Automated Termination Proofs for Java Bytecode with Cyclic Data

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Termination Analysis for Imperative Programs

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- Julia & COSTA
  Termination Analysis of Java Bytecode (JBC)
  Fixed abstraction, via Constraint Logic Programs
  (Spoto, Mesnard, Payet, 10)
  (Albert, Arenas, Codish, Genaim, Puebla, Zanardini, 08)

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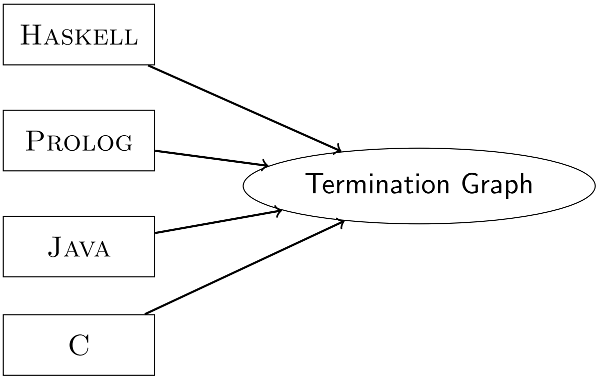

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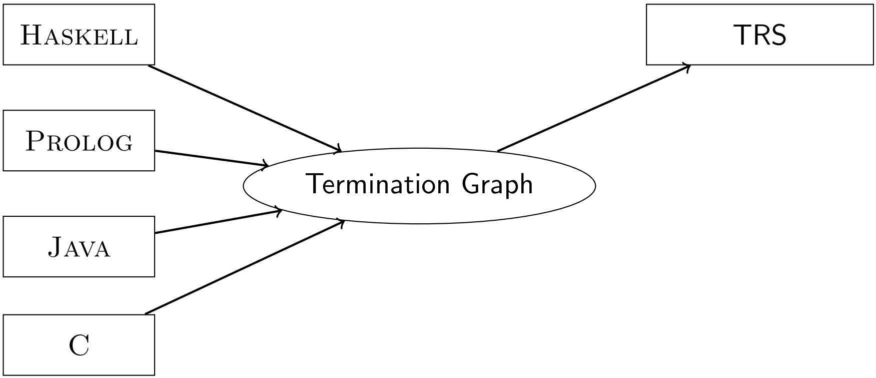

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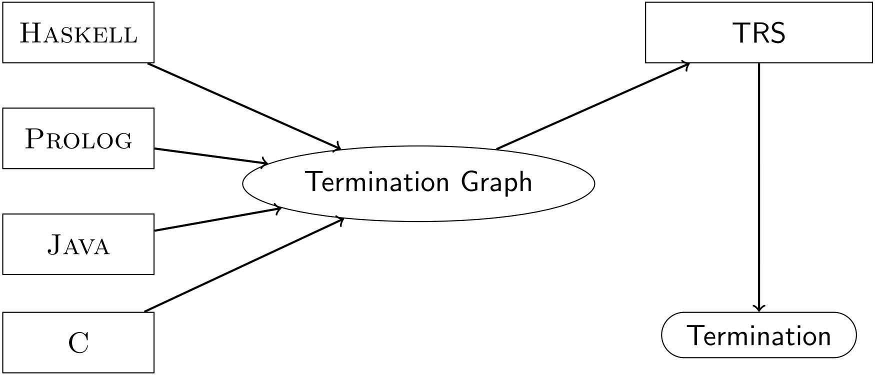
- Term Rewrite Systems (TRSs) generated from Termination Graph
- Prove TRS termination using existing provers



Rewriting-based approach: Advantages

Handling of user-defined data structures:

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public class List {
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- **TRS techniques** search for suitable orders automatically

⇒ Complex orders for user-defined data structures possible

Rewriting-based approach: Challenges

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- Solution:
  1. Find suitable measures on Termination Graph level
  2. Encode (numeric) measures into TRS

Overview

visit the example

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class L {
  int v;    List n;
  static void visit(L x) {
    int e = x.v;
    while (x.v == e) {
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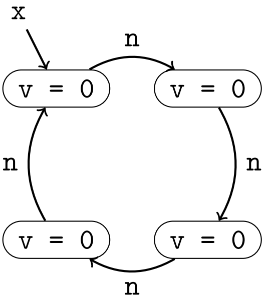

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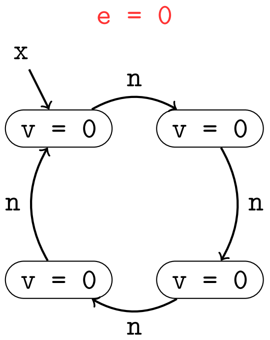

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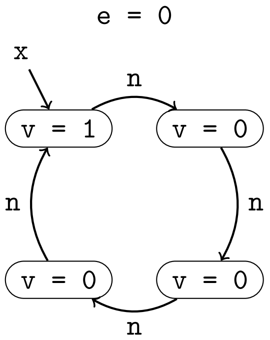

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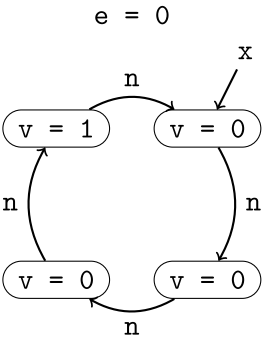

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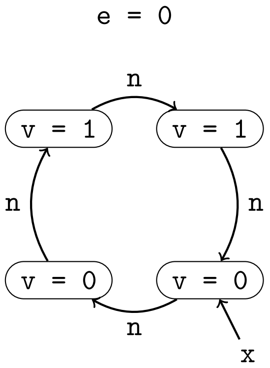

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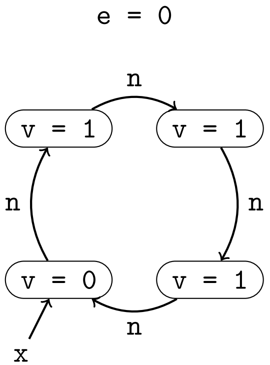

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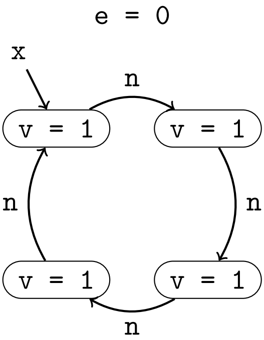

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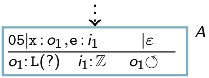
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

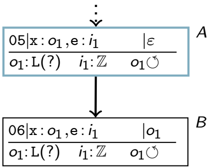


State *A*:

- x some (possibly cyclic) list
- e some integer

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

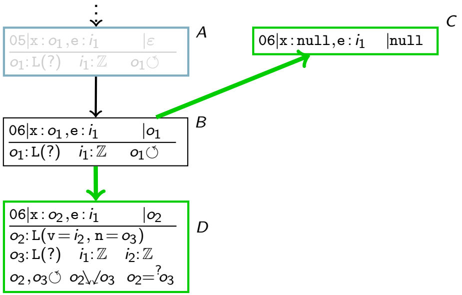


State *B*:

- *Evaluation* between *A* and *B*
- Need field of o_1

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

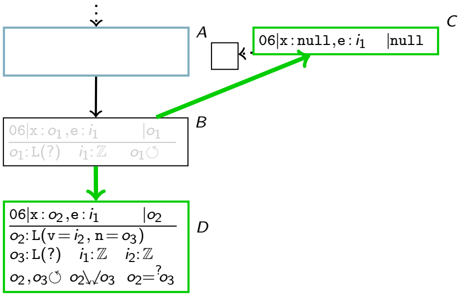



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States *B*, *C*, *D*:

