Complexity Analysis for Java with AProVE

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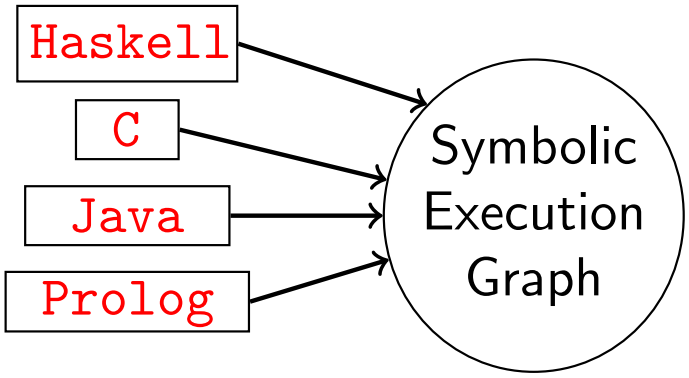
joint work with Florian Frohn

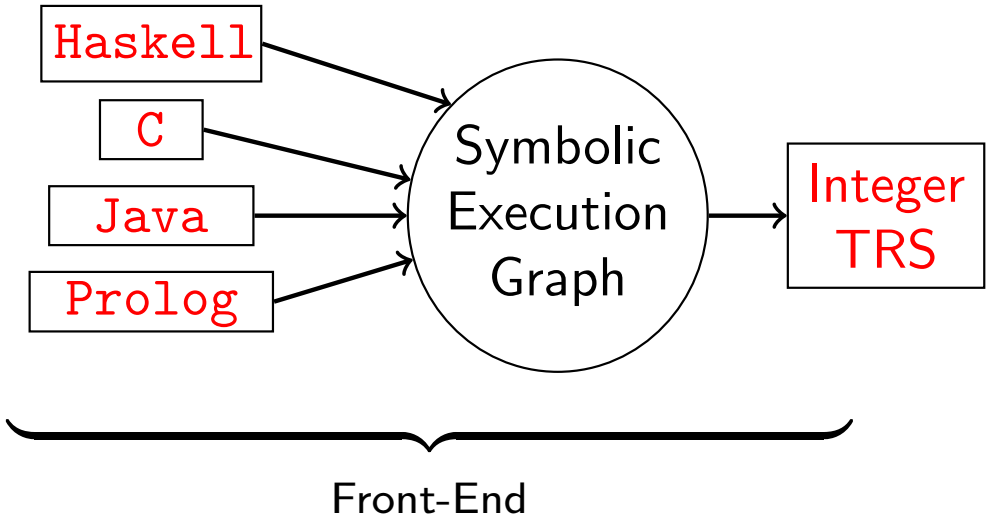
AProVE for Termination Analysis

AProVE for Termination Analysis



