Proving Termination of Heap-Manipulating Java Programs

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Automated Termination Analysis

Imperative Programs:

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- Synthesis of Linear Ranking Functions
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  KITTeL – (*Falke, Kapur, Sinz*, since '11)
- Termination Analysis for Java (via Path Length, CLP backend)
  Julia – (*Spoto, Mesnard, Payet*, since '08)
  COSTA – (*Albert, Arenas, Codish, Genaim, Puebla, Zanardini*, since '08)

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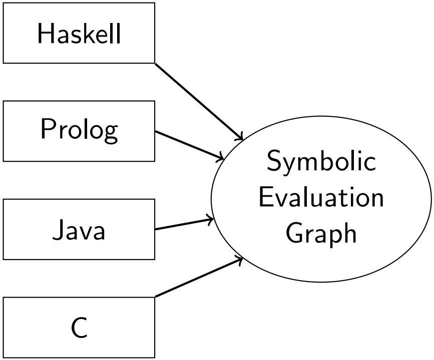

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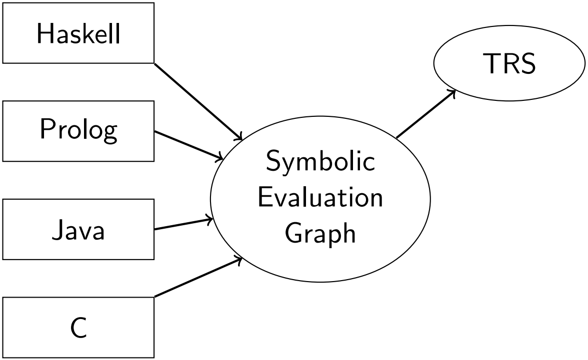

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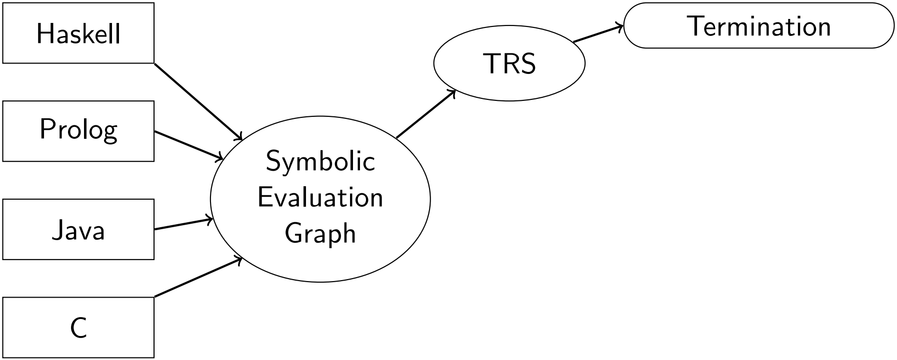

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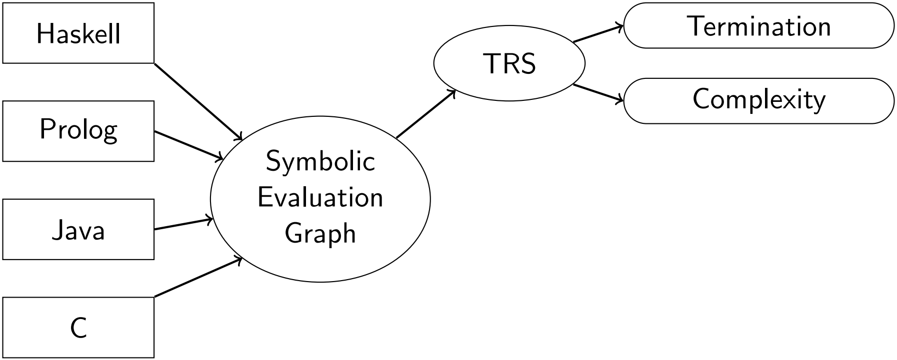

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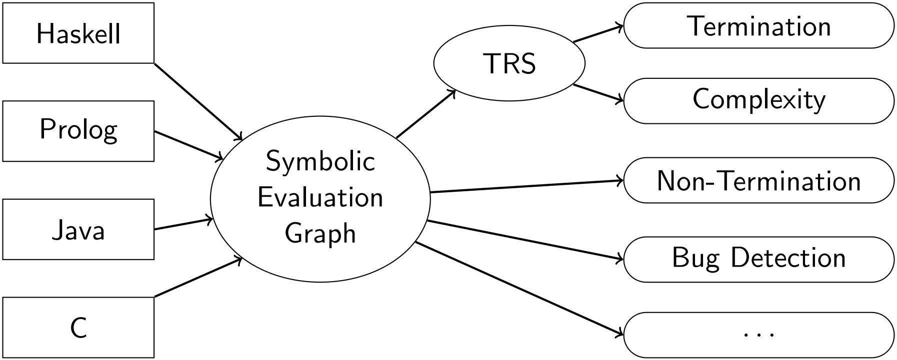

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Handling of user-defined acyclic data structures:

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public class List {
  int value;
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- **TRS techniques** search for suitable orders automatically

⇒ Complex orders for user-defined data structures possible

Overview

length: the example

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class L {
  L p, n;
  static int length(L x) {
    int r = 1;
    while (x != null) {
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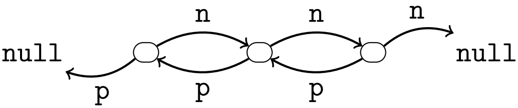

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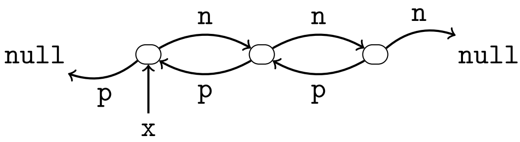

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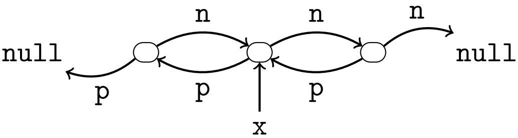

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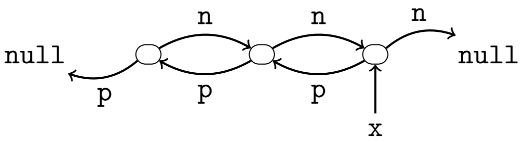

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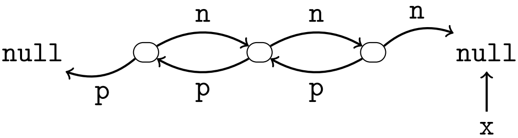

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- Two references may share: $o_1 \searrow\!\!\swarrow o_2$

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- Reference might have cycles containing all fields F: $o_1 \circlearrowleft_F$

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State *A*:

- x some list, might contain cycles using p and n

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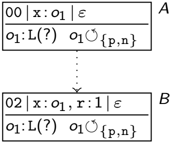

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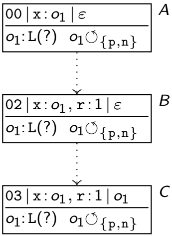

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State *C*:

- x (o_1) null? We do not know!

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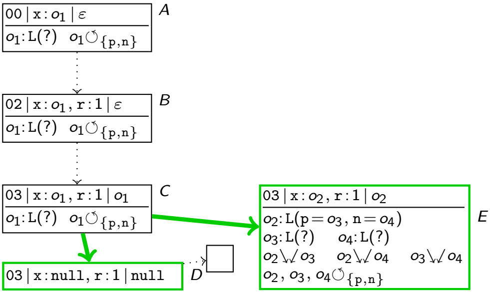

State *A*:

- x some list, might contain cycles using p and n

State *B*:

- Initialized variable r to 1

States *C*, *D*, *E*:

- x (o_1) null? We do not know!

