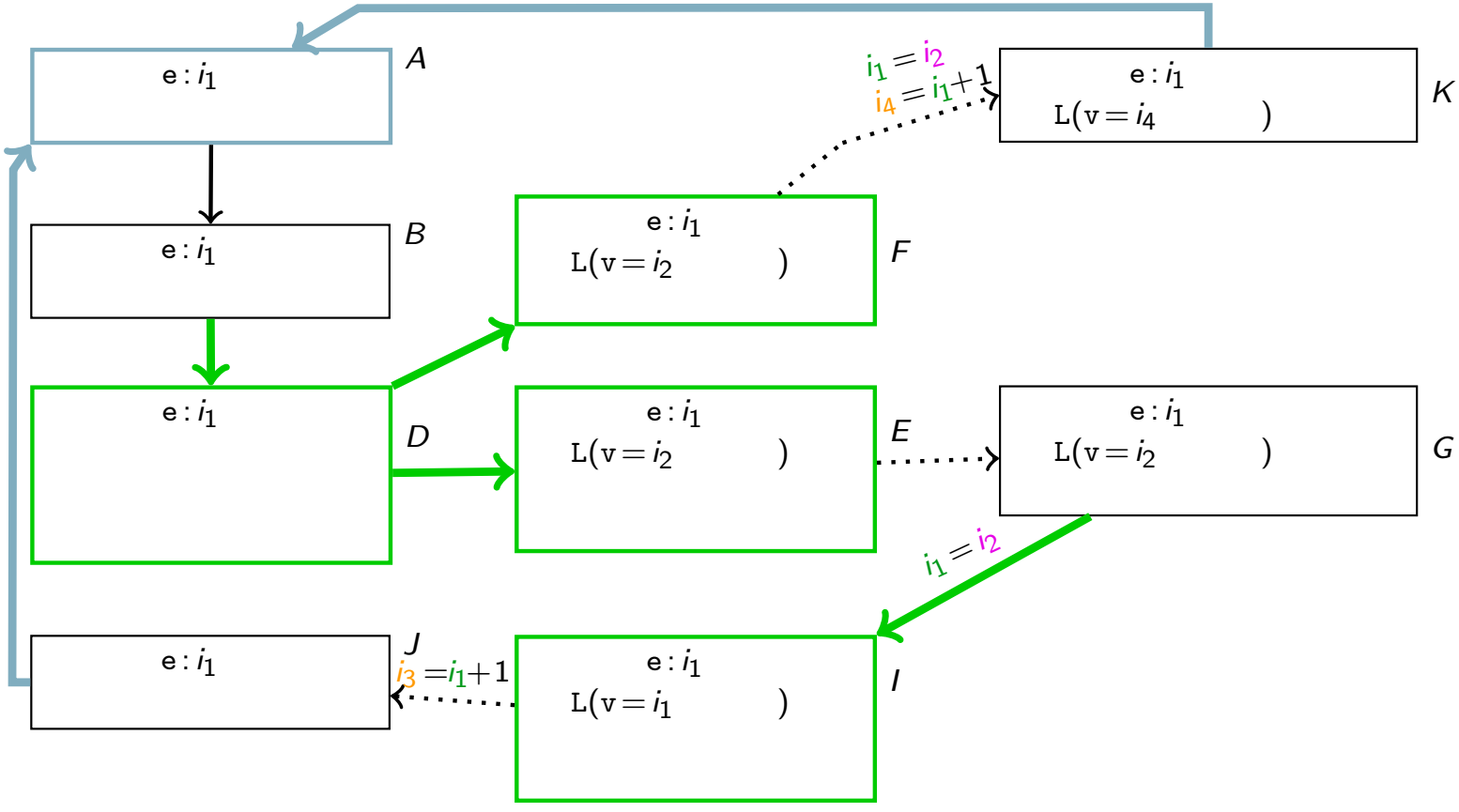

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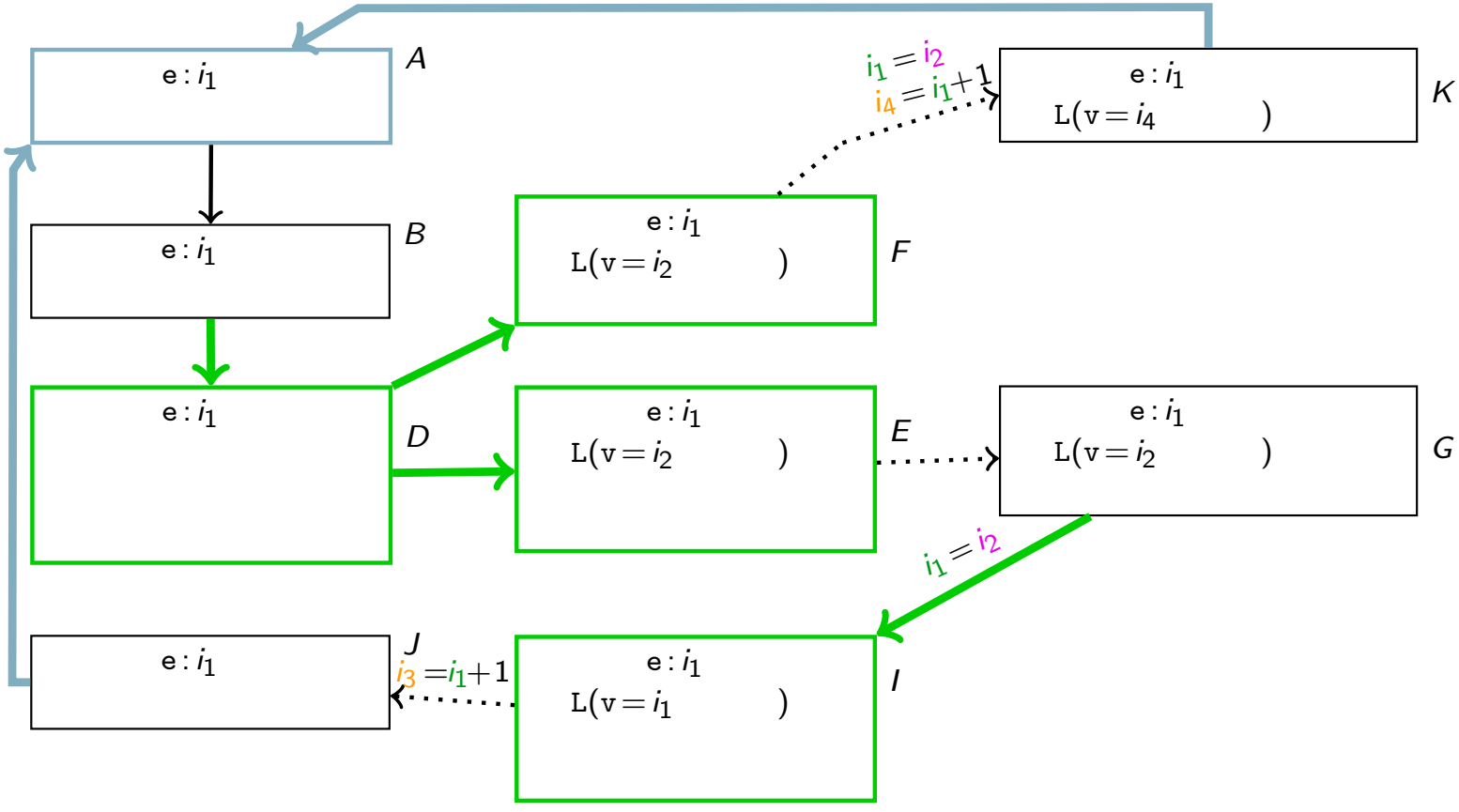

