Equational Termination by Semantic Labelling

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Semantic labelling (Zantema [5]) is a powerful tool for proving termination of term rewrite systems. The usefulness of the extension to equational term rewriting described in [5] is however rather limited. In this talk, we introduce a more powerful version of *equational semantic labelling*, parameterized by three choices: (1) the well-founded order on the underlying algebra (partial order vs. quasi-order), (2) the relation between the algebra and the rewrite system (model vs. quasi-model), and (3) the labelling of the function symbols appearing in the equations (allowed vs. forbidden). We present soundness and completeness results for the various instantiations and analyze the relationships between them.

We present several applications of our equational semantic labelling technique. We give an alternative proof of the correctness of the dependency pair approach (Arts and Giesl [1]) for proving termination of term rewrite systems. We give a short proof of the main result of Ferreira *et al.* [2]: The correctness of a version of dummy elimination for AC-rewriting which completely removes the AC-axioms. We extend some of the results of Zantema [4] concerning distribution elimination to the equational setting.

Every terminating term rewrite system can be transformed by self-labelling (a special case of semantic labelling) into a rewrite system whose termination can be shown by the recursive path order (Middeldorp *et al.* [3]). In the final part of the talk, we discuss to what extent our equational semantic labelling technique is complete in this sense.

References

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