Translating TRSs into OS-TRSs: a result on modularity of termination*

(Abstract)

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An important aspect of the termination property of term rewriting systems (TRSs) is its *modularity*, that is termination is preserved under combinations of TRSs. These combinations vary from TRSs with a disjoint union of signatures to more complex, hierarchical combinations of TRSs, through TRSs which share constructors [Toy87,GG87,TKB89,Pol92,MT93,FJ94,Ohl94,Der95,Gra96]. Very often, non-termination of the combined system arises because of subtle and (semantically) meaningless interactions between the TRSs. The criteria used to guarantee modularity are usually given at the syntactic level by relying on the form of terms which occur in the rules of the TRSs, often using a detailed inspection of the structure of the terms. Frequently, it is difficult to understand intuitively how these syntactic criteria interact.

We present a translation of unsorted TRSs into suitable order-sorted TRSs (OS-TRSs) [Gna92,Mat96,ÖL96]. The aim of such a translation is to add more structure (i.e. the sorts) to the rewriting rules and the terms to be rewritten, in order to enforce termination. The translation builds on the usual distinction between constructors and functions to yield a rewriting relation that induces a particular rewriting strategy, i.e. a bottom-up (innermost) rewriting constrained by the sorts of terms.

The translation is correct and complete, in the sense that the OS-TRS T resulting from the translation of an unsorted TRS R is equivalent to R with respect to terminating derivations. This means that every terminating derivation in the unsorted TRS R can be simulated by a derivation in T that reaches the same normal form. Thus, for the class of systems for which the completeness holds, the translation prunes all the possible infinite derivations of R. Moreover, the OS-TRS T can, on the one hand, have properties that the TRS R does not have (e.g. termination) and, on the other hand, allows a different combination of systems, for which more general modularity results for termination can be proved.

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Our result is the modularity of termination for a subclass of OS-TRSs. This result holds for the OS-TRSs resulting from the translation which, with respect to the original unsorted TRSs, possess a more constrained structure both in the terms and in the rewriting relations. However, the modularity of termination for the combined OS-TRSs provides a non-trivial criterion for modular combinations of a particular class of OS-TRSs. Our current combination only allows common constructors, and results on modularity of termination for such a combination already exist for suitable subclasses of TRSs. We believe that, by adding the sort information and establishing suitable orderings on sorts, it will be possible to achieve modularity of termination for more complex combinations of TRSs, such as hierarchical unions.

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