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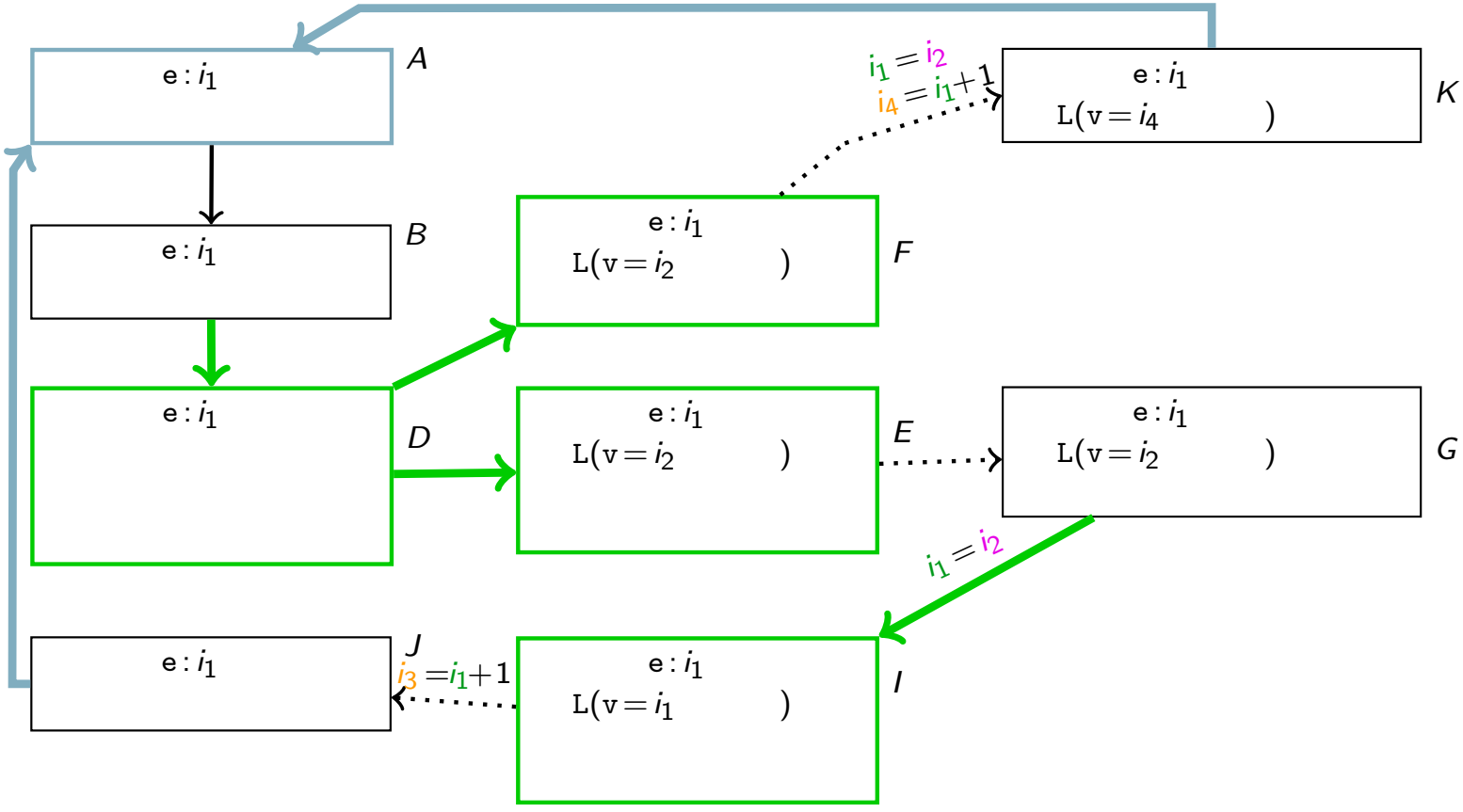

