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LEHR- UND FORSCHUNGSGEBIET INFORMATIK II
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Exercises *Functional Programming* – Sheet 5

Solutions will be collected until Thursday, June 14, 2001.

Exercise 1 (2 points)

Prove that if all $\sqsubseteq_{D_i}$ are reflexive orders, then $\sqsubseteq_{D_1 \times ... \times D_n}$ is also a reflexive order.

Exercise 2 (2 points)

Find all monotonic extensions of the following functions (given their usual interpretation):

(a) $\neg : \mathbb{B} \to \mathbb{B}$

(b) $\wedge, \vee : \mathbb{B}^2 \to \mathbb{B}$

(c) $-, + : \mathbb{N}^2 \to \mathbb{N}$

Exercise 3 (2 points)

Let $(D_1, \sqsubseteq), ..., (D_n, \sqsubseteq), (E, \sqsubseteq)$ be partial orders, $n \geq 1$. A function $g : D_1 \times ... \times D_n \to E$ is called *monotonic in the i-th argument* if whenever $d_i, d'_i \in D_i$ such that $d_i \sqsubseteq d'_i$, then for all

$$(d_1, ..., d_{i-1}, d_{i+1}, ..., d_n) \in D_1 \times ... \times D_{i-1} \times D_{i+1} \times ... \times D_n$$

one has:

$$g(d_1, ..., d_{i-1}, d_i, d_{i+1}, ..., d_n) \sqsubseteq g(d_1, ..., d_{i-1}, d'_i, d_{i+1}, ..., d_n)$$

Show that g is monotonic (in the usual sense) iff it is monotonic in every argument.

Exercise 4 (2 points)

Prove the following:

Let $(D, \sqsubseteq_D)$ be a flat order with the least element $\bot_D$, let $(D_i, \sqsubseteq_{D_i})_{1 \leq i \leq n}$ be an family of flat orders with the least elements $\bot_{D_i}$ and $\phi \in \{D_1 \times ... \times D_n \to D\}$ with $n \geq 1$. Then ϕ is monotonic iff for all $i \in \{1, ..., n\}$ and all $(d_1, ..., d_n) \in D_1 \times ... \times D_n$ either $\phi(d_1, ..., d_{n-1}, \bot_{D_i}, d_{n+1}, ..., d_n) = \bot_D$ or $\phi(d_1, ..., d_{n-1}, \bot_{D_i}, d_{n+1}, ..., d_n) = \phi(d_1, ..., d_n)$.

Exercise 5 (2 points)

Let $D_1, ..., D_n, D$ be domains where $D_1, ..., D_n$ are flat. Let $f : D_1 \times ... \times D_n \to D$ be strict. Show that f is monotonic.

Exercise 6 (2 points)

(a) Present the partial order $(\mathbb{B}_\perp \to \mathbb{B}_\perp, \sqsubseteq)$ graphically. How many elements does it have? How many elements are maximal?

(b) How many elements does the partial order $(\mathbb{B}_\perp^2 \to \mathbb{B}_\perp, \sqsubseteq)$ have?

Exercise 7 (1 point)

Show that each chain of a partial order has at most one least upper bound.

Exercise 8

Let D_1 and D_2 be domains and let S be a set of functions from D_1 to D_2. For all $i \in D_1$ let $S_i = \{f(i) \mid f \in S\}$. Show that

- $\sqcup S$ exists iff $\sqcup S_i$ exists for all $i \in D_1$
- if $\sqcup S$ exists then $(\sqcup S)(i) = \sqcup S_i$

Exercise 9 (2 points)

Consider the non strict function `cond` defined as follows:

```
cond :: (Bool, Int, Int) -> Int
cond (True,  x, y) = x
cond (False, x, y) = y
```

As interpretation of `cond` we have the function $f : \mathbb{B}_\perp \times \mathbb{Z}_\perp \times \mathbb{Z}_\perp \to \mathbb{Z}_\perp$ with

$$f(b, x, y) = \begin{cases} x, & \text{if } b = \texttt{True} \\ y, & \text{if } b = \texttt{False} \\ \perp_{\mathbb{Z}}, & \text{if } b = \perp_{\mathbb{B}} \end{cases}$$

Because $f(\texttt{True}, 2, \perp_{\mathbb{Z}}) = 2$, f is non strict. Show that f is monotonic.