- *Evaluation* between *A* and *B*
- Need field of o_1 ⇒ Refinement:
  - In *C*: o_1 is `null`
  - In *D*: o_1 renamed to o_2, pointing to L-object with successor o_3:
    - o_3 possibly cyclic
    - o_3 possibly equal to o_2 and may reach o_2

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

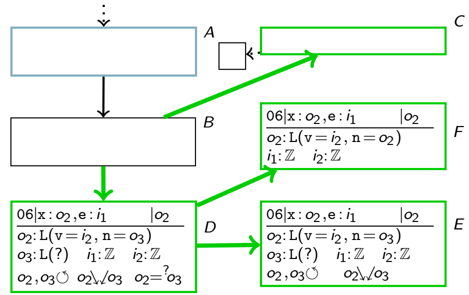


States *B*, *C*, *D*:

- *Evaluation* between *A* and *B*
- Need field of o_1 $\Rightarrow$ Refinement:
  - In *C*: o_1 is `null` (program crashes)
  - In *D*: o_1 renamed to o_2, pointing to L-object with successor o_3:
    - o_3 possibly cyclic
    - o_3 possibly equal to o_2 and may reach o_2

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

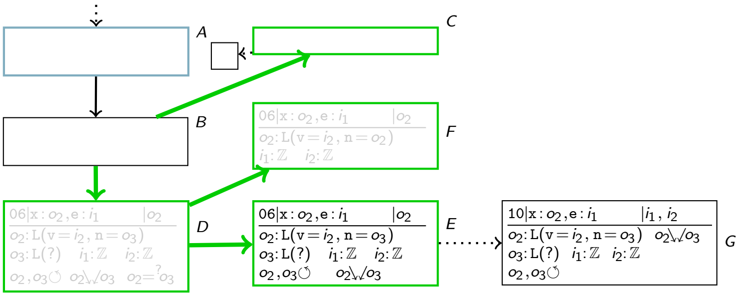


States E, F:

- Need to read field of o_2 $\Rightarrow$ Refinement
  - In E: $o_2 \neq o_3$
  - In F: $o_2 = o_3$

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

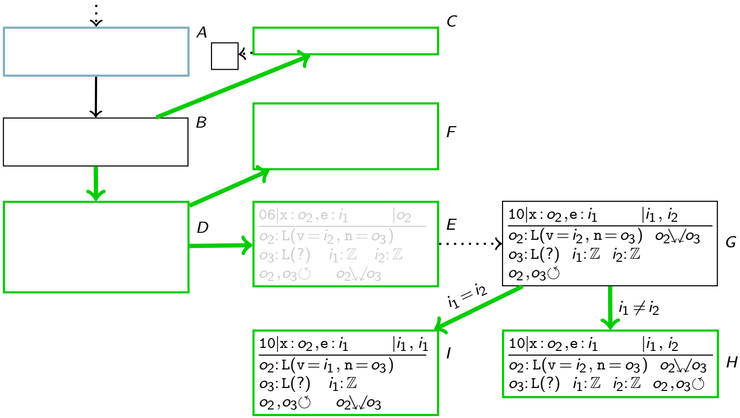


State G:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2$

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```




```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States G, I, H:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2 \Rightarrow$ Refinement:
  - In I: $i_1 = i_2$
  - In H: $i_1 \neq i_2$

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
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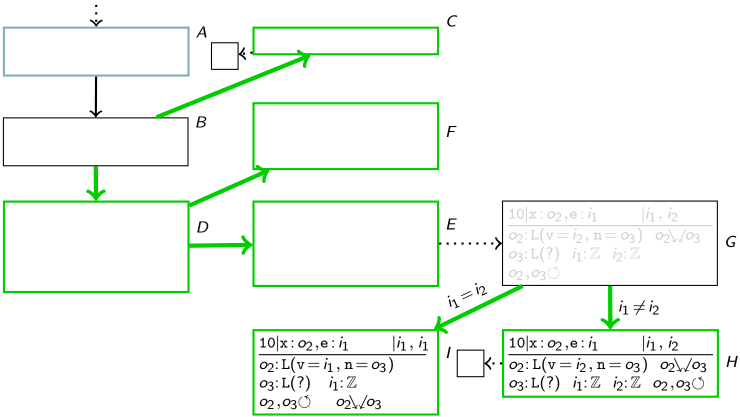
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

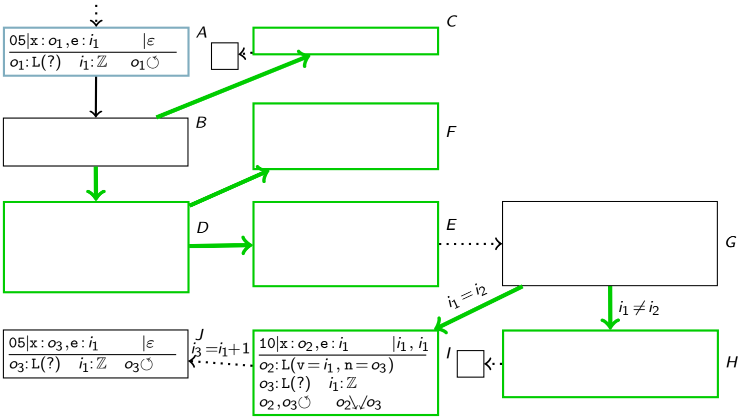



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States G, I, H:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2 \Rightarrow$ Refinement:
  - In I: $i_1 = i_2$ (program ends)
  - In H: $i_1 \neq i_2$

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

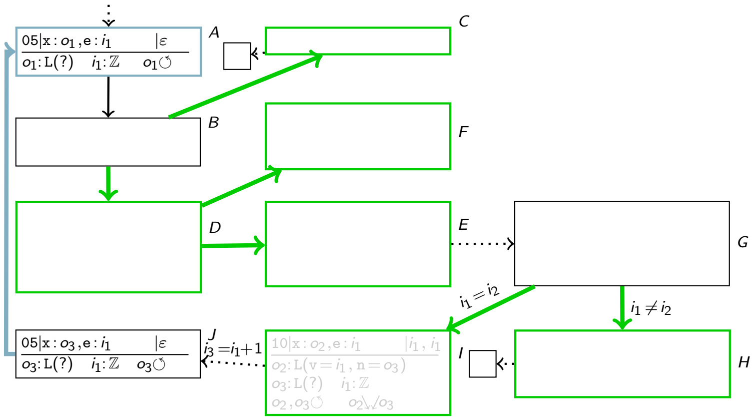


States G, I, H:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2 \Rightarrow$ Refinement:
  - In I: $i_1 = i_2$ (program ends)
  - In H: $i_1 \neq i_2$
- State J reached by evaluation

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```

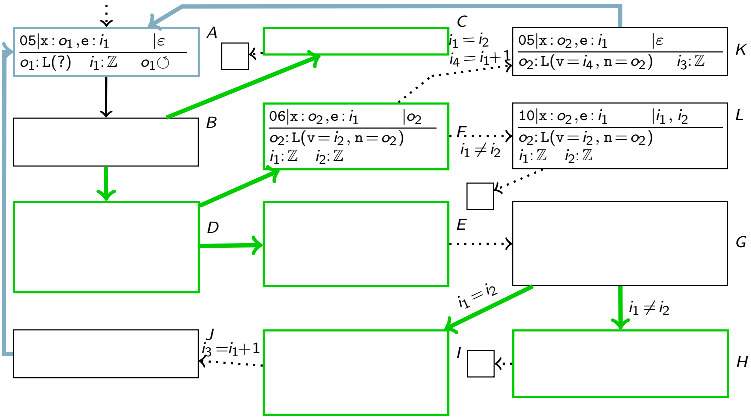