⇒ Refinement

- In *D*: o_1 is null (↷ program ends)
- In *E*: o_1 replaced by o_2, which exists and has fields:
  - Field values can share (↷ add $\searrow\!\!\swarrow$)
  - Field values can be cyclic again (↷ add ↻)

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int length(L x) {
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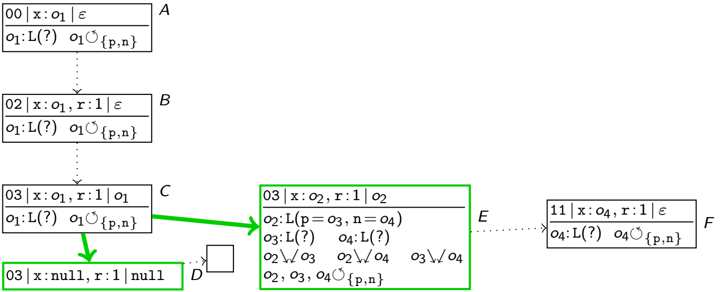

State *F*:

- Stored `x.n` to `x` (allowing for GC)

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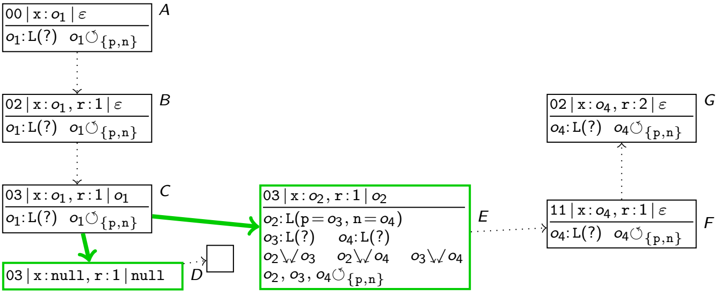

State *F*:

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State *G*:

- Incremented `r`, back to position 02 (as *B*)

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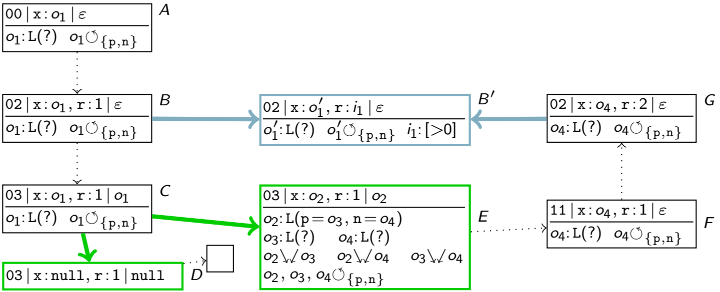

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⇒ Generalization: "Merge" states B, G

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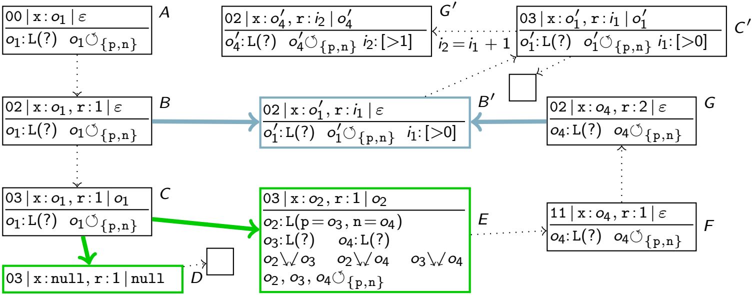

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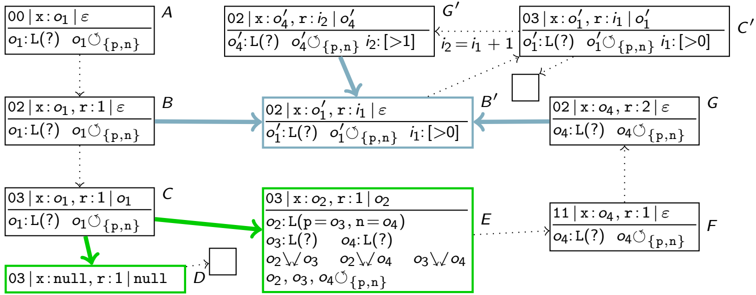

State *F*:

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States *G*, *B′*:

- Incremented `r`, back to position 02 (as *B*)

⇒ Generalization: "Merge" states *B*, *G*

States *C′*, *G′*:

- Repetition of *C*, *G*

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Orientation: Term Rewriting

- Generalized Functional Programming

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- Rules $\mathcal{R}$ define rewrite relation:

$$\mathsf{app}(\mathsf{Cons}(x, xs), ys) \to \mathsf{Cons}(x, \mathsf{app}(xs, ys)) \quad (1)$$

$$\mathsf{app}(\mathsf{Nil}, ys) \to ys \quad (2)$$

- Rewriting of term t with rule $l \to r$:
  1. Find subterm s of t
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$$\begin{aligned} & \underline{\mathsf{app}(\mathsf{Cons}(1, \mathsf{Nil}), \mathsf{Cons}(2, \mathsf{Nil}))} && \text{with } (1), x = 1, xs = \mathsf{Nil}, \\ & && ys = \mathsf{Cons}(2, \mathsf{Nil}) \\ \to\ & \mathsf{Cons}(1, \underline{\mathsf{app}(\mathsf{Nil}, \mathsf{Cons}(2, \mathsf{Nil}))}) && \text{with } (2), ys = \mathsf{Cons}(2, \mathsf{Nil}) \\ \to\ & \mathsf{Cons}(1, \mathsf{Cons}(2, \mathsf{Nil})) && \end{aligned}$$

Transforming values to terms

$$\frac{03 \mid \mathtt{x} : o_2, \mathtt{r} : 1 \mid o_2}{\begin{array}{l} o_2 \colon \mathtt{L}(\mathtt{p} = o_3, \mathtt{n} = o_4) \\ o_3 \colon \mathtt{L}(?) \quad o_4 \colon \mathtt{L}(?) \\ o_2 \between o_3 \quad o_2 \between o_4 \quad o_3 \between o_4 \\ o_2, o_3, o_4 \circlearrowleft_{\{\mathtt{p},\mathtt{n}\}} \end{array}} \quad E$$

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- Known integers transformed to themselves

1

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- Known integers transformed to themselves
- Unknown values transformed to variables

$o_3, o_4 \quad 1$

Transforming values to terms

$$\begin{array}{l} 03 \mid \mathtt{x}: o_2, \mathtt{r}: 1 \mid o_2 \\ \hline \color{red}{o_2: \mathtt{L}(\mathtt{p} = o_3, \mathtt{n} = o_4)} \\ o_3: \mathtt{L}(?) \quad o_4: \mathtt{L}(?) \\ o_2 \searrow\!\!\swarrow o_3 \quad o_2 \searrow\!\!\swarrow o_4 \quad o_3 \searrow\!\!\swarrow o_4 \\ o_2, o_3, o_4 \circlearrowleft_{\{\mathtt{p},\mathtt{n}\}} \end{array} \quad E$$