Termination

AProVE for Termination Analysis



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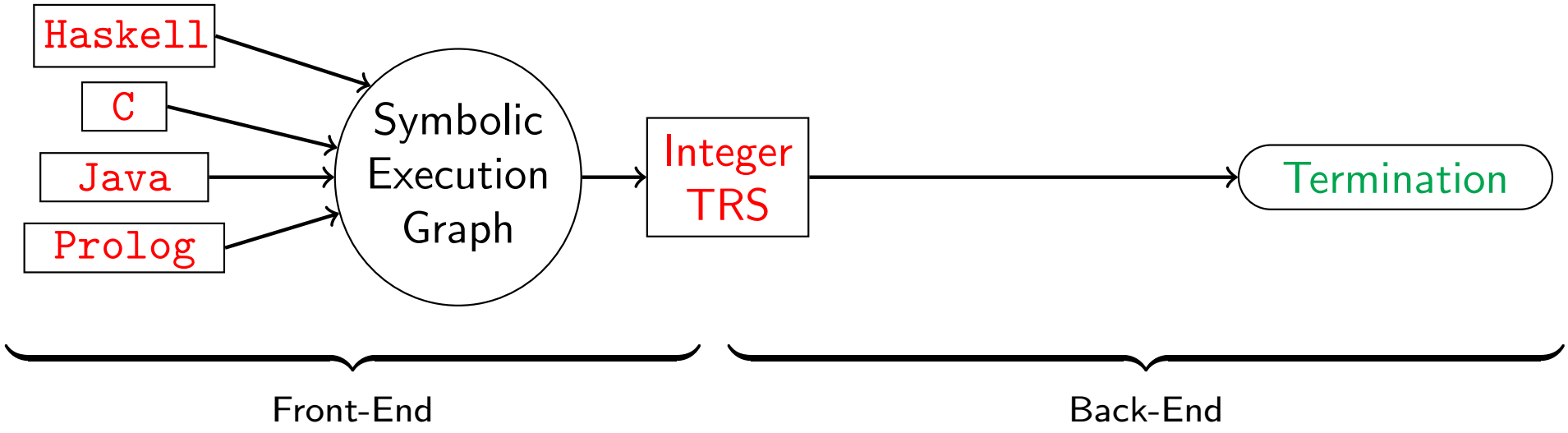

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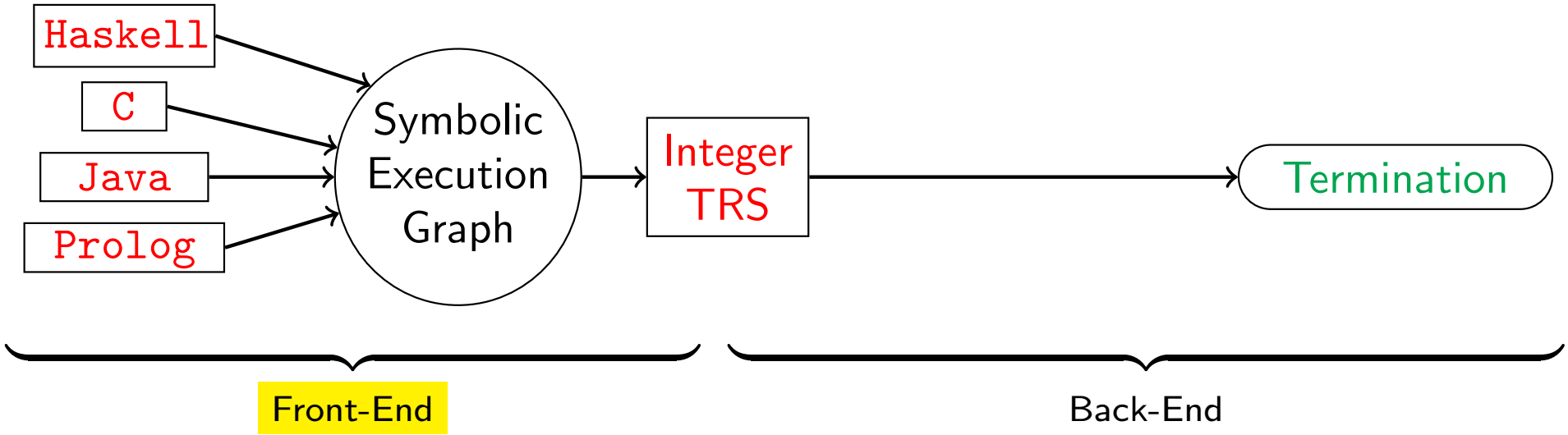
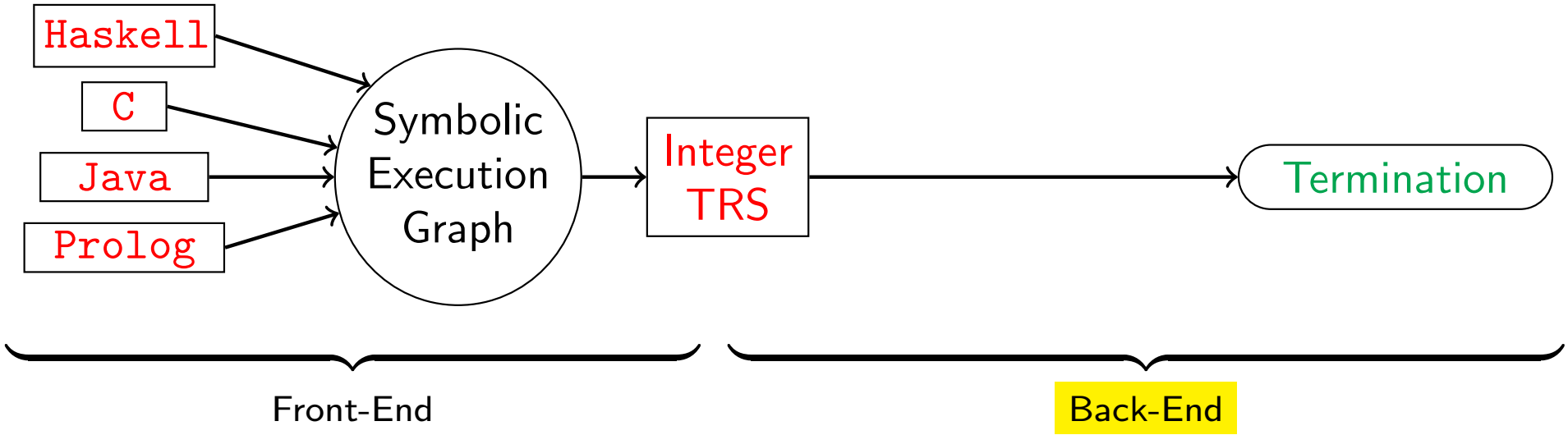