```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

States G, I, H:

- Evaluation: Read v, loaded e
- Need to decide $i_1 \neq i_2 \Rightarrow$ Refinement:
  - In I: $I_1 = i_2$ (program ends)
  - In H: $i_1 \neq i_2$
- State J reached by evaluation, represented by (instance of) A

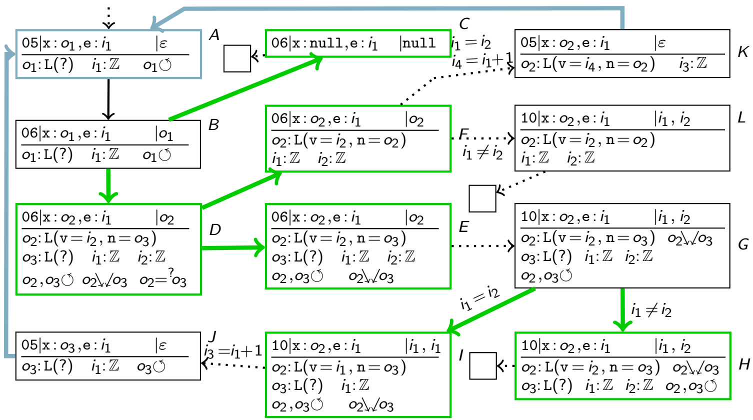
```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



States K, L: Analogous for one-element list

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

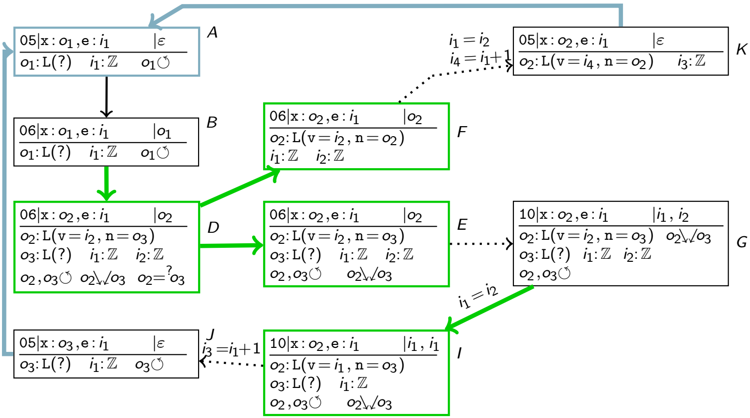
```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



- All leaves program ends $\Rightarrow$ Graph finished
- How can we prove termination?

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

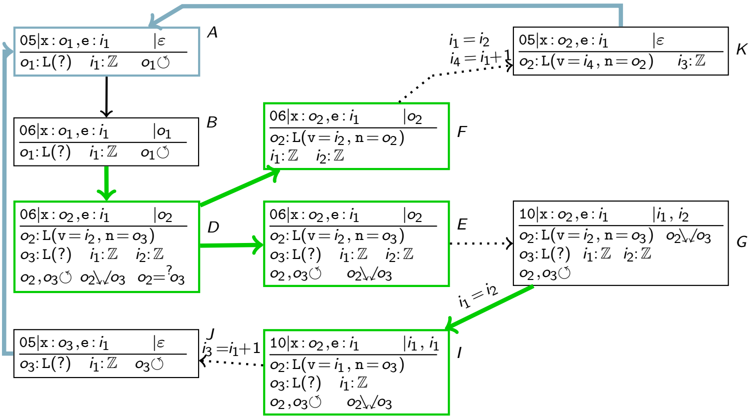
```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



- All leaves program ends ⇒ Graph finished
- How can we prove termination?
- Only consider SCCs

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



- All leaves program ends ⇒ Graph finished
- How can we prove termination?
- Only consider SCCs

High-level argument: Number of unvisited elements strictly decreasing

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
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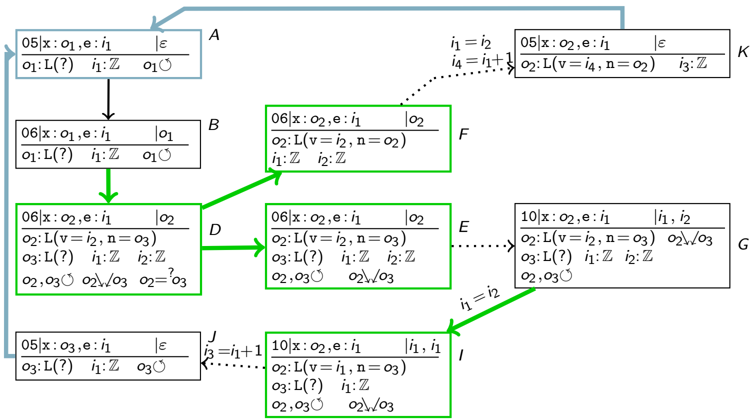
```



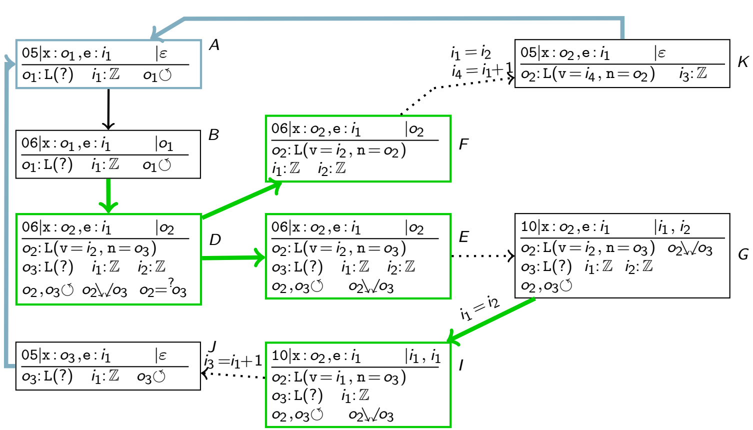
- All leaves program ends ⇒ Graph finished
- How can we prove termination?
- Only consider SCCs

  High-level argument: Number of unvisited elements strictly decreasing

... Let's drag that down to our level!

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

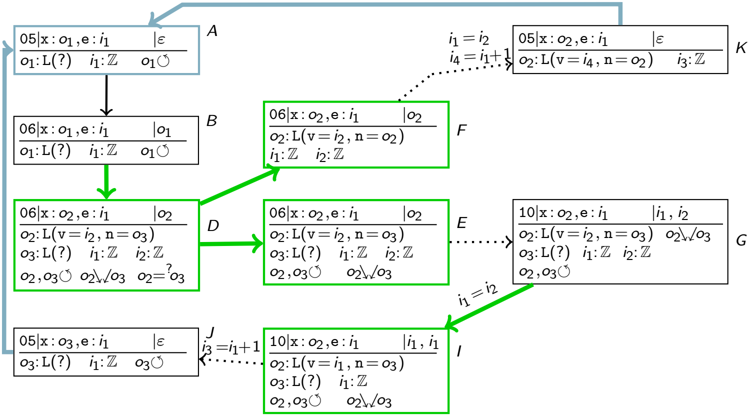
```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



Q: What is an "unvisited element", formally?

```java
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



Q: What is an “unvisited element”, formally?