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- Data structures transformed to nested constructor terms:
  Class `Cl` with n fields $\curvearrowright$ symbol Cl of arity n

$$\overbrace{\mathsf{L}(o_3, o_4)}^{\color{red}{o_2}}\ 1$$

Transforming states to terms

$$
\begin{array}{l}
03 \mid \texttt{x}: o_2, \texttt{r}: 1 \mid o_2 \\
\hline
o_2 : \texttt{L}(\texttt{p} = o_3, \texttt{n} = o_4) \\
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\end{array}
\quad E
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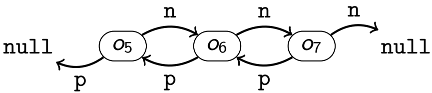
- Known integers transformed to themselves
- Unknown values transformed to variables
- Data structures transformed to nested constructor terms:
  Class `Cl` with n fields $\curvearrowright$ symbol Cl of arity n
- Encoding cycles: Special symbol $\bigcirc$ for repetition

$$o_5 : \texttt{L}(\texttt{p} = \texttt{null}, \texttt{n} = o_6)$$
$$o_6 : \texttt{L}(\texttt{p} = o_5, \texttt{n} = o_7)$$
$$o_7 : \texttt{L}(\texttt{p} = o_6, \texttt{n} = \texttt{null})$$



Encoding of o_5: $\mathsf{L}(\mathsf{null}, \mathsf{L}(\bigcirc, \mathsf{L}(\bigcirc, \mathsf{null})))$

Encoding of o_6: $\mathsf{L}(\mathsf{L}(\mathsf{null}, \bigcirc), \mathsf{L}(\bigcirc, \mathsf{null}))$

Transforming edges to rules

$03 \mid \mathtt{x}: o_2, \mathtt{r}: 1 \mid o_2$

$o_2: \mathtt{L}(\mathtt{p} = o_3, \mathtt{n} = o_4)$

$o_3: \mathtt{L}(?) \quad o_4: \mathtt{L}(?)$

$o_2 \curlyvee o_3 \quad o_2 \curlyvee o_4 \quad o_3 \curlyvee o_4$

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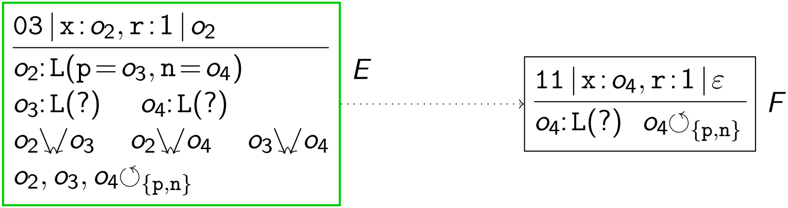
E

- State s transformed to term with symbol f_s
- All local variables, stack entries as arguments

$$\mathsf{f}_E(\overbrace{\mathsf{L}(o_3, o_4)}^{o_2}, 1, \overbrace{\mathsf{L}(o_3, o_4)}^{o_2})$$

Transforming edges to rules



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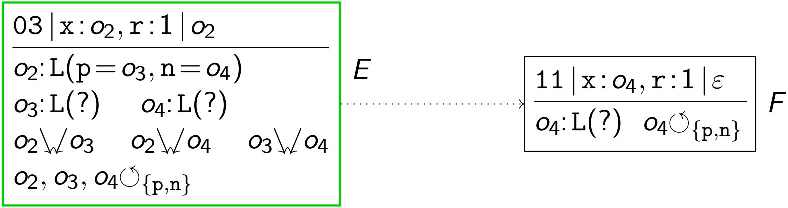

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- Evaluation edges: Encode states, put in $\to$

$$\mathsf{f}_E(\overbrace{\mathsf{L}(o_3, o_4)}^{o_2}, 1, \overbrace{\mathsf{L}(o_3, o_4)}^{o_2}) \quad \to \quad \mathsf{f}_F(o_4\ , 1)$$

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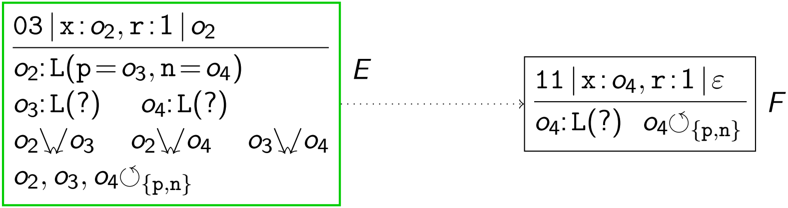

- State s transformed to term with symbol f_s
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- Problem: Cycle encoding changes $\curvearrowright$ free var on rhs

$$\mathsf{f}_E(\overbrace{\mathsf{L}(o_3, o_4)}^{o_2}, 1, \overbrace{\mathsf{L}(o_3, o_4)}^{o_2}) \quad \rightarrow \quad \mathsf{f}_F(o_4{}', 1)$$

Transforming edges to rules

E:

$03 \mid \mathtt{x} : o_2, \mathtt{r} : 1 \mid o_2$

$o_2 : \mathrm{L}(\mathtt{p} = o_3, \mathtt{n} = o_4)$

$o_3 : \mathrm{L}(?) \quad o_4 : \mathrm{L}(?)$

$o_2 \searrow\!\!\!\swarrow o_3 \quad o_2 \searrow\!\!\!\swarrow o_4 \quad o_3 \searrow\!\!\!\swarrow o_4$

$o_2, o_3, o_4 \circlearrowleft_{\{\mathtt{p},\mathtt{n}\}}$

$E \;\cdots\!\!\rightarrow\; F$

F:

$11 \mid \mathtt{x} : o_4, \mathtt{r} : 1 \mid \varepsilon$

$o_4 : \mathrm{L}(?) \quad o_4 \circlearrowleft_{\{\mathtt{p},\mathtt{n}\}}$

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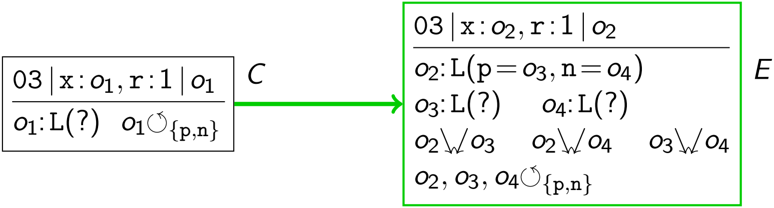
$$\mathsf{f}_E(\overbrace{\mathsf{L}(\quad o_4)}^{o_2}, 1, \overbrace{\mathsf{L}(\quad o_4)}^{o_2}) \quad \rightarrow \quad \mathsf{f}_F(o_4\ , 1)$$

Transforming edges to rules



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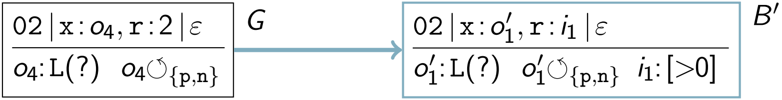
$$\mathsf{f}_C(\mathsf{L}(o_4), 1, \mathsf{L}(o_4)) \;\to\; \mathsf{f}_E(\mathsf{L}(o_4), 1, \mathsf{L}(o_4))$$

Transforming edges to rules



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- Instantiation edges: Encode source state twice, relabel

$$\mathsf{f}_G(o_4, 2) \quad \to \quad \mathsf{f}_{B'}(o_4, 2)$$

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- Function symbols interpreted as polynomials over $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{N}^n, \ldots$

