RWTH RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHR- UND FORSCHUNGSGEBIET INFORMATIK II
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Exercises *Functional Programming* – Sheet 6

Solutions will be collected until Friday, June 22, 2001.

Exercise 1 (1 point)

Show that each chain of a partial order has at most one least upper bound.

Exercise 2

Let D_1 and D_2 be domains and let S be a set of functions from D_1 to D_2. For all $i \in D_1$ let $S_i = \{f(i) \mid f \in S\}$. Show that

- $\sqcup S$ exists iff $\sqcup S_i$ exists for all $i \in D_1$
- if $\sqcup S$ exists then $(\sqcup S)(i) = \sqcup S_i$

Exercise 3 (2 points)

Let D_1 and D_2 be domains. Show that if $\sqsubseteq_{D_2}$ is a cpo on D_2 then $\sqsubseteq_{D_1 \to D_2}$ is a cpo on the set of functions from D_1 to D_2.

Hint: Use the lemma from exercise 2.

Exercise 4 (2 points)

Let $\sqsubseteq_{D_1}$ and $\sqsubseteq_{D_2}$ be complete partial orderings on D_1 and D_2 respectively, and let $f : D_1 \to D_2$ be a monotonic function such that for each chain S of D_1 we have $f(\sqcup S) \sqsubseteq \sqcup f(S)$. Show that f is continuous.

Exercise 5 (1 point)

Let D_1, D_2, and D_3 be cpo's and let $f : D_1 \to D_2$ and $g : D_2 \to D_3$ be continuous functions. Show that the composition $g \circ f : D_1 \to D_3$ is also continuous.

Exercise 6 (3 points)

Consider the function $ff : \langle \mathbb{Z}_\perp \to \mathbb{Z}_\perp \rangle \to \langle \mathbb{Z}_\perp \to \mathbb{Z}_\perp \rangle$ defined as follows:

$$(ff(g))(x) = \begin{cases} 1, & \text{if } x \leq 0 \\ g(x - 1) * x, & \text{otherwise} \end{cases}$$

Here, $-$ and $*$ are the strict extensions of the subtraction and the multiplication function. Show that

(a) ff is monotonic.

(b) ff is continuous.

Hint: Use the lemmas from exercise 2 and 4.

Exercise 7 (3 points)

Consider the following functions:

```
one :: Int -> Int
one x = 1

non_term :: Int -> Int
non_term x = non_term (x + 1)

inf :: Int -> Int
inf x = 1 + inf x
```

For the functions `one`, `non_term`, and `inf` give the corresponding higher-order functions $\Phi_{\texttt{one}}$, $\Phi_{\texttt{non\_term}}$, and $\Phi_{\texttt{inf}}$. (Here, Φ_f corresponds to f in the same way as ff from exercise 5 corresponds to `fact`, i.e., $\Phi_{\texttt{fact}}$ is ff.) Give all their fixpoints and determine the least one. What does the function $\Phi_f^n(\perp)$ compute for $n \in \mathbb{N}$, $f \in \{\texttt{one}, \texttt{non\_term}, \texttt{inf}\}$ (where $\Phi_f^n(\perp)$ denotes n applications of Φ_f to $\perp$)?