- language-specific features when generating symbolic execution graph

AProVE for Termination Analysis



- language-specific features when generating symbolic execution graph
- back-end analyzes Term Rewrite Systems with built-in Integers

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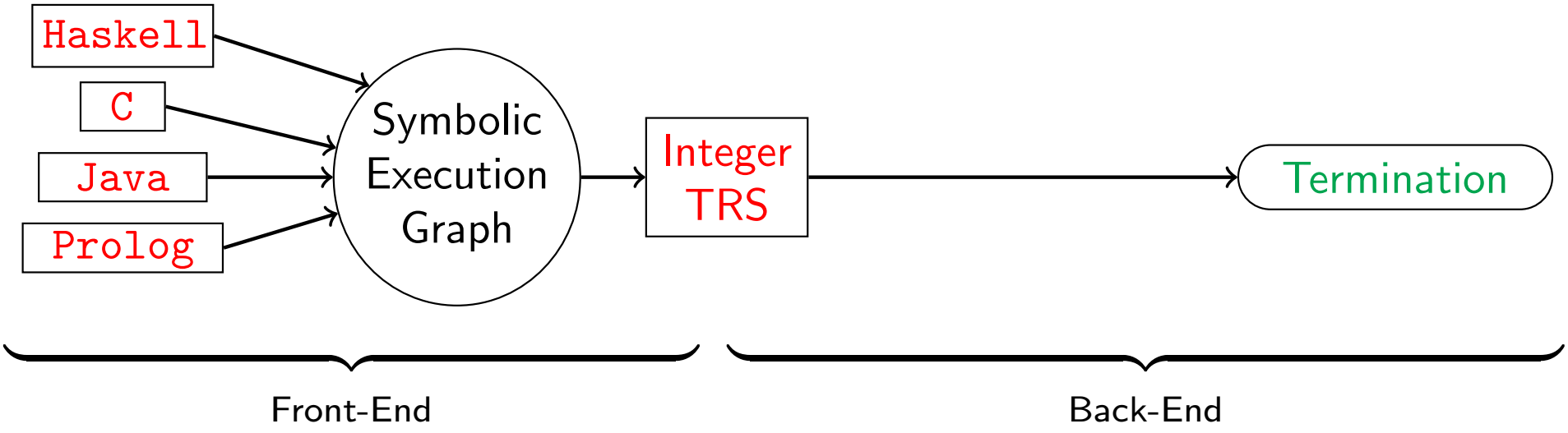
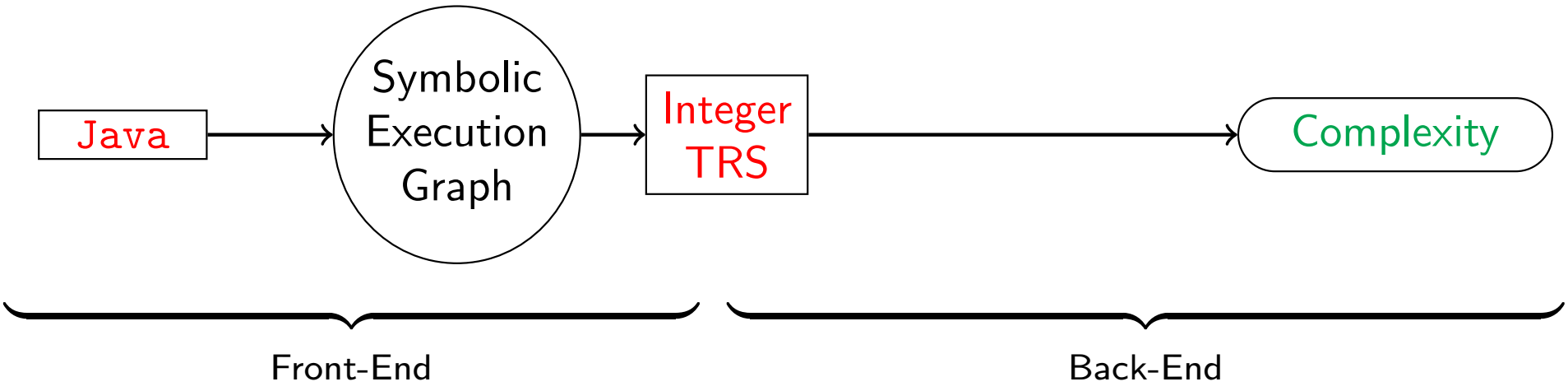


