RHEINISCH-
WESTFÄLISCHE
TECHNISCHE
HOCHSCHULE
AACHEN

LEHR- UND FORSCHUNGSGEBIET
INFORMATIK 2
RWTH Aachen · D-52056 Aachen · GERMANY
http://verify.rwth-aachen.de/

LuFG Informatik II

Functional Programming
Exam, March 19, 2010

Prof. Dr. Jürgen Giesl
Carsten Fuhs

**First name:** ______________________________

**Last name:** ______________________________

**Matr. number:** ______________________________

**Course of study (please mark exactly one):**

- ○ Master of Informatik
- ○ Bachelor of Informatik – for Master's studies

- On every sheet please give your **first name**, **last name**, and **matriculation number**.
- You must solve the exam **without** consulting any **extra documents** (e.g., course notes).
- Make sure your answers are readable. Do not use **red pens or pencils**.
- Please answer the exercises on the **exercise sheets**. If needed, also use the back sides of the exercise sheets.
- Answers on extra sheets can only be accepted if they are clearly marked with your name, your matriculation number, and the **exercise number**.
- **Cross out** text that should not be considered in the evaluation.
- Students that try to cheat **do not pass** the exam.
- At the end of the exam, please return **all sheets together with the exercise sheets**.

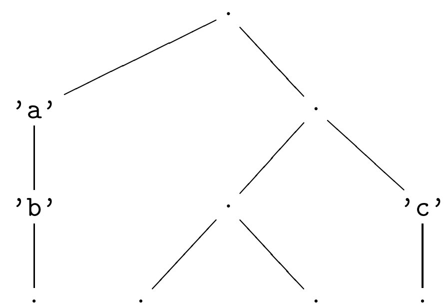
| | Total number of points | Number of points obtained |
|---|---|---|
| Exercise 1 | 18 | |
| Exercise 2 | 9 | |
| Exercise 3 | 6 | |
| Exercise 4 | 6 | |
| Exercise 5 | 9 | |
| Exercise 6 | 10 | |
| Total | 58 | |
| Grade | - | |

| First name | Last name | Matriculation number |
|---|---|---|
| | | |

## Exercise 1 (4 + 5 + 4 + 5 = 18 points)

The following data structure represents polymorphic binary trees that contain values only in special `Value` nodes that have a single successor:
`data Tree a = Leaf | Node (Tree a) (Tree a) | Value a (Tree a)`

Consider the tree `t` of characters on the right-hand side. The representation of `t` as an object of type `Tree Char` in Haskell would be:




`(Node (Value 'a' (Value 'b' Leaf)) (Node (Node Leaf Leaf) (Value 'c' Leaf)))`

Implement the following functions in Haskell.

(a) The function `foldTree` of type

`(a -> b -> b) -> (b -> b -> b) -> b -> Tree a -> b`

works as follows: `foldTree f g h x` replaces all occurrences of the constructor `Value` in the tree `x` by `f`, it replaces all occurrences of the constructor `Node` in `x` by `g`, and it replaces all occurrences of the constructor `Leaf` in `x` by `h`. So for the tree `t` above,

`foldTree (:) (++) [] t`

should compute

`((++) ((:) 'a' ((:) 'b' [])) ((++) ((++) [] []) ((:) 'c' [])))`,

which in the end results in `"abc"` (i.e., in the list `['a','b','c']`). Here, `Value` is replaced by `(:)`, `Node` is replaced by `(++)`, and `Leaf` is replaced by `[]`.

| First name | Last name | Matriculation number |
|---|---|---|
| | | |



(b) Use the `foldTree` function from (a) to implement the `average` function which has the type `Tree Int -> Int` and returns the average of the values that are stored in the tree. This should be accomplished as follows:

- Use `foldTree` with suitable functions as arguments in order to compute the *sum* of the values stored in the trees.
- Use `foldTree` with suitable functions as arguments in order to compute the *number of* `Value`*-objects in the tree.*
- Perform integer division with the pre-defined function `div :: Int -> Int -> Int` on these values to obtain the result.

