

$f : D_1 \rightarrow D_2$ is *monotonic* iff $f(d) \sqsubseteq_{D_2} f(d')$ for all $d \sqsubseteq_{D_1} d'$
 $\{d_1, d_2, \dots\}$ is a *chain* iff $d_1 \sqsubseteq d_2 \sqsubseteq d_3 \sqsubseteq \dots$

$\{\text{fact}_0, \text{fact}_1, \dots\}$ is a chain where

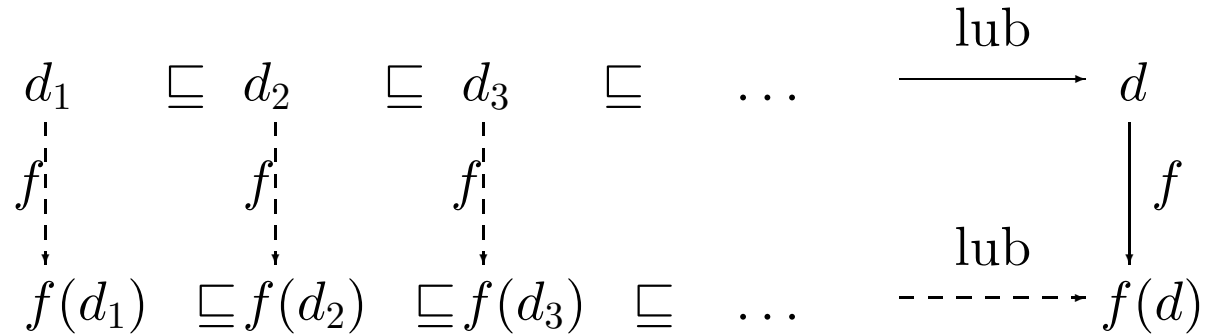
$$\begin{aligned} \text{fact}_0(x) &= \perp \text{ for all } x \in \mathbb{Z}_\perp \\ \text{fact}_1(x) &= \begin{cases} x!, & \text{for } 0 \leq x < 1 \\ 1, & \text{for } x < 0 \\ \perp, & \text{for } x = \perp \text{ or } 1 \leq x \end{cases} \\ \text{fact}_2(x) &= \begin{cases} x!, & \text{for } 0 \leq x < 2 \\ 1, & \text{for } x < 0 \\ \perp, & \text{for } x = \perp \text{ or } 2 \leq x \end{cases} \\ &\vdots \end{aligned}$$

Least upper bound: $\sqcup\{\text{fact}_0, \text{fact}_1, \text{fact}_2, \dots\} = \text{fact}$ with

$$\text{fact}(x) = \begin{cases} x!, & \text{for } 0 \leq x \\ 1, & \text{for } x < 0 \\ \perp, & \text{for } x = \perp \end{cases}$$

A reflexive partial ordering \sqsubseteq on a set D is *complete* iff

- (1) D has a smallest element \perp_D
- (2) every chain S of D has a least upper bound $\sqcup S \in D$



$f : D_1 \rightarrow D_2$ is *continuous* if $f(\sqcup S) = \sqcup f(S)$ for every chain S of D_1 .

f is *continuous* \Rightarrow f is *monotonic*

\sqsubseteq is a cpo on:

- Base Domains $\mathbb{Z}_\perp, \mathbb{B}_\perp, C_\perp, F_\perp$
- Product Domains $D_1 \times \dots \times D_n$
- Function Domains $\langle D_1 \rightarrow D_2 \rangle$ (*continuous* functions)