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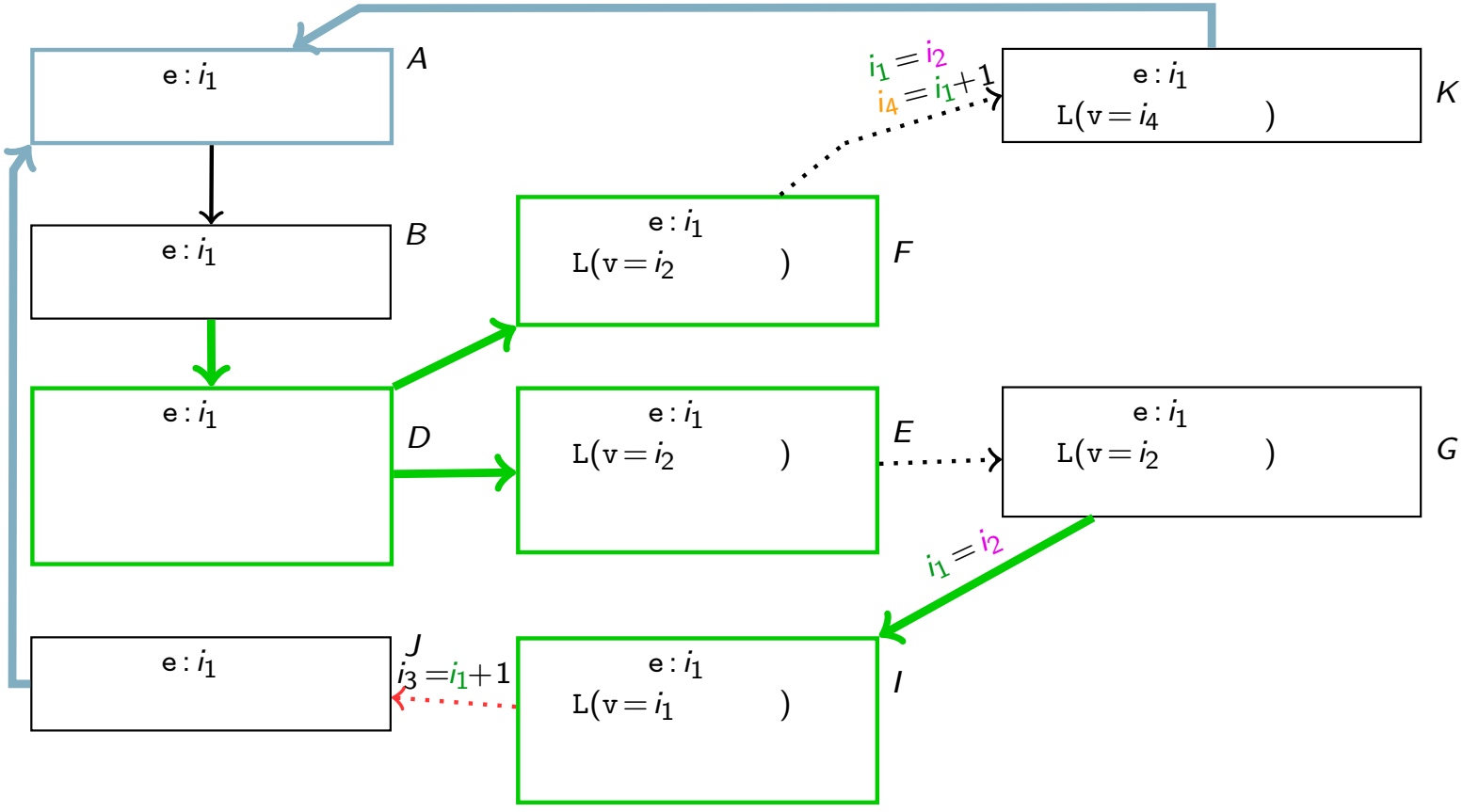

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