- language-specific features when generating symbolic execution graph
- back-end analyzes Term Rewrite Systems with built-in Integers
- powerful termination analysis
  - Termination Competition since 2004 (Java, C, Haskell, Prolog, TRS)
  - SV-COMP since 2014 (C)

AProVE for Complexity Analysis of Java



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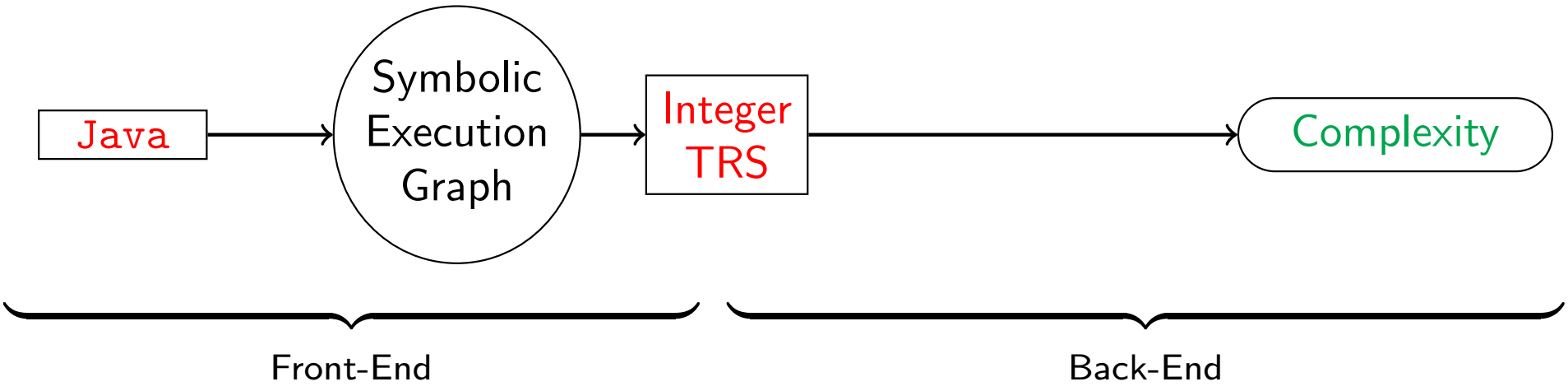