A: One with $\mathtt{L.v} = i_1 = \mathtt{e}$

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
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16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```
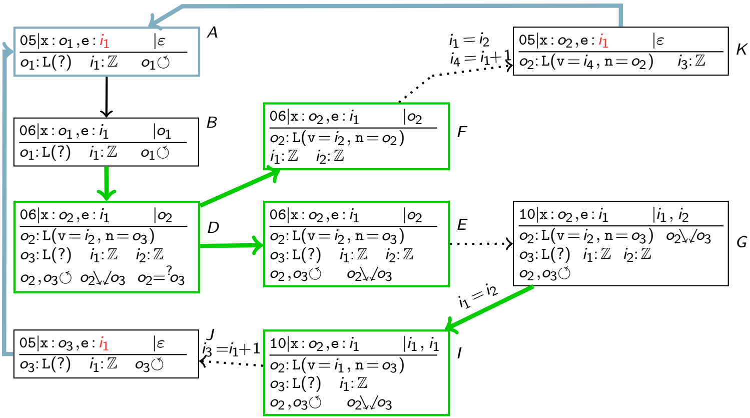



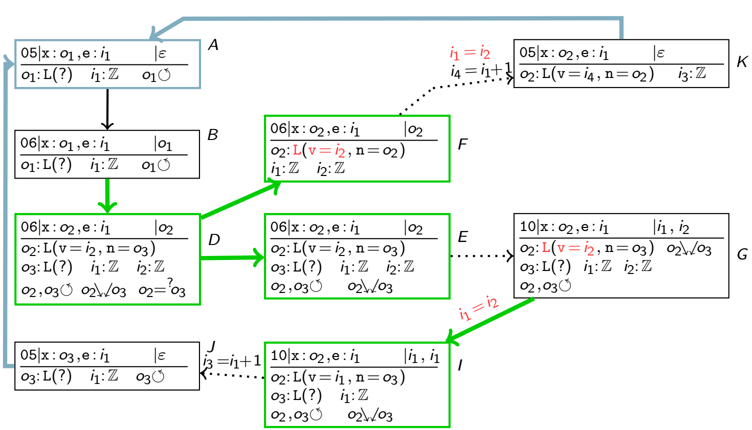
Q: What is an "unvisited element", formally?

A: One with $\mathrm{L.v} = i_1 = \mathrm{e}$

- Automatically finding this relation:
  1. Identify constant c in SCC

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



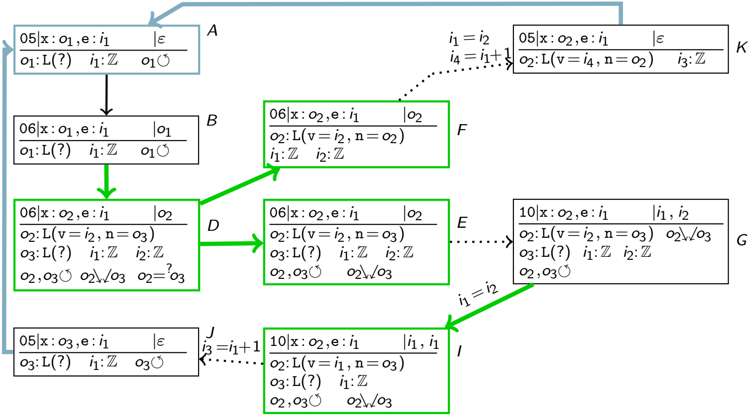

```
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

Q: What is an "unvisited element", formally?

A: One with $\mathtt{L.v} = i_1 = \mathtt{e}$

- Automatically finding this relation:
  1. Identify constant c in SCC
  2. Search property $M = \mathtt{C.f} \bowtie c$ checked on all cycles

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```




```java
static void visit(L x) {
  int e = x.v;
  while (x.v == e) {
    x.v = e + 1;
    x = x.n; }}
```

Q: What is an "unvisited element", formally?

A: One with $\mathtt{L.v} = i_1 = \mathtt{e}$

- Automatically finding this relation:
  1. Identify constant c in SCC
  2. Search property $M = \mathtt{C.f} \bowtie c$ checked on all cycles
- Track number of objects where $\mathtt{C.f} \bowtie c$ holds ($\#_M$)

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
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```

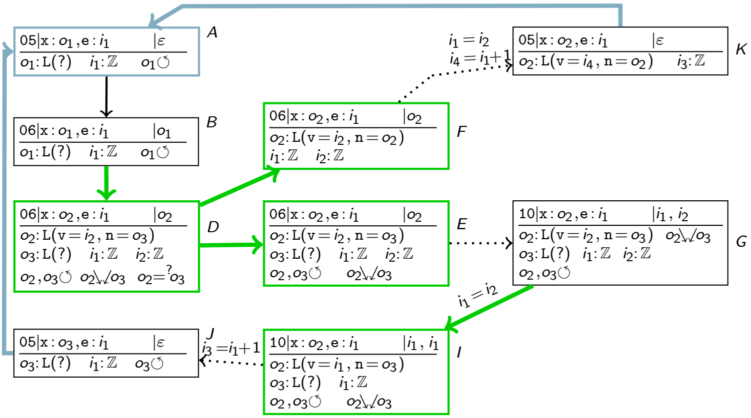

Property $M = \texttt{C.f} \bowtie \mathsf{c}$ (here: $\mathsf{c} = i_1$). When does $\#_M$ change?

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
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```

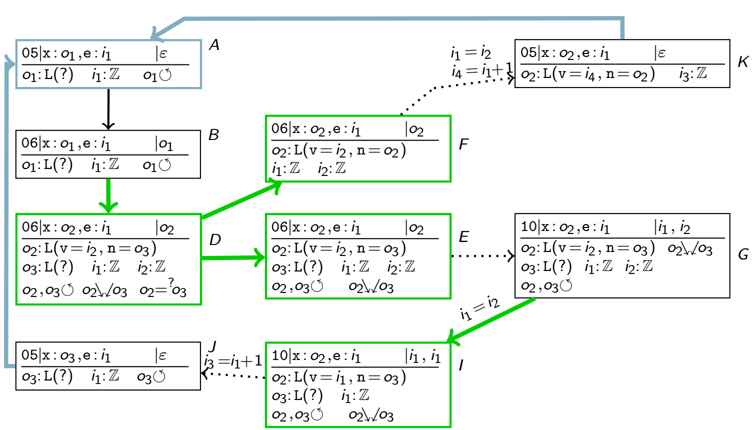

Property $M = \texttt{C.f} \bowtie \mathsf{c}$ (here: $\mathsf{c} = i_1$). When does $\#_M$ change?

- C.f written (old value u, new value w):

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
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09: iload_1
10: if_icmpne 28
13: aload_0
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```

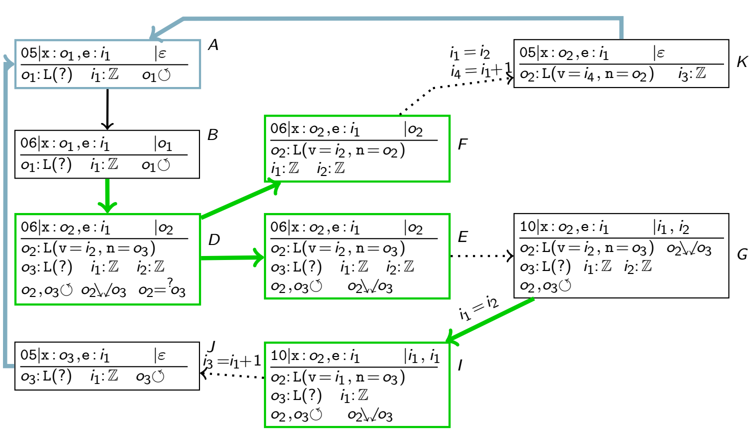

Property $M = \texttt{C.f} \bowtie \texttt{c}$ (here: $\texttt{c} = i_1$). When does $\#_M$ change?

- C.f written (old value u, new value w):
  - $u \bowtie c \wedge \neg w \bowtie c$ tautology $\Rightarrow \#_M$ decremented by 1

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
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28: return
```

