A slow growing analysis of the canonical rewrite system for the Ackermann function

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Abstract

Let $R_A := \{A(0,y) \to S(y); A(S(x),y) \to A(x,S(y)), A(S(x),S(y))\}$ be the canonical rewrite system for the Ackermannfunction. It is well known the the ordinal notation system T for the ordinals less than Γ_0 yields a canonical termination proof for R_A. In addition the slow growing hierarchy up to Γ_0 classifies the R_A derivation lengths (more precisely, the derivation lengths of A in the system R_A). Let T^- be the subsystem of T in which the ordinal addition function is omitted. Then – due to de Jongh – the order type of T^- is ε_0. Nevertheless T^- yields the termination of R_A. We investigate the behaviour of the associated slow growing hierarchy for T^- and obtain the following two results (to appear in JSL):
1. If the elements of T^- are considered as tree ordinals then the associated slow growing hierarchy classifies the R_A-derivation lengths.
2. If the elements of T^- are considered as set-theoretic ordinals then the associated slow growing hierarchy classifies the Kalmar-elementary recursive functions.

These results indicate that sometimes tree ordinals reflect the computational behaviour of rewrite systems better than set-theoretic ordinals.