Problems

- transformation from Symbolic Execution Graph to Integer TRSs not complexity preserving

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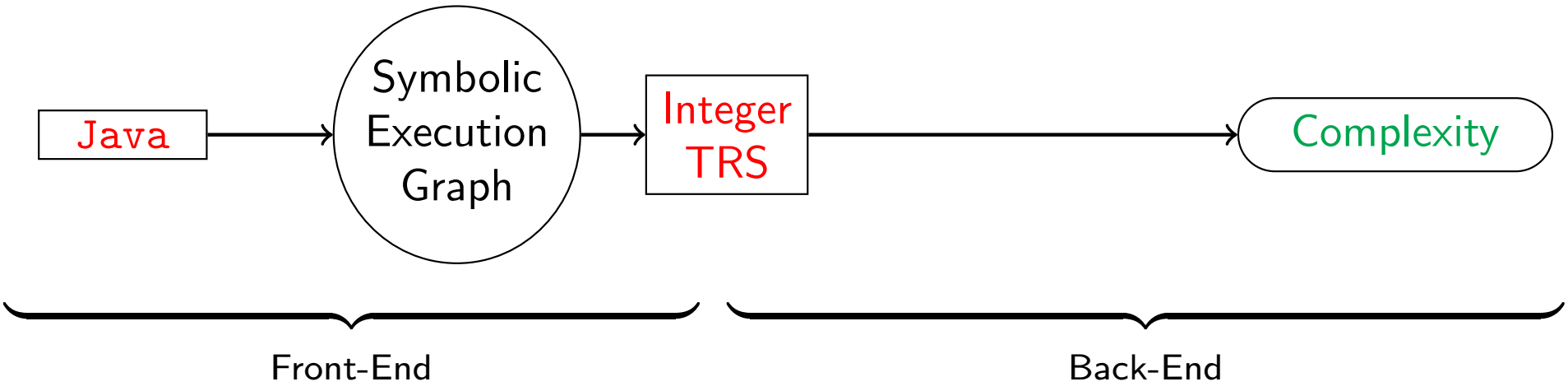

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- transformation from Symbolic Execution Graph to Integer TRSs not complexity preserving
- no back-end complexity analyzers for Integer TRSs

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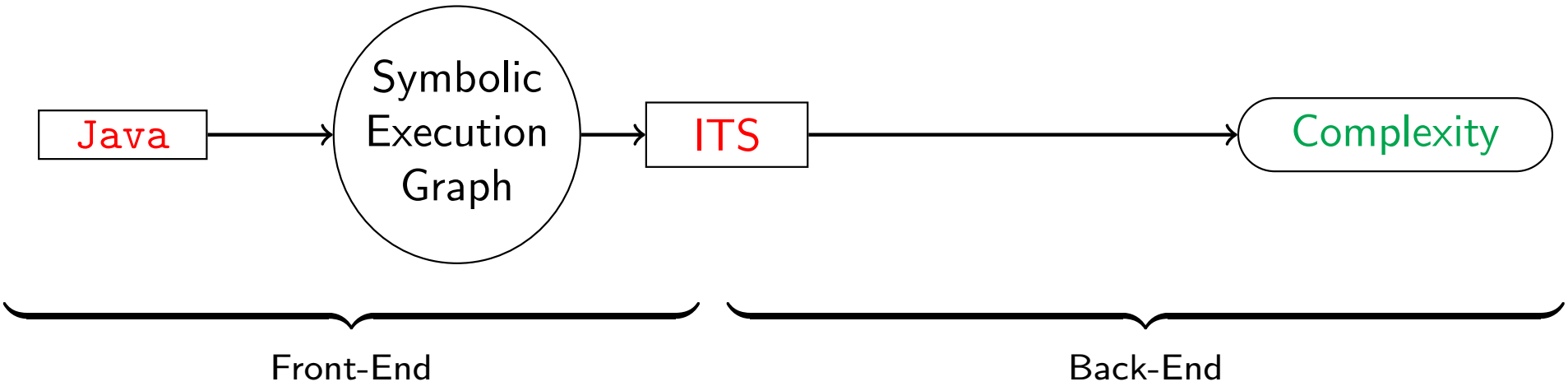