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Choose $[\![\mathsf{app}]\!] = (x, y) \mapsto 1 + 2 \cdot x$, $[\![\mathsf{Cons}]\!] = (x, y) \mapsto 1 + y$,

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Rule:	$\mathsf{app}(\mathsf{Cons}(x, xs), ys) \to \mathsf{Cons}(x, \mathsf{app}(xs, ys))$
Interpretation:	$1 + 2 + 2 \cdot xs > 1 + 1 + 2 \cdot xs$

Choose $[\![\mathsf{app}]\!] = (x, y) \mapsto 1 + 2 \cdot x$, $[\![\mathsf{Cons}]\!] = (x, y) \mapsto 1 + y$,

The example TRS

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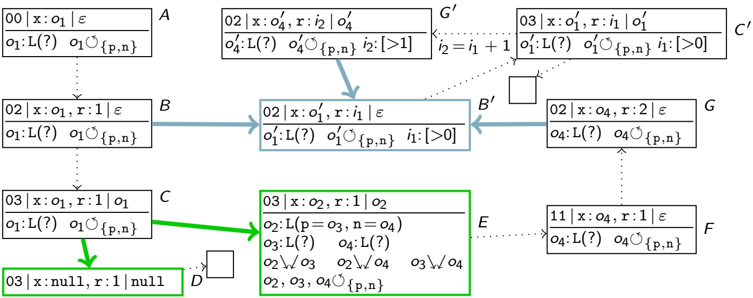

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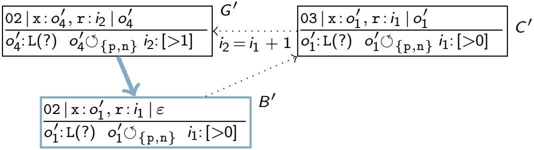

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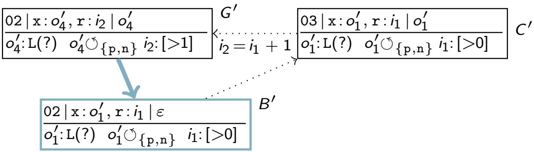

1. Only consider SCCs!
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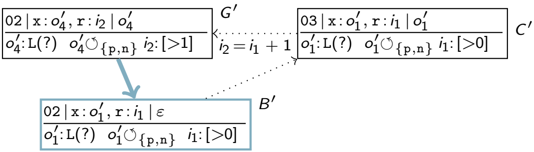

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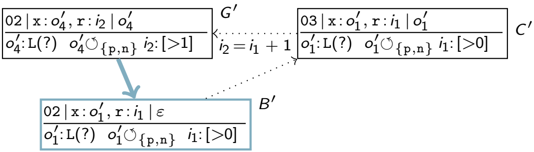

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visit: the example

```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
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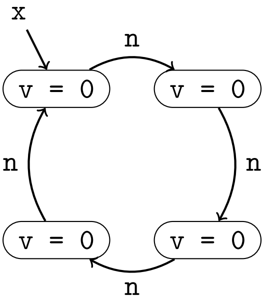

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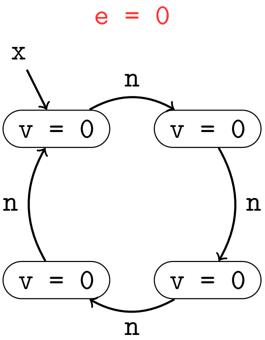

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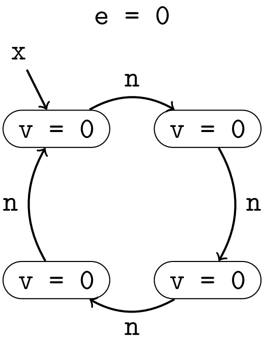
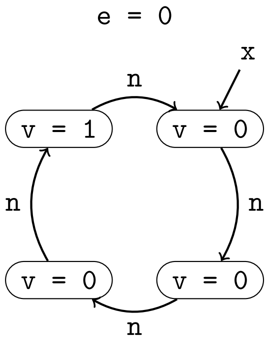


1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

visit: the example

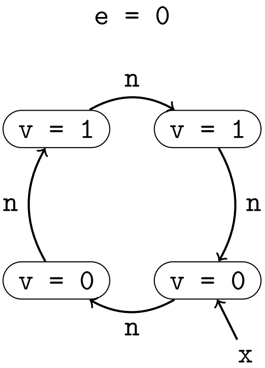
```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```



1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

visit: the example

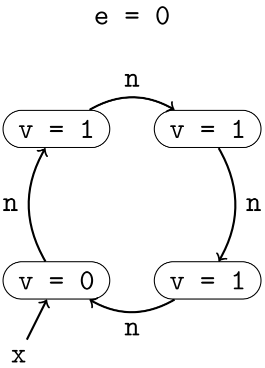
```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```



1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

visit: the example

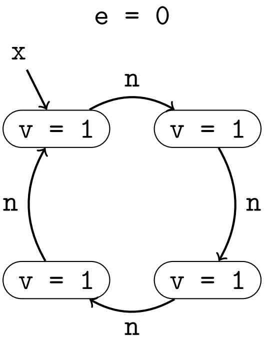
```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```



1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

visit: the example

```
class L {
  int v;     L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```

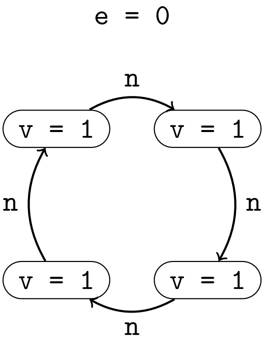


1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

visit: the example

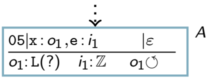
```
class L {
  int v;    L n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
      x.v = e + 1;
      x = x.n; }}}
```