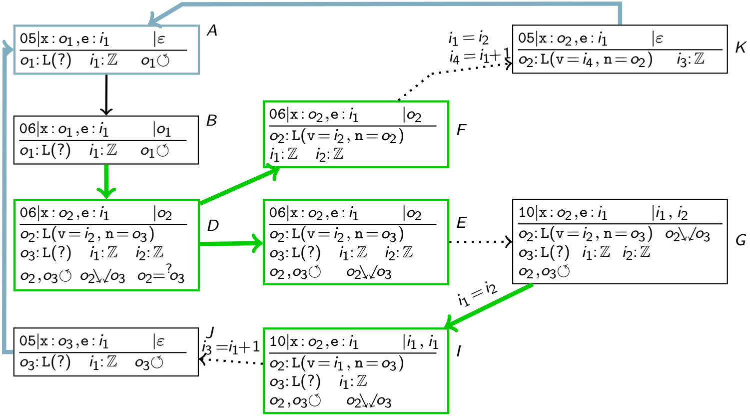

Property $M = \texttt{C.f} \bowtie c$ (here: $c = i_1$). When does $\#_M$ change?

- `C.f` written (old value u, new value w):
  - $u \bowtie c \wedge \neg w \bowtie c$ tautology $\Rightarrow$ $\#_M$ decremented by 1
  - $u \bowtie c \leftrightarrow w \bowtie c$ tautology $\Rightarrow$ $\#_M$ unchanged

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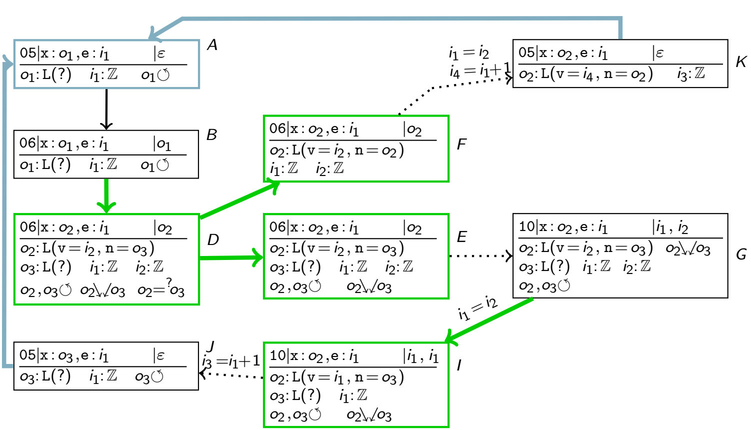
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



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25: goto 5
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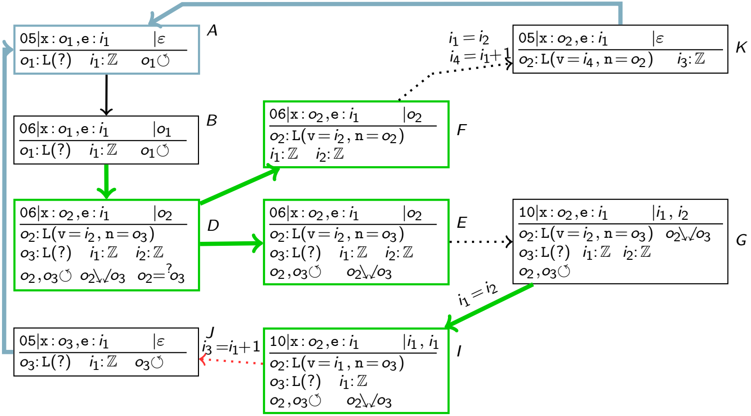

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  - $u \bowtie c \leftrightarrow w \bowtie c$ tautology $\Rightarrow \#_M$ unchanged
  - Otherwise: $\#_M$ incremented by 1.

In example: $I \to J$: i_1 old, i_3 new

$\Rightarrow$ $i_1 = i_1 \wedge \neg i_3 = i_1$

```
00: aload_0
01: getfield v
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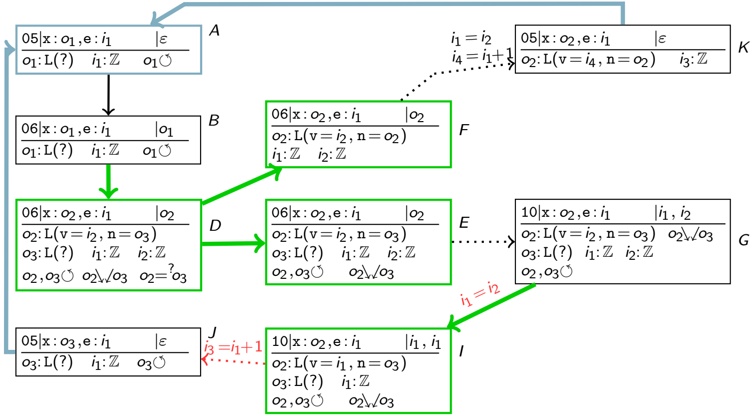
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



Property $M = \mathtt{C.f} \bowtie c$ (here: $c = i_1$). When does $\#_M$ change?

- `C.f` written (old value u, new value w):
  - $u \bowtie c \wedge \neg w \bowtie c$ tautology $\Rightarrow \#_M$ decremented by 1
  - $u \bowtie c \leftrightarrow w \bowtie c$ tautology $\Rightarrow \#_M$ unchanged
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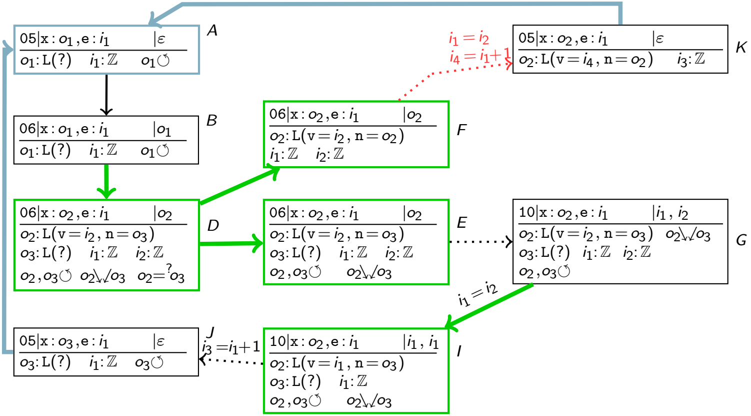
In example: $I \to J$: i_1 old, i_3 new

$\Rightarrow i_1 = i_2 \wedge i_3 = i_1 + 1 \ \rightarrow\ i_1 = i_1 \wedge \neg i_3 = i_1$

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



Property $M = \texttt{C.f} \bowtie \texttt{c}$ (here: $\texttt{c} = i_1$). When does $\#_M$ change?

- $\texttt{C.f}$ written (old value u, new value w):
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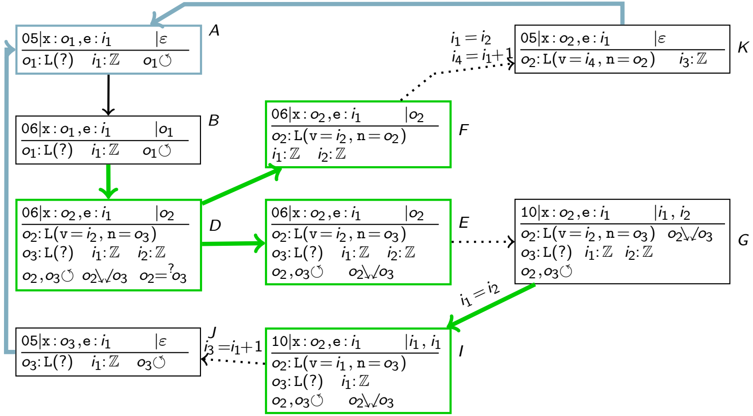
  - $u \bowtie c \leftrightarrow w \bowtie c$ tautology $\Rightarrow \#_M$ unchanged
  - Otherwise: $\#_M$ incremented by 1.

In example: $F \to K$: i_1 old, i_4 new

$\Rightarrow i_1 = i_2 \wedge i_4 = i_1 + 1 \;\to\; i_1 = i_1 \wedge \neg i_4 = i_1$

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
15: iconst_1
16: iadd
17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
28: return
```



