Transforming Context-Sensitive Rewrite Systems

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Context-sensitive rewriting (Lucas [1, 2]) is a variant of term rewriting in which for every function symbol one indicates which arguments may be evaluated and a redex contraction is allowed only if it is does not take place in a forbidden argument of a function symbol above it. In this talk we are concerned with the problem of showing termination of context-sensitive rewriting. More precisely, we consider transformations from context-sensitive rewrite systems to ordinary term rewrite systems that are *sound* with respect to termination: termination of the transformed term rewrite system implies termination of the original context-sensitive rewrite system. The advantage of such an approach is that all techniques for proving termination of term rewriting can be used to infer termination of context-sensitive rewriting. Two such transformations are reported in the literature, by Lucas [1] and by Zantema [3]. We add two more. Our first transformation is simple, its soundness is easily established, and it improves upon the transformations of [1, 3]. To be precise, we prove that the class of terminating context-sensitive rewrite systems for which our transformation succeeds is larger than that of Lucas' transformation and we claim that the same holds for Zantema's transformation. None of these three transformations succeeds in transforming every terminating context-sensitive rewrite system into a terminating term rewrite system. In other words, they all lack *completeness*. We analyze the failure of completeness for our first transformation, resulting in a second transformation with is both sound and complete. This latter result can be interpreted as stating that from a termination perspective there is no reason to study context-sensitive rewriting.

References

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[2] S. Lucas. Context-sensitive computations in functional and functional logic programs. *Journal of Functional and Logic Programming*, 1:1–61, 1998.

[3] H. Zantema. Termination of context-sensitive rewriting. In *Proceedings of the 8th International Conference on Rewriting Techniques and Applications*, volume 1232 of *LNCS*, pages 172–186, 1997.