The Semantics of Rational Contractions

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Abstract

The *Logic of Theory Change* developed by Alchourrón, Gärdenfors and Makinson is concerned with the revision of beliefs in the face of new and possibly contradicting information. This nonmonotonic process consists of a contraction and an expansion transforming one belief into another. Beliefs are represented by consistent deductively closed sets of formulas. To achieve minimal change Alchourrón, Gärdenfors and Makinson suggested widely accepted postulates that rational contractions have to fulfill.

In practical applications, e.g. knowledge representation, deductively closed sets of formulas have to be representable in a finite way. Therefore our main interest is in *rational finite contractions*, i.e. rational contractions that transform sets of formulas possessing a finite base into finitely representable sets again.

We have formulated a semantical characterization of rational finite contractions which provides an insight into the true nature of these operations and shows all possibilities to define concrete functions of this kind.

Semantically, the rational finite contraction of a set Φ by a formula φ means extending the models M of Φ by some set of models of $\neg\varphi$. This set has to be uniquely determined by its restriction to a finite subsignature.

By means of this characterization we have examined the relationship of the concrete contractions known from literature and have found that they are all defined according to the same semantical strategy. Its aim is to extend the former set of models M by those models of $\neg\varphi$ that can be obtained by a *"small" change* of M.

This strategy results in maintaining those formulas of Φ which belong to the subsignature not affected by the change of M. But as the number of "important" formulas in Φ is not equal for different subsignatures of the same size we argue that this strategy leads to a contraintuitive behaviour[3].

We have discovered that the syntactical goal of keeping as many important formulas as possible in the contracted set corresponds to the following semantical strategy: M has to be extended by some models I of $\neg\varphi$ such that the *number of "big" changes* of M which result in I is *as large as possible*. Using our characterization we suggest a new rational finite contraction defined according to this strategy.

[3] When restricting ourselves to clauses instead of formulas a clause is the more important the less literals it consists of. If Φ is the deductive closure of $\{a, b \vee c\}$ the subsignatures $\{a, c\}$ and $\{b, c\}$ have the same size, but the most important clause of Φ does not belong to the latter one.