Problems

- transformation from Symbolic Execution Graph to Integer TRSs
  not complexity preserving
- no back-end complexity analyzers for Integer TRSs

Solution

- new transformation from Symbolic Execution Graph to Integer Transition Systems

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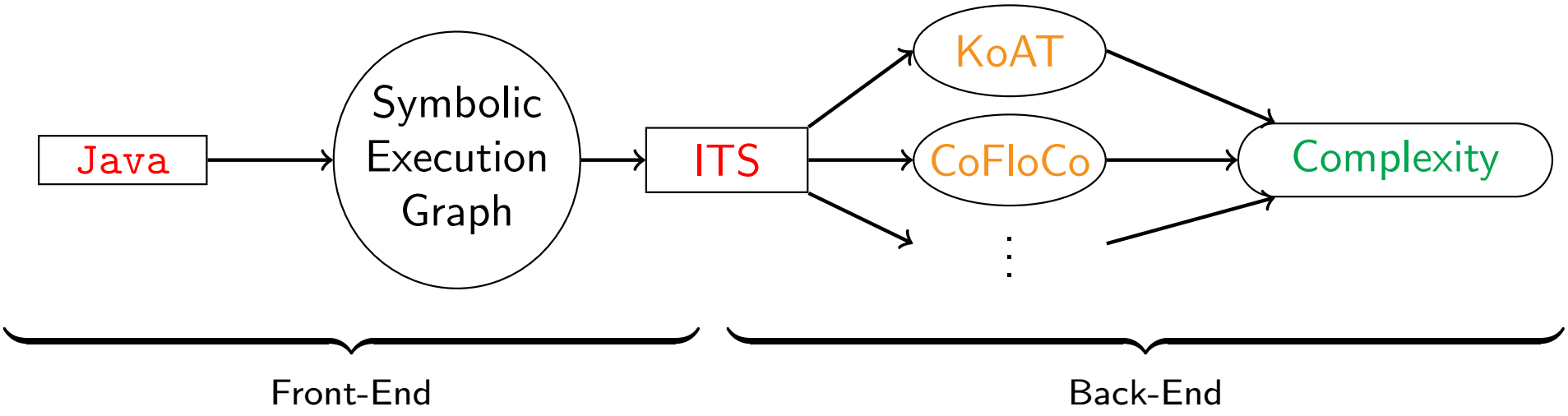