Property $M = \texttt{C.f} \bowtie \texttt{c}$ (here: $\texttt{c} = i_1$). When does $\#_M$ change?

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  - Otherwise: $\#_M$ incremented by 1.

- New `L` object is created: Same for default value

```
00: aload_0
01: getfield v
04: istore_1
05: aload_0
06: getfield v
09: iload_1
10: if_icmpne 28
13: aload_0
14: iload_1
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17: putfield v
20: aload_0
21: getfield n
24: astore_0
25: goto 5
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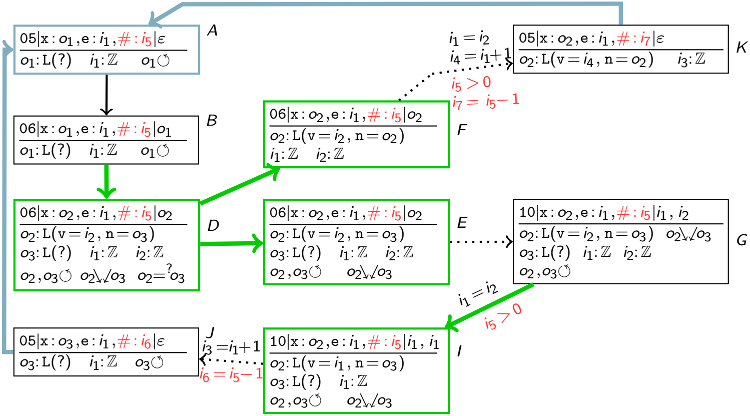
```



- Add variable for counter to states, changes to edges
- Require counter > 0 at checks

```
00: aload_0
01: getfield v
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21: getfield n
24: astore_0
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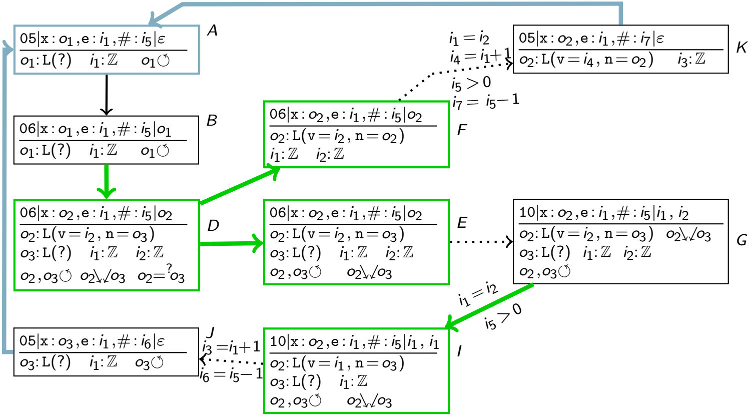
```



- Add variable for counter to states, changes to edges
- Require counter > 0 at checks
- Termination proof via TRS now trivial:

$$f_A(\dots, i_5) \to f_I(\dots, i_5) \quad | \; i_5 > 0$$

$$f_I(\dots, i_5) \to f_J(\dots, i_5 - 1)$$

$$f_J(\dots, i_6) \to f_A(\dots, i_6)$$

$$f_A(\dots, i_5) \to f_F(\dots, i_5)$$

$$f_F(\dots, i_5) \to f_K(\dots, i_5 - 1) \quad | \; i_5 > 0$$

$$f_F(\dots, i_7) \to f_A(\dots, i_7)$$

Overview

The iterate example

```
00: aload_0        #load this
01: getfield n     #get n from this
04: astore_1       #store to x
05: aload_1        #load x
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```
class L {
  L n;
  void iterate() {
    L x = this.n;
    while (x != this)
      x = x.n; }}
```

1. Keep first element in `this`
2. Iterate until reaching it again

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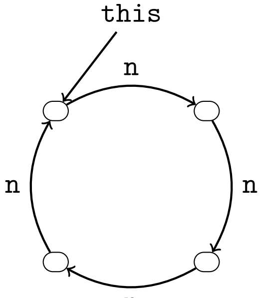

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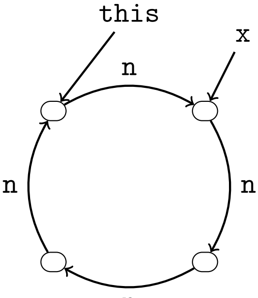

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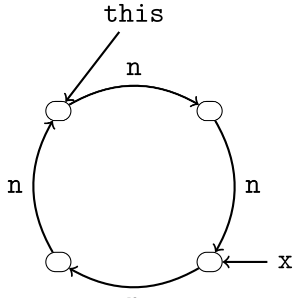

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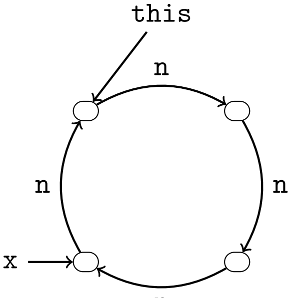

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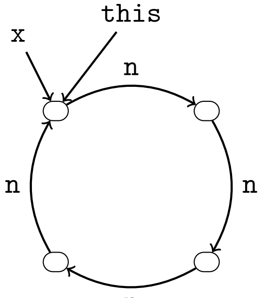