Here your function is required to work correctly only on those trees that contain the constructor `Value` at least once.

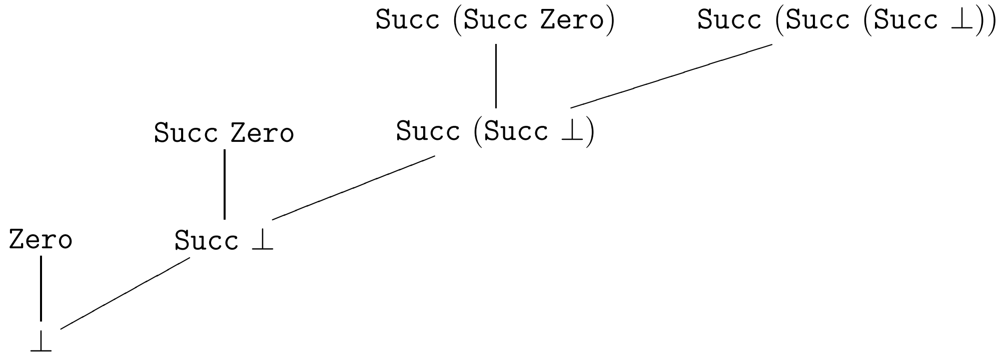
| First name | Last name | Matriculation number |
|---|---|---|
| | | |



(c) Consider the following data type declaration for natural numbers:

```
data Nats = Zero | Succ Nats
```

A graphical representation of the first four levels of the domain for `Nats` could look like this:




Sketch a graphical representation of the first three levels of the domain for the data type `Tree Bool`.

| First name | Last name | Matriculation number |
| --- | --- | --- |
| | | |



(d) Write a Haskell function `printStars` that first reads a string from the user, then prints this string on the console, converts the string to a number `n` (using the pre-defined function `read`) and in the end also prints `n` times the character `'*'` on the console. Also give the type declaration for your function.

You may use the `do`-notation, but you are not obliged to use it. You do not have to check whether the input string is really a number. Some of the following pre-defined functions can be helpful:

- `getLine :: IO String` reads a string from the user
- `read :: String -> Int` converts a string to a number
- `putStr :: String -> IO ()` writes a string to the console

An example run should look as given below. Here the string “7” was read from the user.

```
Main> printStars
7
7*******
```

| First name | Last name | Matriculation number |
|---|---|---|
| | | |



## Exercise 2 (4 + 5 = 9 points)

Consider the following Haskell declarations for the `fib` function, which for a natural number $x$ computes the value *fibonacci*$(x)$:

```
fib :: Int -> Int
fib 0     = 0
fib 1     = 1
fib (x+2) = fib (x+1) + fib x
```

(a) Please give the Haskell declarations for the higher-order function `f_fib` corresponding to `fib`, i.e., the higher-order function `f_fib` such that the least fixpoint of `f_fib` is `fib`. In addition to the function declaration(s), please also give the type declaration of `f_fib`. Since you may use full Haskell for `f_fib`, you do not need to translate `fib` into simple Haskell.

(b) We add the Haskell declaration `bot = bot`. For each $n \in \mathbb{N}$ please determine which function is computed by $\texttt{f\_fib}^n\ \texttt{bot}$. Here "$\texttt{f\_fib}^n\ \texttt{bot}$" represents the $n$-fold application of `f_fib` to `bot`, i.e., it is short for $\underbrace{\texttt{f\_fib}\ (\texttt{f\_fib}\ \ldots\ (\texttt{f\_fib}}_{n \text{ times}}\ \texttt{bot})\ldots)$.