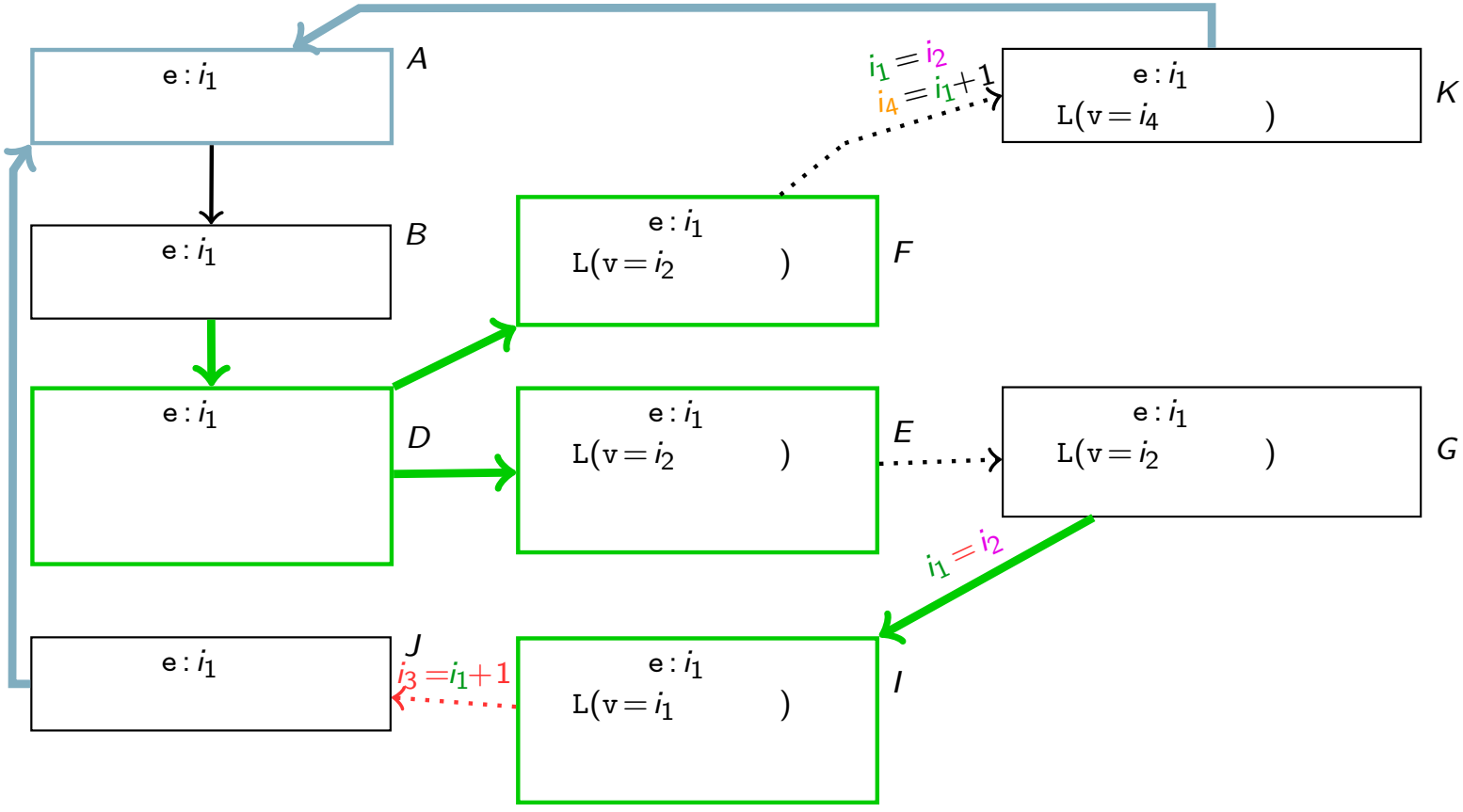

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