Complexity classes within KBO

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Abstract

Termination proofs of rewrite systems using the Knuth-Bendix ordering, KBO, imply multiply-recursive bounds on the lengths of derivations. Hofbauer has studied two restrictions of KBO. The first is a restriction to the use of positive weights, the second is a restriction on the signature which is only allowed to contain at most unary symbols. In both cases, Hofbauer proves the derivation lengths to be bounded exponentially. We prove that in the first case computations are LINSPACE and that in the second computations are ETIME.

The theme of our investigations is that of complexity characterisations of classes of functions computed by rewrite systems. Comparisons are achieved with respect to computations with a Turing Machine. Actually, we concentrate on classes of feasible complexity.

There are several examples where a fine analysis of termination proofs gives upper bounds on the complexity of the computed function. Concerning path orderings for instance, see Hofbauer [Hof92] for the case of the *multiset path ordering*, MPO, and Weiermann [Wei95] for the case of the *lexicographic path ordering*, LPO. In both cases, the authors answer the question as to what complexity restrictions are imposed by the termination proof method.

We are concerned here with a similar point of view. Given a termination proof method and a complexity class, are all functions of this class computed by a rewrite system whose termination can be established by the method? Following on from work of Cichon and Lescanne in [CL92], Bonfante, Cichon, Marion and Touzet, in [BCMT98], have analysed the situation for termination proofs using polynomial interpretations in terms of time complexity classes. We consider the same problem for the Knuth-Bendix ordering, KBO.

KBO is a syntactic ordering on terms invented by Knuth and Bendix [KB70] in the late 1960's and it has been proved decidable whether or not a rewrite system admits a KBO ordering termination proof ([Lan79, Mar87, Alt89]). As a consequence, KBO can be used by automatic proof search systems.

In [HL89], Hofbauer shows that the upper bounds on derivation lengths of rewrite systems with a KBO termination proof are not primitive recursive. In fact, they are multiply-recursive [Hof9-, Tou97]. However, there are two natural restrictions of the order for which the bounds are exponential. The first requires that the weights be strictly positive (which gives a restriction on size) and the second restricts to computations on words (or strings).

¿From our point of view, these two restrictions lead to very different classes of functions. We show that the restriction on weights leads to a characterisation of LINSPACE and that the restriction on the signature leads to a characterisation of ETIME.

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