Definite Reachability

```
00: aload_0        #load this
01: getfield n     #get n from this
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New annotation: Definite reachability $\overset{F}{\dashrightarrow}!$

- $o \overset{F}{\dashrightarrow}! o' \Rightarrow$
  All paths from o using fields from F reach o'

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  Allow (not enforce) sharing/shapes

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15: goto 05
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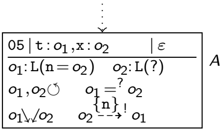
- $o \overset{F}{\dashrightarrow}^{!} o' \Rightarrow$
  All paths from o using fields from F reach o'
- $=^?$, $\bigvee\!\!\bigvee$, $\circlearrowleft$ *extending* annotations:
  Allow (not enforce) sharing/shapes
- $\overset{F}{\dashrightarrow}^{!}$ *restricting* annotation:
  Enforce sharing

$$\begin{array}{l} 05 \mid \mathtt{t}: o_1, \mathtt{x}: o_2 \mid \varepsilon \\ \hline o_1 : \mathtt{L}(\mathtt{n} = o_2) \quad o_2 : \mathtt{L}(?) \\ o_1, o_2 \circlearrowleft \quad o_1 =^? o_2 \\ o_1 \bigvee\!\!\bigvee o_2 \quad o_2 \overset{\{\mathtt{n}\}}{\dashrightarrow}^{!} o_1 \end{array}$$

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```

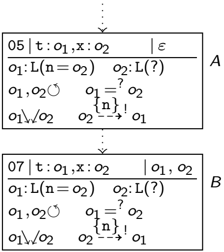


State *A*:

- t some definitely cyclic list
- x second element in list

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```



State *A*:

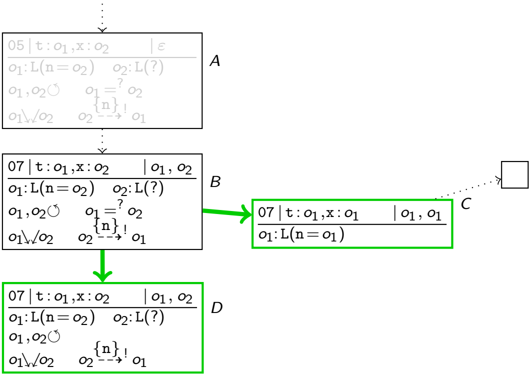
- t some definitely cyclic list
- x second element in list

State *B*:

- First equals second element?

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```




```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

State *A*:

- t some definitely cyclic list
- x second element in list

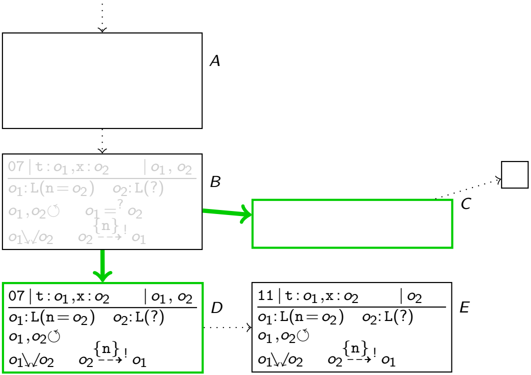
State *B*:

- First equals second element?

⇒ Refinement

- In *C*: References equal (↷ program ends)
- In *D*: References not equal

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```

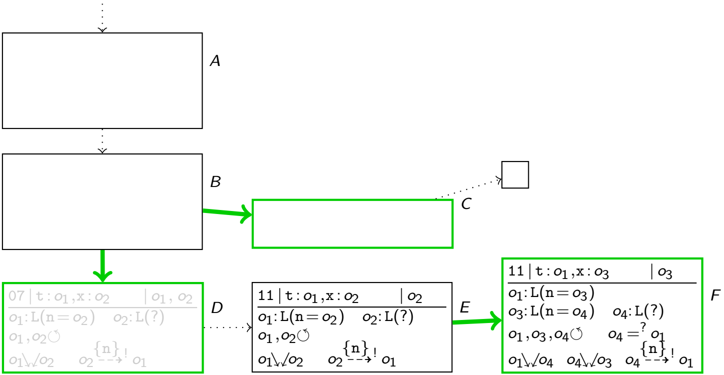


State *E*:

- Access to unknown object o_2

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```

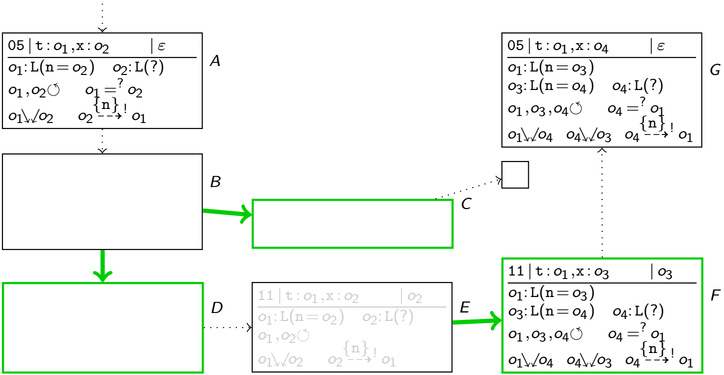



```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

States E, F:

- Access to unknown object o_2
  $\Rightarrow$ Refinement
- Case $o_2 = \texttt{null}$ not possible (implies o_2 not reaching o_1)

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```



State G:

- Same program position as A $\Rightarrow$ Instantiate

  In A: $\texttt{this} = o_1 \xrightarrow{\texttt{n}} o_2 = \texttt{x}$

  In G: $\texttt{this} = o_1 \xrightarrow{\texttt{n}} o_3 \xrightarrow{\texttt{n}} o_4 = \texttt{x}$

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
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10: aload_1
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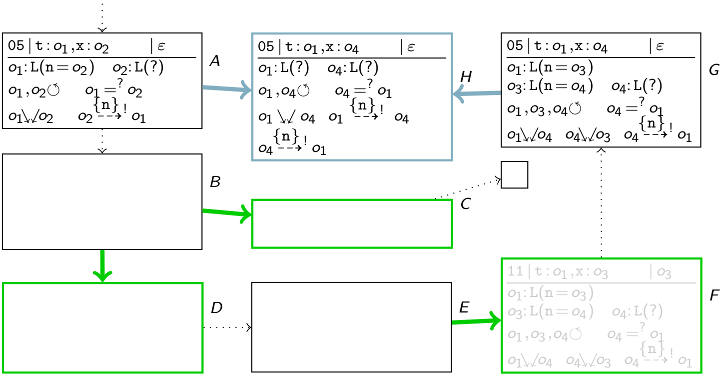
15: goto 05
18: return
```



States G, H:

- Same program position as $A \Rightarrow$ Instantiate

  In A: $\texttt{this} = o_1 \xrightarrow{\texttt{n}} o_2 = \texttt{x}$

  In G: $\texttt{this} = o_1 \xrightarrow{\texttt{n}} o_3 \xrightarrow{\texttt{n}} o_4 = \texttt{x}$

$\Rightarrow$ In H: Abstract to $\texttt{this} = o_1 \overset{\{\texttt{n}\}}{\dashrightarrow}! \; o_4 = \texttt{x}$

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

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04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```

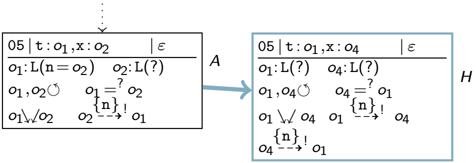

States G, H:

- Same program position as $A \Rightarrow$ Instantiate

  In A: $\texttt{this} = o_1 \stackrel{\texttt{n}}{\rightarrow} o_2 = \texttt{x}$

  In G: $\texttt{this} = o_1 \stackrel{\texttt{n}}{\rightarrow} o_3 \stackrel{\texttt{n}}{\rightarrow} o_4 = \texttt{x}$

$\Rightarrow$ In H: Abstract to $\texttt{this} = o_1 \stackrel{\{\texttt{n}\}}{\dashrightarrow}{}^{!}\ o_4 = \texttt{x}$

- Restart construction from more general state

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
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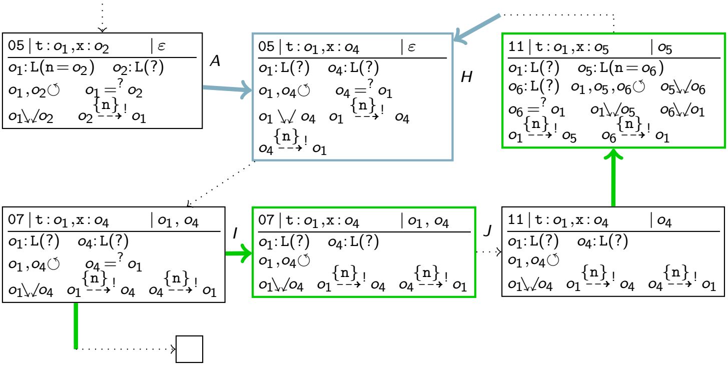