```
00: aload_0        #load x
01: getfield v     #get v from x
04: istore_1       #store to e
05: aload_0        #load x
06: getfield v     #get v from x
09: iload_1        #load e
10: if_icmpne 28 #jump if x.v != e
13: aload_0        #load x
14: iload_1        #load e
15: iconst_1       #load 1
16: iadd           #add e and 1
17: putfield v     #store to x.v
20: aload_0        #load x
21: getfield n     #get n from x
24: astore_0       #store to x
25: goto 5
28: return
```



1. Store first v
2. Continue if object unvisited
3. Change v
4. Go to next element

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

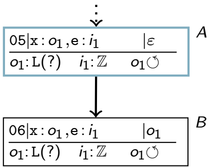


State *A*:

- x some (possibly cyclic) list
- e some integer

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

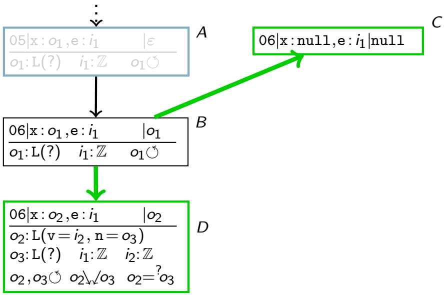


State *B*:

- *Evaluation* between *A* and *B*
- Need field of o_1

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

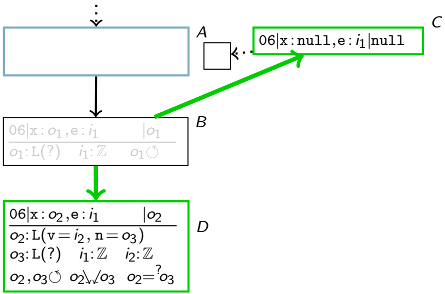



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States B, C, D:

- *Evaluation* between A and B
- Need field of o_1 $\Rightarrow$ Refinement:
  - In C: o_1 is `null`
  - In D: o_1 renamed to o_2, pointing to L-object with successor o_3:
    - o_3 possibly cyclic
    - o_3 possibly equal to o_2 and may reach o_2

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

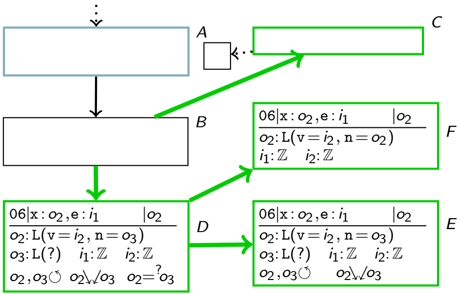



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States *B*, *C*, *D*:

- *Evaluation* between *A* and *B*
- Need field of o_1 $\Rightarrow$ Refinement:
  - In *C*: o_1 is `null` (program crashes)
  - In *D*: o_1 renamed to o_2, pointing to L-object with successor o_3:
    - o_3 possibly cyclic
    - o_3 possibly equal to o_2 and may reach o_2

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

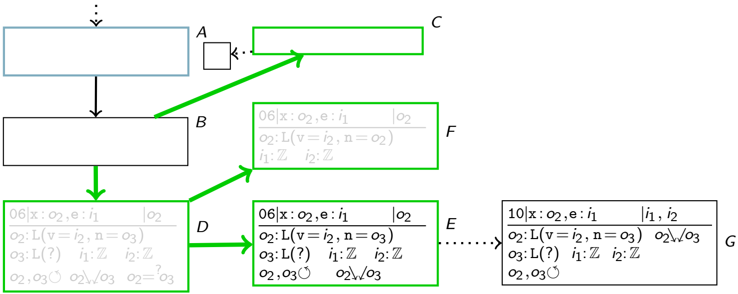


States E, F:

- Need to read field of $o_2 \Rightarrow$ Refinement
  - In E: $o_2 \neq o_3$
  - In F: $o_2 = o_3$

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

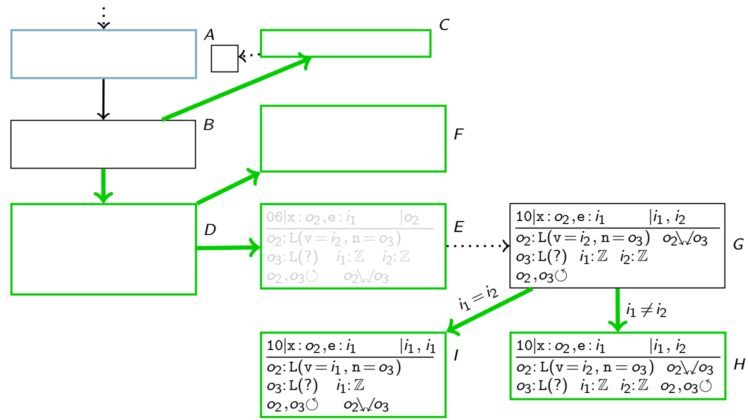


State G:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2$

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

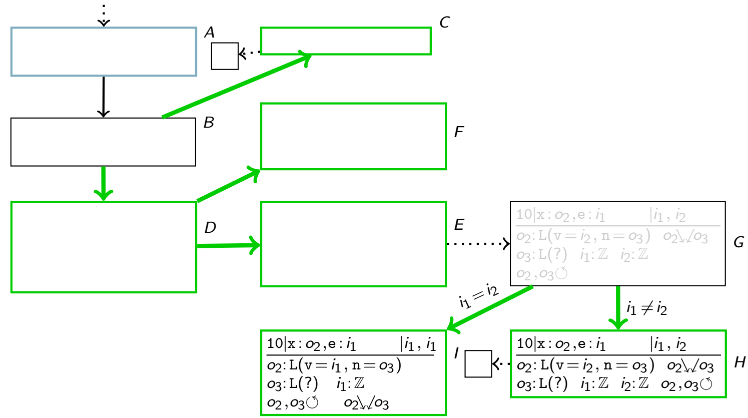



```java
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States G, I, H:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2 \Rightarrow$ Refinement:
  - In I: $i_1 = i_2$
  - In H: $i_1 \neq i_2$

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

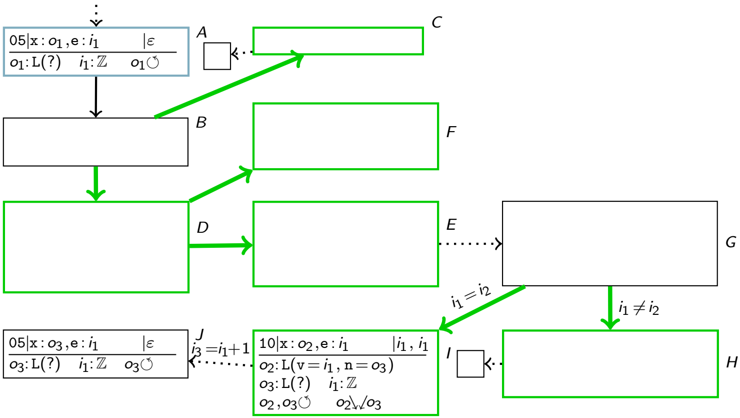



```java
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States G, I, H:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2$ $\Rightarrow$ Refinement:
  - In I: $I_1 = i_2$ (program ends)
  - In H: $i_1 \neq i_2$

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

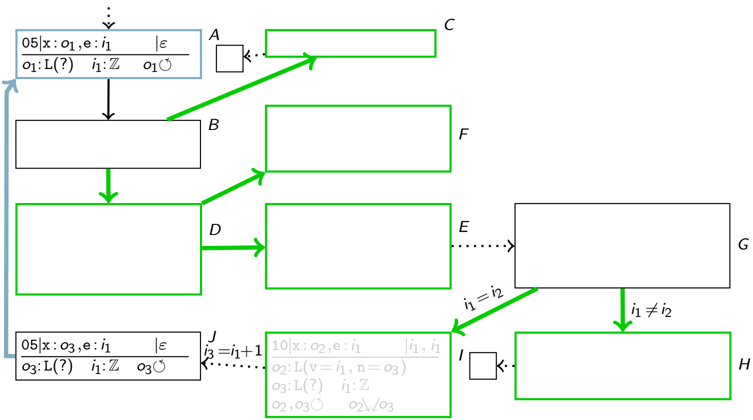



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States G, I, H:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2 \Rightarrow$ Refinement:
  - In I: $i_1 = i_2$ (program ends)
  - In H: $i_1 \neq i_2$
- State J reached by evaluation

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

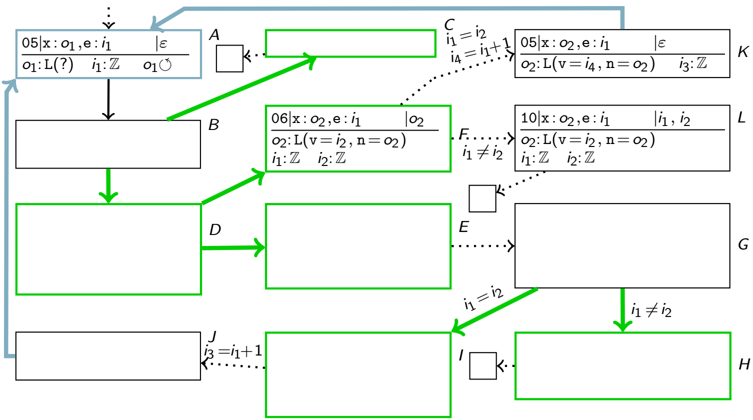