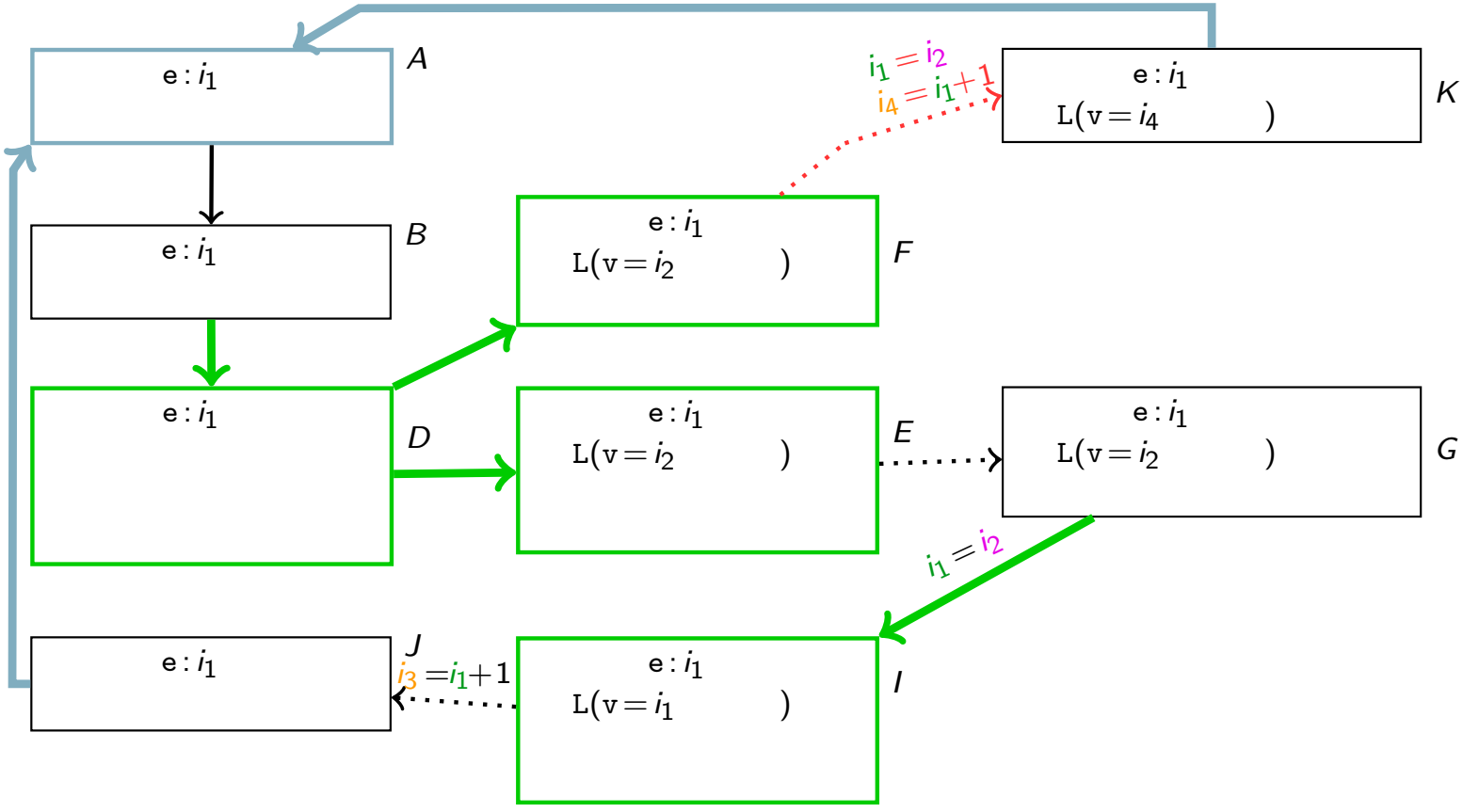

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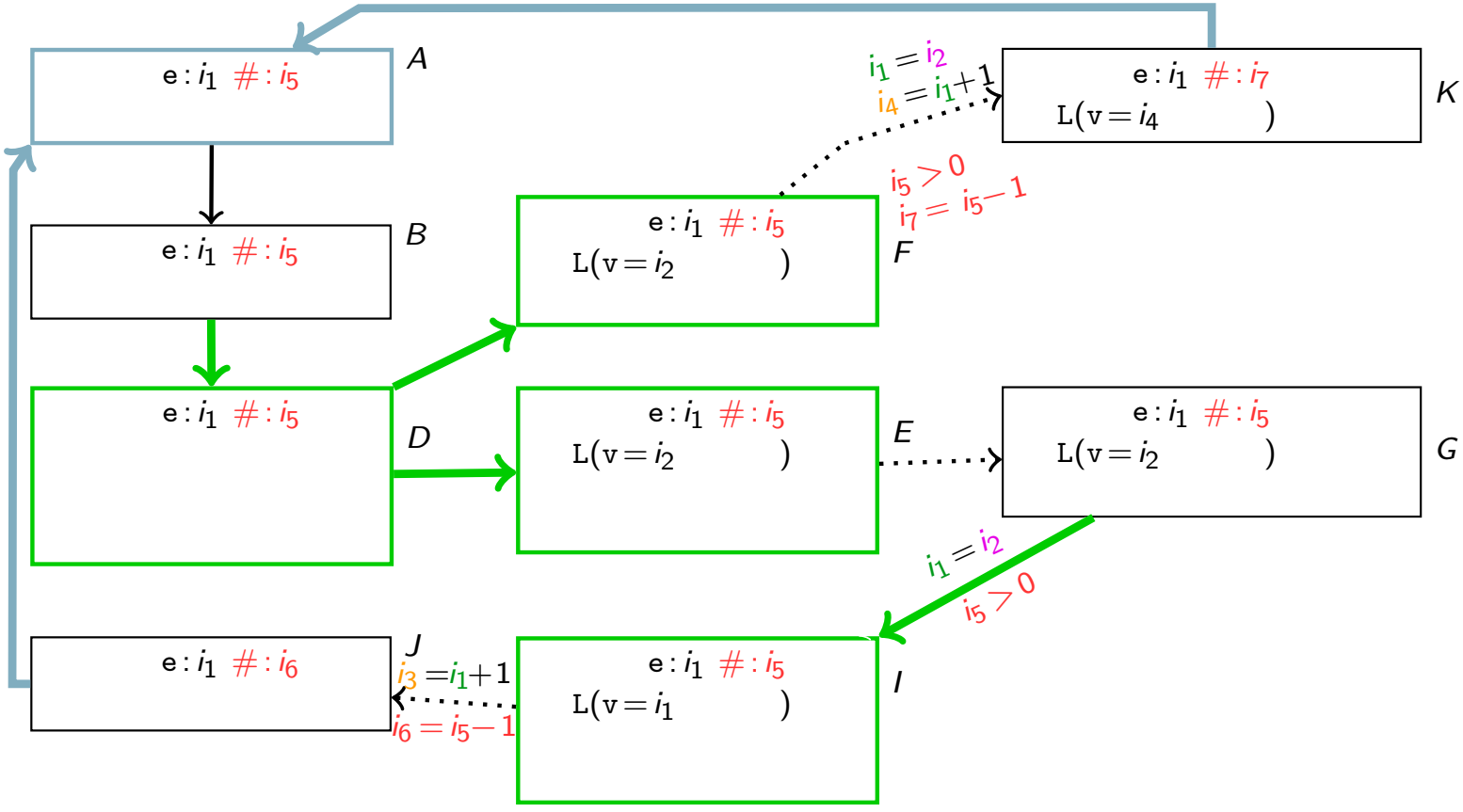

