A Totally Terminating Rewrite System whose Complexity is not Γ_0-recursive

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Weiermann [2] has shown that the complexity of a rewrite system which is known to terminate by Kruskal's Theorem is bound by $(s^{\omega})^{\bar{\theta}_{\Omega^{\omega}}(0)}$. This result appeared to be of rather theoretical nature, because all known examples had multiply recursive complexities, until recently Touzet [1] gave examples of (even totally) terminating rewrite systems whose complexities are not multiply recursive. This was established by simulating the famous Hydra battle for ordinals below ε_0.

Based on the work of Touzet we define a totally terminating rewrite system which is able to simulate Hydra battles for all ordinals below Γ_0, the first infinite ordinal which is closed under $+$ and the binary Veblen φ-function. In a second step, we show how the ternary φ-function can be approached by our method.

Since the bound of Weiermann is the first infinite ordinal closed under $+$ and all n-ary φ-functions, we are led to the conjecture that this bound is sharp.

References

[1] H. Touzet. Encoding the Hydra Battle as a Rewrite System. `http://www.loria.fr/~touzet/mfcs98.ps.gz`, 1998.

[2] A. Weiermann. Complexity Bounds for some Finite Forms of Kruskal's Theorem. *Journal of Symbolic Computation*, 18:463–488, 1994.

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