```java
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States G, I, H:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2 \Rightarrow$ Refinement:
  - In I: $I_1 = i_2$ (program ends)
  - In H: $i_1 \neq i_2$
- State J reached by evaluation, represented by (instance of) A

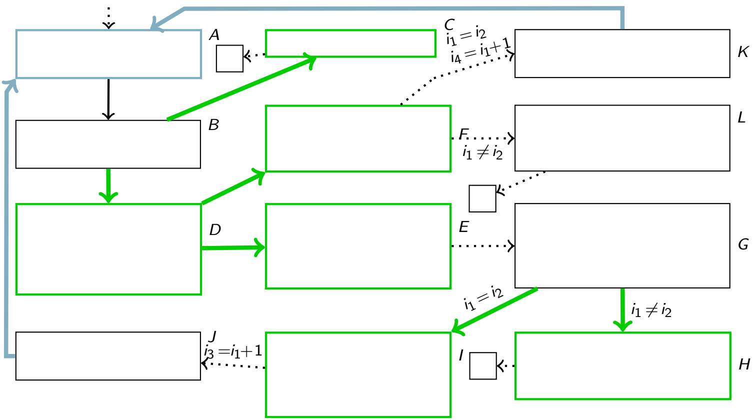
```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



States K, L: Analogous for one-element list

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

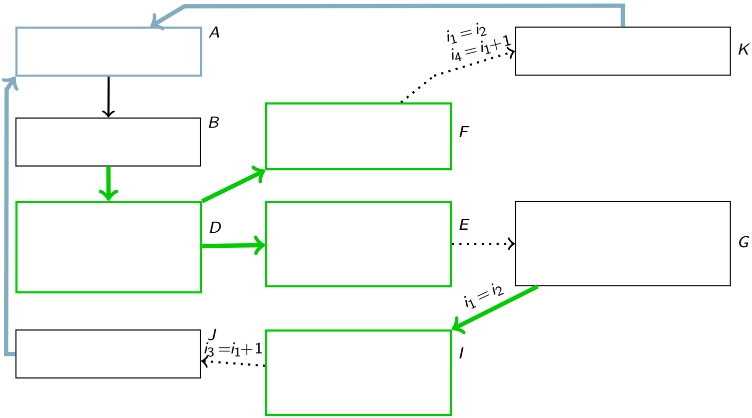
```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



- All leaves program ends ⇒ Graph finished
- How can we prove termination?

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

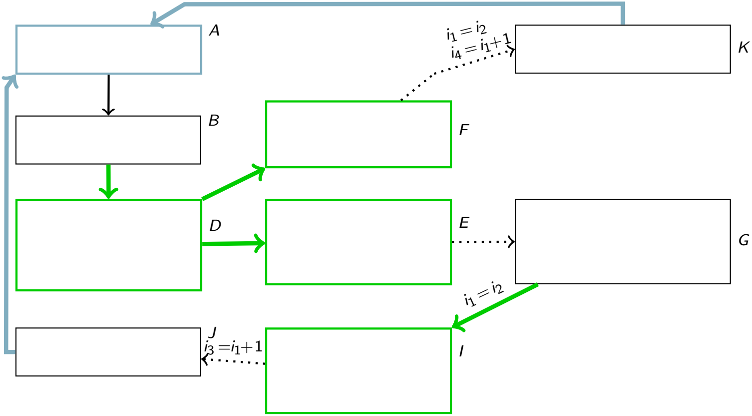
```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



- All leaves program ends ⇒ Graph finished
- How can we prove termination?
- Only consider SCCs

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

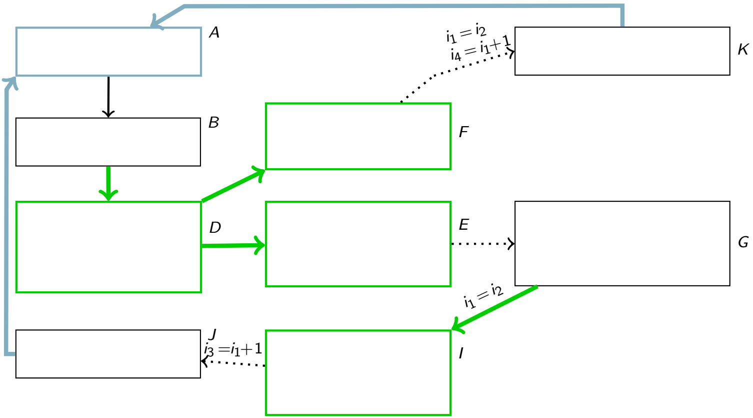


- All leaves program ends ⇒ Graph finished
- How can we prove termination?
- Only consider SCCs

  High-level argument: Number of unvisited elements strictly decreasing

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

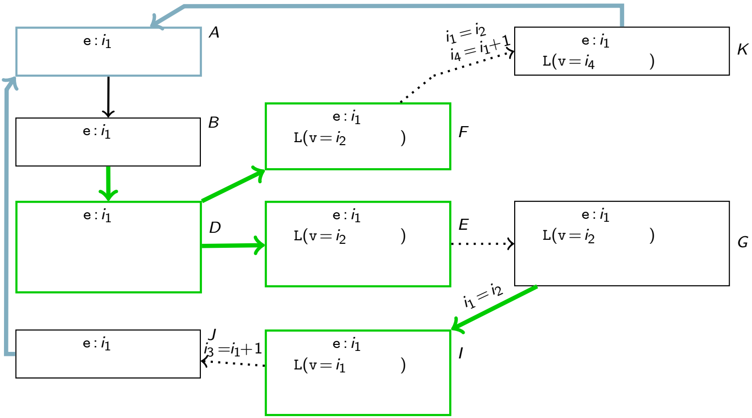
```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



Q: What is an “unvisited element”, formally?

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

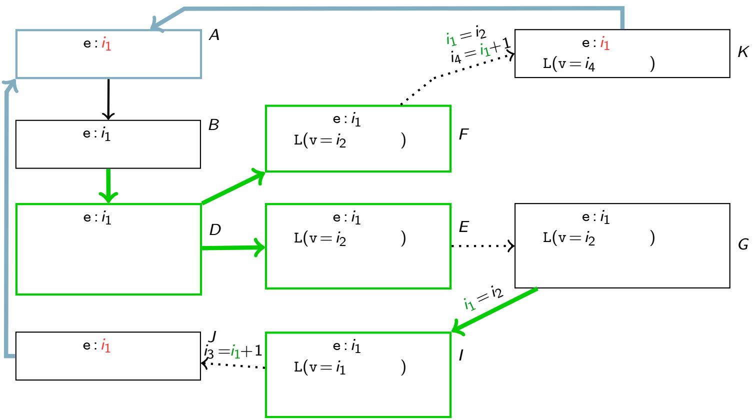


Q: What is an "unvisited element", formally?