Let $f_n : \mathbb{Z}_\perp \to \mathbb{Z}_\perp$ be the function that is computed by $\texttt{f\_fib}^n\ \texttt{bot}$.
Give $f_n$ in **closed form**, i.e., using a non-recursive definition. In this definition, you may use the function *fibonacci* $: \mathbb{N} \to \mathbb{N}$ where *fibonacci*$(x)$ computes the $x$-th Fibonacci number. Here it suffices to give the result of your calculations. You do not need to present any intermediate steps.

| First name | Last name | Matriculation number |
|---|---|---|
| | | |



## Exercise 3 (3 + 3 = 6 points)

Let $D_1, D_2, D_3$ be domains with corresponding complete partial orders $\sqsubseteq_{D_1}, \sqsubseteq_{D_2}, \sqsubseteq_{D_3}$. As we know from the lecture, then also $\sqsubseteq_{(D_2 \times D_3)_\perp}$ is a complete partial order on $(D_2 \times D_3)_\perp$.

Now let $f : D_1 \to D_2$ and $g : D_1 \to D_3$ be functions.
We then define the function $h : D_1 \to (D_2 \times D_3)_\perp$ via $h(x) = (f(x), g(x))$.

(a) Prove or disprove: If $f$ and $g$ are *strict* functions, then also $h$ is a strict function.

(b) Prove or disprove: If $f$ and $g$ are *monotonic* functions, then also $h$ is a monotonic function.

| First name | Last name | Matriculation number |
|---|---|---|
| | | |



## Exercise 4 (6 points)

We define the following data structures for natural numbers and polymorphic lists:

```
data Nats = Zero | Succ Nats
data List a = Nil | Cons a (List a)
```

Consider the following expression in complex Haskell:

```
let get n        Nil         = Zero
    get Zero     (Cons x xs) = x
    get (Succ n) (Cons x xs) = get n xs
    in get
```

Please give an equivalent expression `let get = ...  in get` in **simple** Haskell.

Your solution should use the functions defined in the transformation from the lecture such as $\texttt{sel}_{n,i}$, $\texttt{isa}_{\underline{\texttt{constr}}}$, and $\texttt{argof}_{\underline{\texttt{constr}}}$. However, you do not have to use the transformation rules from the lecture.

| First name | Last name | Matriculation number |
|---|---|---|
| | | |



## Exercise 5 (4 + 5 = 9 points)

Consider the following data structure for polymorphic lists:

```
data List a = Nil | Cons a (List a)
```

(a) Please translate the following Haskell expression into an equivalent lambda term (e.g., using $\mathcal{L}am$). Recall that pre-defined functions like `odd` or `(+)` are translated into constants of the lambda calculus.

It suffices to give the result of the transformation.

```
let f = \x -> if (odd x) then (\y -> x) else f ((+) x 3)
    in f
```

| First name | Last name | Matriculation number |
| --- | --- | --- |
| | | |



(b) Let $\delta$ be the set of rules for evaluating the lambda terms resulting from Haskell, i.e., $\delta$ contains at least the following rules:

$$\begin{array}{rcl} \texttt{fix} & \rightarrow & \lambda f.\ f\ (\texttt{fix}\ f) \\ \texttt{times}\ 3\ 2 & \rightarrow & 6 \end{array}$$

Now let the lambda term $t$ be defined as follows:

$$t = (\lambda x.\,(\texttt{fix}\ \ \lambda g.\,x))\ \ (\lambda z.\,(\texttt{times}\ 3\ 2))$$

Please reduce the lambda term $t$ by WHNO-reduction with the $\rightarrow_{\beta\delta}$-relation. You have to give **all** intermediate steps until you reach **weak head normal form** (and no further steps).

| First name | Last name | Matriculation number |
|---|---|---|
| | | |



## Exercise 6 (10 points)

Use the type inference algorithm $\mathcal{W}$ to determine the most general type of the following lambda term under the initial type assumption $A_0$. Show the results of all sub-computations and unifications, too. If the term is not well typed, show how and why the $\mathcal{W}$-algorithm detects this.

$$(\,(\texttt{Cons}\ \ \lambda x.\,x)\ y\,)$$

The initial type assumption $A_0$ contains at least the following:

$$\begin{array}{lcl} A_0(\texttt{Cons}) & = & \forall a.\ (a \rightarrow (\texttt{List}\ a \rightarrow \texttt{List}\ a)) \\ A_0(x) & = & \forall a.\ a \\ A_0(y) & = & \forall a.\ a \end{array}$$