Top Termination of CL(S)

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Introduction. In a term rewriting system (TRS) that is both confluent and terminating, normal forms are unique (each computation has a unique result) and thus the word problem is decidable. Even if the TRS is not terminating, we would like to obtain partial results of computations, and to compare terms. We lift the restriction that redexes must eventually vanisch (that is, termination) and require only that each initial segment of the term eventually becomes free from redexes.

A TRS is called *top terminating* iff in each infinite derivation, the redex is at the root of the term only finitely often. If a TRS is top terminating and confluent, we declare the (unique) limit of a fair reduction starting from a term X to be the normal form of X. [DKP91] [KKSdV95]

Main result. The fragment of Combinatory Logic with the only combinator $\mathbf{S}xyz \to xz(yz)$ is top terminating.

This seems to require a rather technical proof, that can, however, be supported by machine calculations.

Aim of the talk. While the result perhaps is nice, but certainly not earth shattering, we mainly present it to invite a discussion on general methods for proving top termination that could be applied here, resulting in a) a nice pen-and-paper proof or b) a fully automated proof.

Open Problems. Describe infinite normal forms in $\mathbf{CL}(\mathbf{S})$, and thus solve the word problem.

Related Work. The word problem for $\mathbf{CL}(\mathbf{L})$ with $\mathbf{L}xy \to x(yy)$ is shown decidable in [Sta89], [SWB93]. Normalization is decidable for $\mathbf{CL}(\mathbf{S})$ [Wal98].

References

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