A: One with $\mathrm{L.v} = i_1 = \mathrm{e}$

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



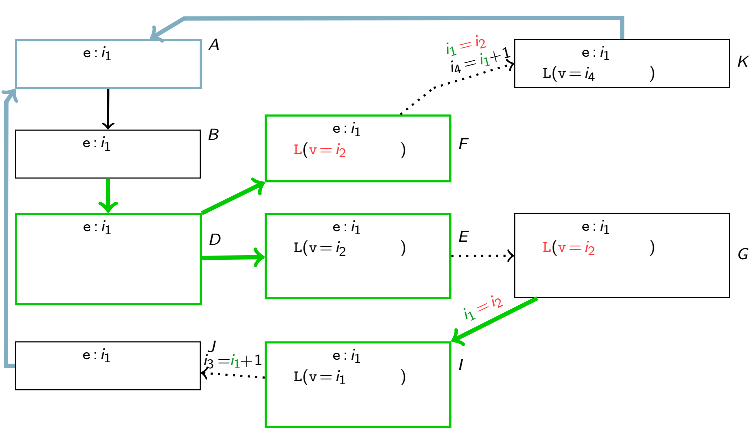
Q: What is an "unvisited element", formally?

A: One with $\mathtt{L.v} = i_1 = \mathtt{e}$

- Automatically finding this relation:
  1. Identify constant c in SCC

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



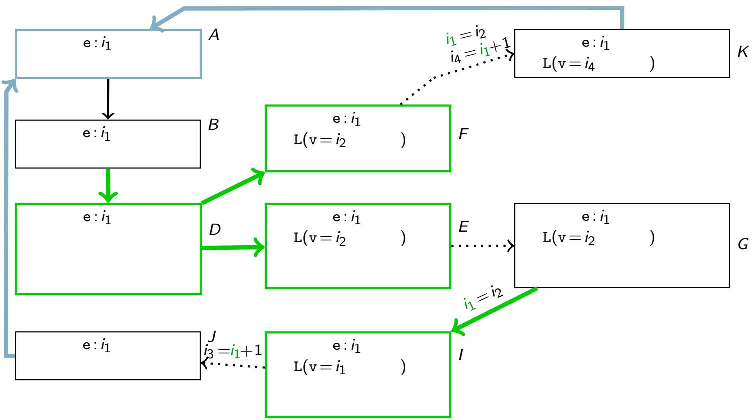
Q: What is an "unvisited element", formally?

A: One with $\mathtt{L.v} = i_1 = \mathtt{e}$

- Automatically finding this relation:
  1. Identify constant c in SCC
  2. Search property $M = \mathtt{C.f} \bowtie c$ checked on all cycles

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```




```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

Q: What is an "unvisited element", formally?

A: One with L.v $= i_1 =$ e

- Automatically finding this relation:
  1. Identify constant c in SCC
  2. Search property $M = \mathtt{C.f} \bowtie \mathtt{c}$ checked on all cycles
- Track number of objects where $\mathtt{C.f} \bowtie \mathtt{c}$ holds ($\#_M$)

```
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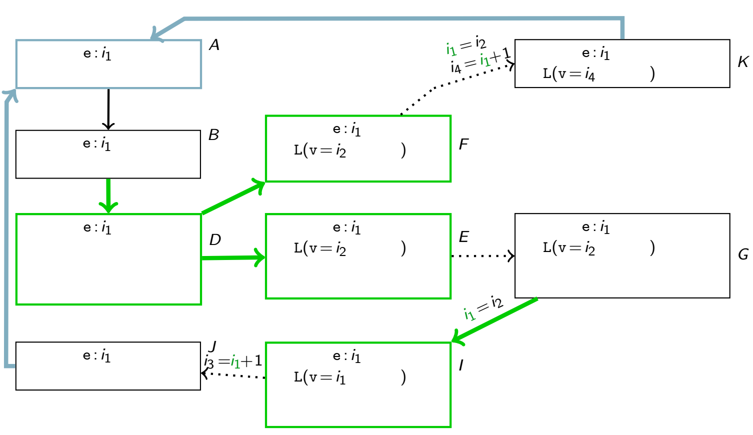

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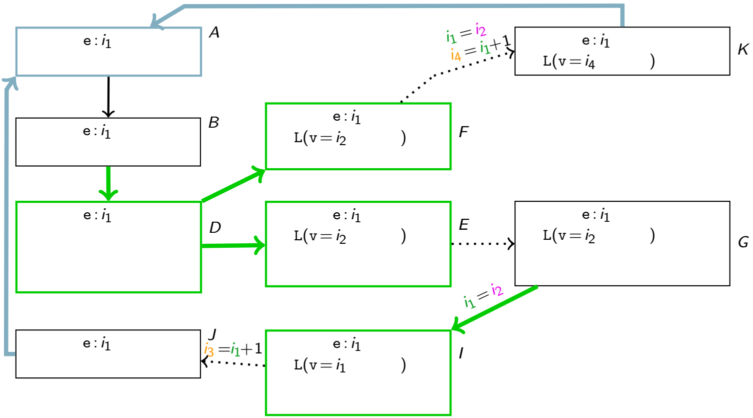

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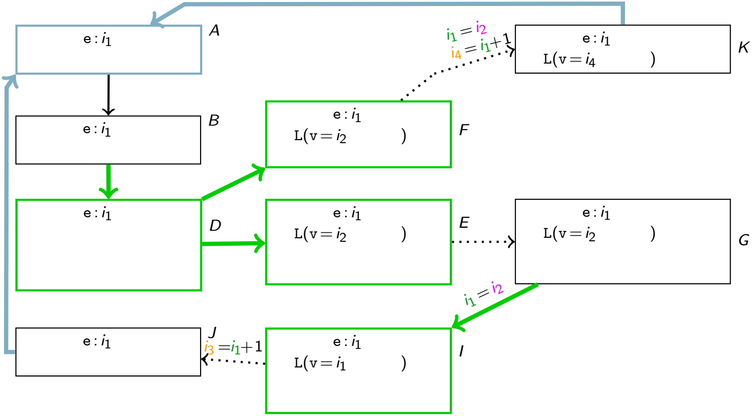

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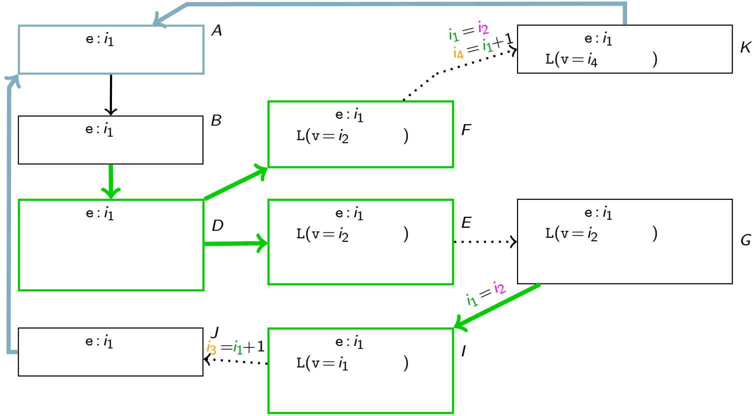