States G, H:

- Same program position as $A \Rightarrow$ Instantiate

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$\Rightarrow$ In H: Abstract to $\mathtt{this} = o_1 \overset{\{\mathtt{n}\}}{\dashrightarrow}{}^{!}\, o_4 = \mathtt{x}$

- Restart construction from more general state

States I, J, K, L: As before

```
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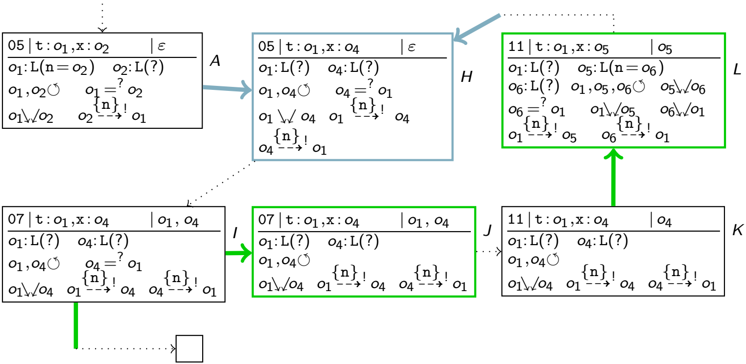

Proving termination with $R = o \overset{F}{\dashrightarrow}{}^{!}\, o'$:

1. Associate length ℓ_R with each R

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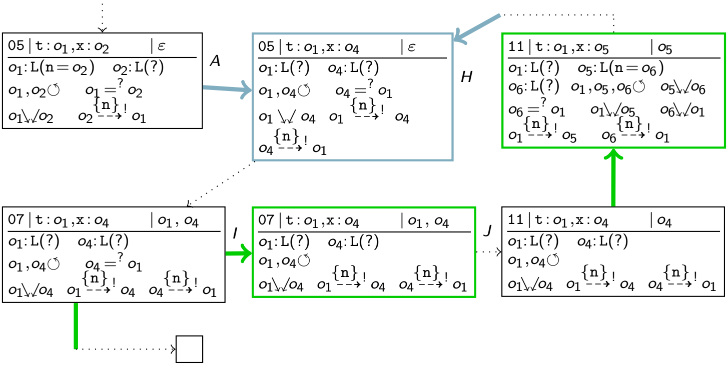

Proving termination with $R = o \overset{F}{\dashrightarrow}^{!} o'$:

1. Associate length ℓ_R with each R
2. In refinements:
   - $o =^{?} o'$ resolved to $o \neq o'$: $\ell_R > 0$

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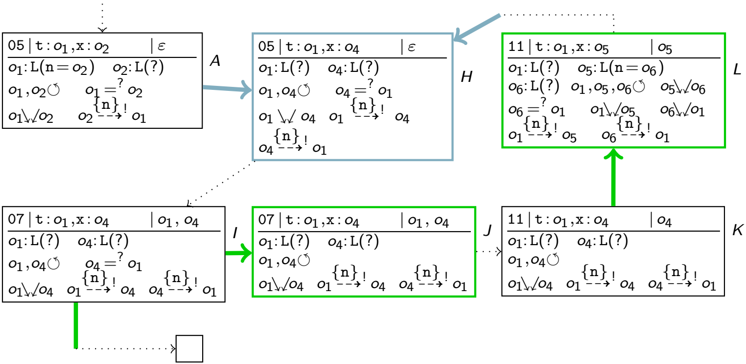

Proving termination with $R = o \overset{F}{\dashrightarrow}! \; o'$:

1. Associate length ℓ_R with each R
2. In refinements:
   - $o =^? o'$ resolved to $o \neq o'$: $\ell_R > 0$
   - $\tilde{o}$ new F-child of o: $\ell_{R'} = \ell_R - 1$ (for $R' = \tilde{o} \overset{F}{\dashrightarrow}! \; o'$)

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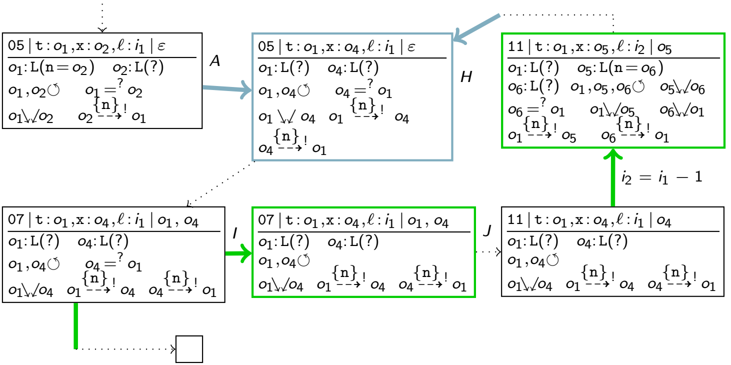

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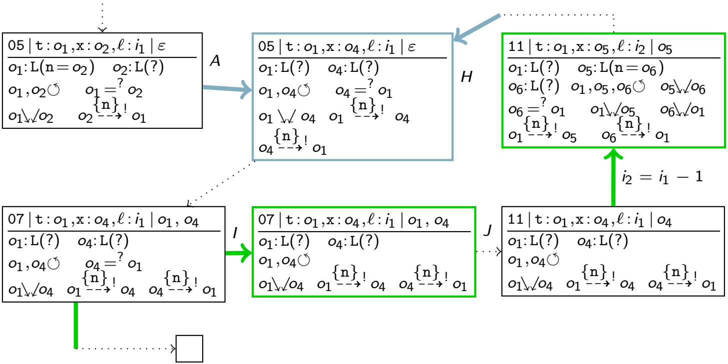

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```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

Resulting TRS:

$$f(\ldots, \ell_{o_4 \overset{\{n\}}{\dashrightarrow}! o_1}) \to f(\ldots, \ell_{o_4 \overset{\{n\}}{\dashrightarrow}! o_1} - 1) \quad | \; \ell_{o_4 \overset{\{n\}}{\dashrightarrow}! o_1} > 0$$

Automated Termination Proofs for Java Bytecode with Cyclic Data

- Implemented in AProVE for full single-threaded Java

Automated Termination Proofs for Java Bytecode with Cyclic Data

- Implemented in AProVE for full single-threaded JAVA
- Evaluated on collection of 387 programs (including the *Termination Problem Data Base*):

	Y	N	F	T	R	Y	N	F	T	R
AProVE	267	81	11	28	9.5	51	0	6	3	15.8
AProVE '11	225	81	45	36	11.4	23	0	29	8	18.3
Julia	191	22	174	0	4.7	32	0	28	0	8.2
COSTA	160	0	181	46	11.0	29	0	5	26	30.4

all examples (first Y–R group) LinkedList + HashMap (second Y–R group)

Automated Termination Proofs for Java Bytecode with Cyclic Data

- Implemented in AProVE for full single-threaded JAVA
- Evaluated on collection of 387 programs (including the *Termination Problem Data Base*):

	all examples					`LinkedList` + `HashMap`				
	Y	**N**	**F**	**T**	**R**	**Y**	**N**	**F**	**T**	**R**
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- Termination depending on cyclic data requires early abstraction