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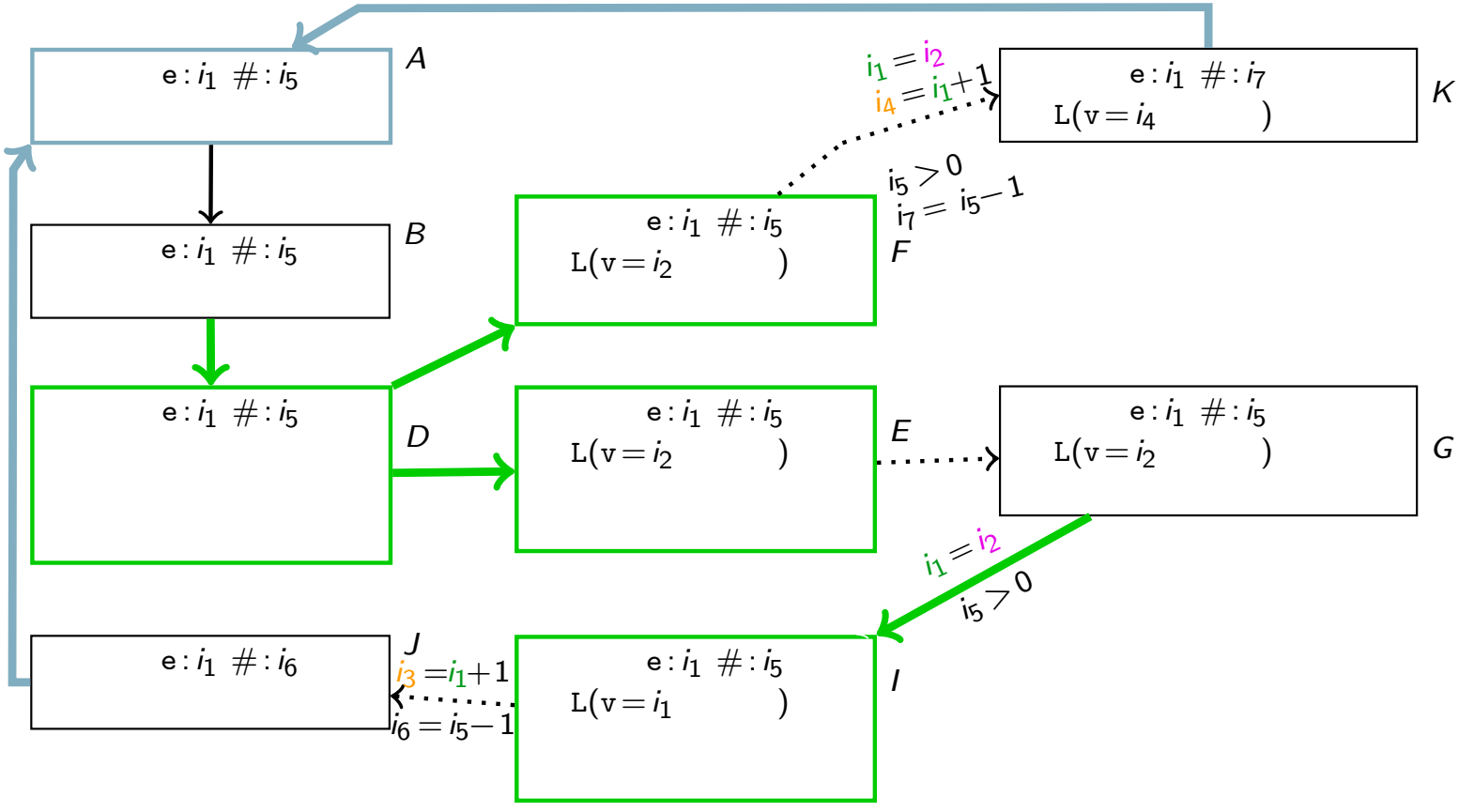

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$$\begin{aligned} f_A(\ldots, i_6) &\to f_A(\ldots, i_6 - 1) && \mid i_6 > 0 \\ f_A(\ldots, i_7) &\to f_A(\ldots, i_7 - 1) && \mid i_7 > 0 \end{aligned}$$

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