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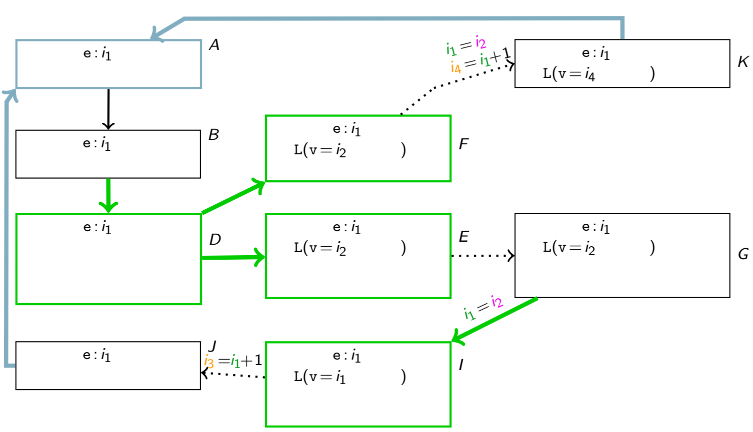

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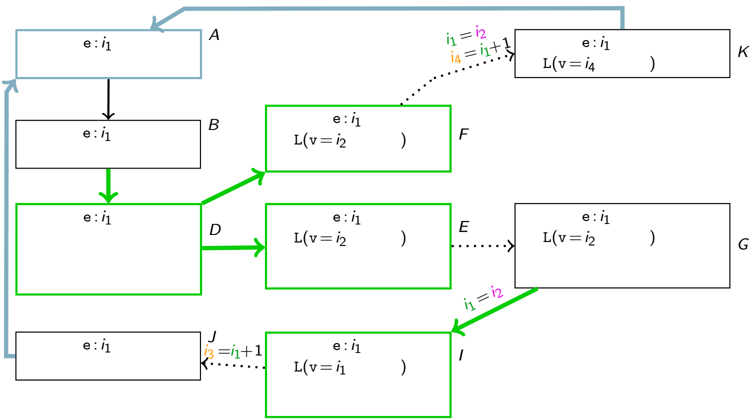

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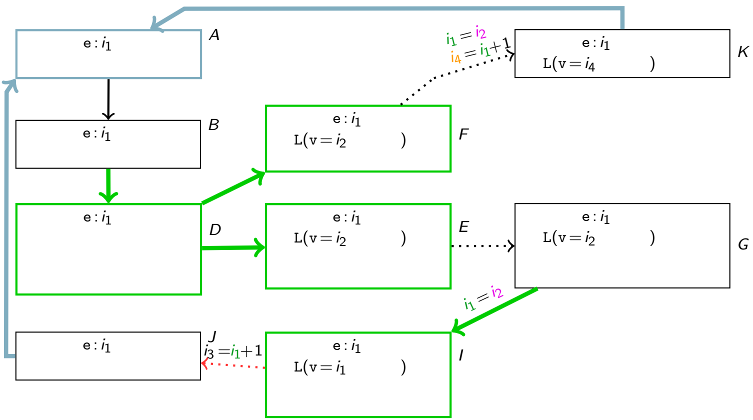

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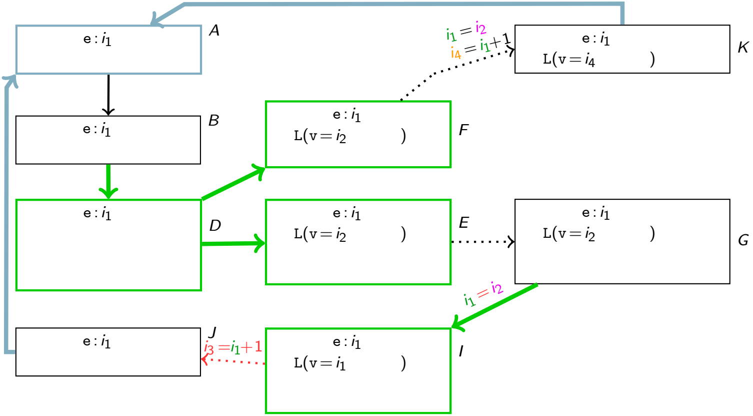

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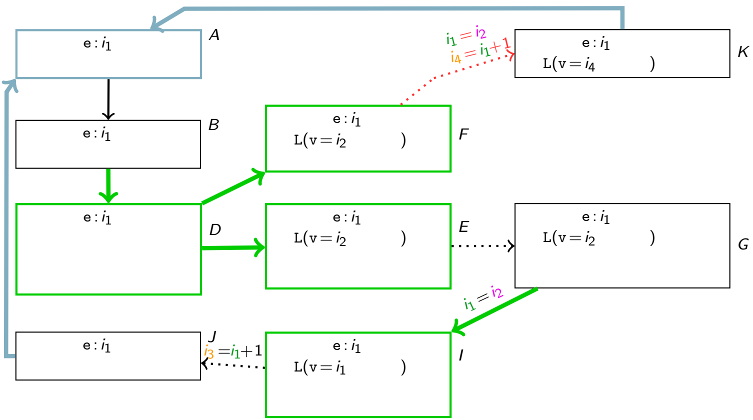

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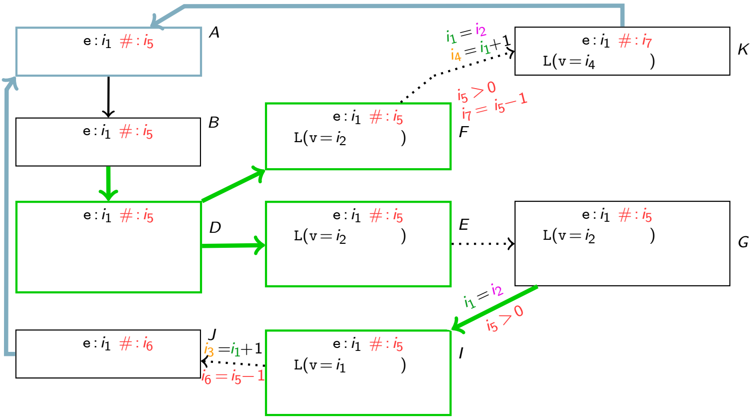

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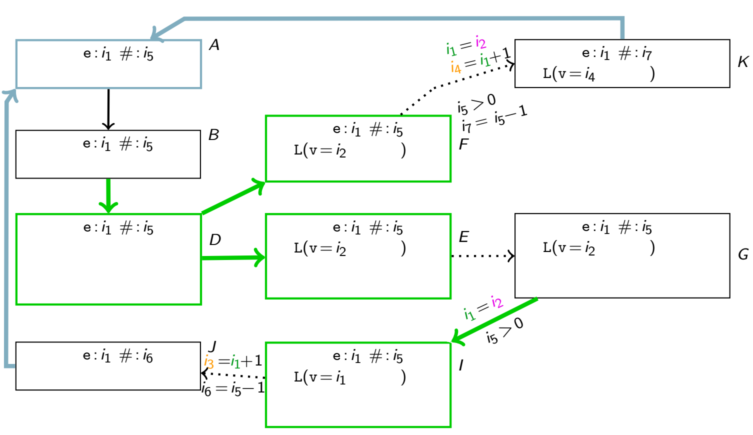

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- Termination proof via TRS now trivial:

$$
\begin{array}{ll}
f_A(\ldots, i_6) \to f_A(\ldots, i_6 - 1) & \mid i_6 > 0 \\
f_A(\ldots, i_7) \to f_A(\ldots, i_7 - 1) & \mid i_7 > 0
\end{array}
$$

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Proving Termination of Heap-Manipulating Java Programs

- Evaluated on collection of 387 programs (including the *Termination Problem Data Base*):

	Yes	**No**	**Fail**	**Run** (s)
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