Recursive Upper-Bounds for Overlay Term Rewrite Systems

Elias TAHHAN BITTAR
Dpto. de Matemáticas Púras y Aplicadas.
Universidad Simón Bolívar
Caracas, VENEZUELA.
etahhan@usb.ve
http://thor.ma.usb.ve/~etahhan/

January 30, 1999

In [Gra], Gramlich extended an O'Donnell's result by proving that finite innermost terminating overlay term rewrite systems with joinable critical pairs are terminating. We study the complexity aspect of this result and we prove that :

Theorem Given a finite overlay term rewrite system $\mathcal{R}$ and a term $t = f(u_0, \ldots, u_m)$ such that :

- each immediate sub-term u_i of t is strongly uniquely normalising (we denote $\downarrow(u_i)$ its unique normal form),
- and if $\downarrow_i(t) := f(\downarrow(u_0), \ldots, \downarrow(u_m))$ is strongly uniquely normalising,

then t is strongly uniquely normalising and there is a constant M such that

$$\lambda(t) \leq \lambda(\downarrow_i(t)) + M^{\lambda(\downarrow_i(t))} \times (\lambda(u_0) + \cdots + \lambda(u_m))$$

where $\lambda(v)$ denotes the length of a worst derivation of a strongly normalising term v.

This theorem implies Gramlich result stated above and extends a result proved in [Tah] for constructor orthogonal rewrite systems.

This result allows to estimate upper bounds for primitive recursive functions and to give a syntactic characterisation of the Grzegorczyk hierarchy as done in [Cich-Tah].

References

[Gra] B. Gramlich. Relating innermost, weak, uniform and modular termination of term rewriting systems. LNAI, pp 285–296. Springer-Verlag, 1992.

[Cich-Tah] E. A. Cichon & E. Tahhan Bittar. Strictly orthogonal left linear rewrite systems and primitive recursion. Submitted 1998.

[Tah] E. TAhhan Bittar. Bornes supérieures de terminaison de sistèmes de réécriture strictement orthogonaux. Prépublication 17, LLAIC1. Université d'Auvergne (FRANCE) 1993.