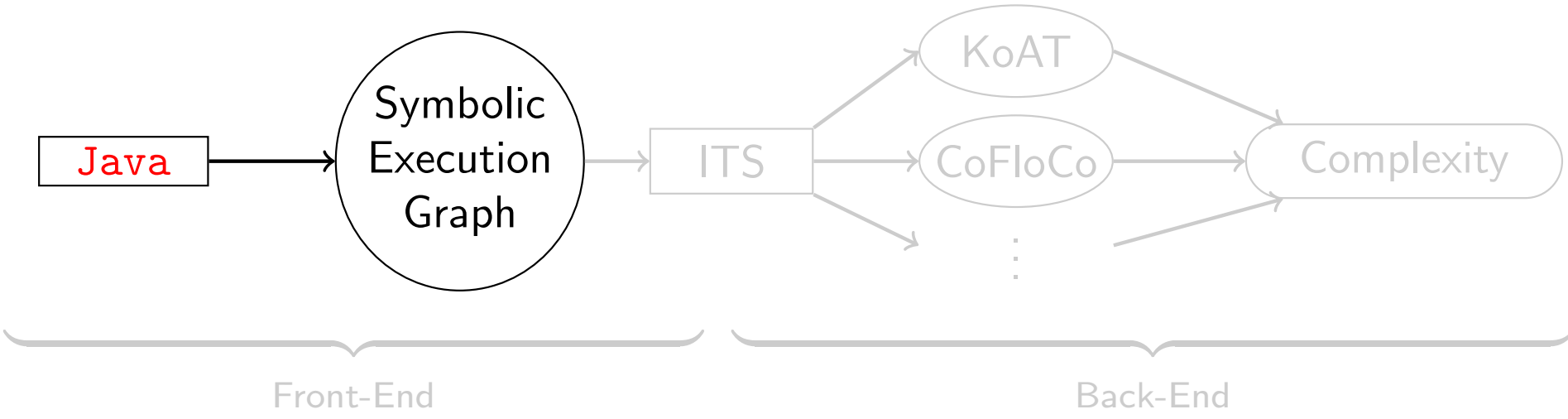
Problems

- transformation from Symbolic Execution Graph to Integer TRSs not complexity preserving
- no back-end complexity analyzers for Integer TRSs

Solution

- new transformation from Symbolic Execution Graph to Integer Transition Systems
- use existing complexity analyzers for ITSs

AProVE for Complexity Analysis of Java



Example: `max`

```
class List {
  int v;
  List n;

  static int max(List l) {
    int m = 0;
    while (l != null) {
      if (l.v > m) {
        m = l.v;
      }
      l = l.n;
    }
    return m;
  }

}
```

Example: max

```
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06: getfield v     // load l.v to opstack
07: iload_1        // load m to opstack
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Abstract States of the JVM

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$$\texttt{01} \quad | \quad \texttt{l}:o_1 \quad | \quad \varepsilon$$

$$o_1 : \texttt{List}(\texttt{v}:i_1, \texttt{n}:o_2)$$

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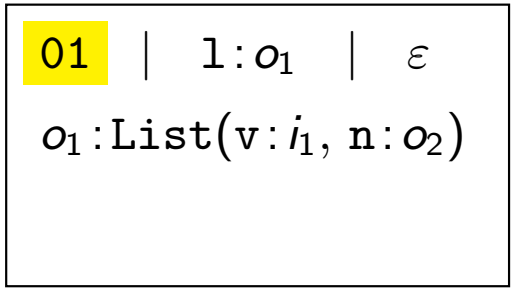

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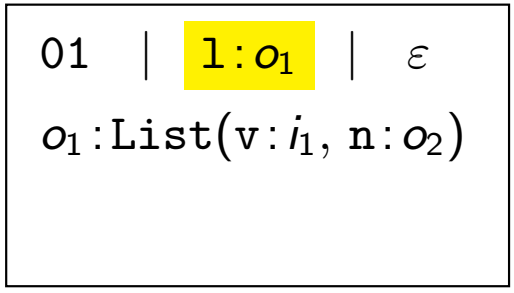

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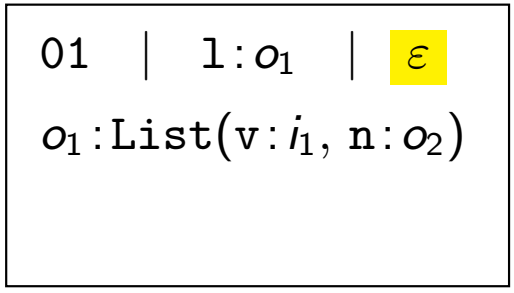

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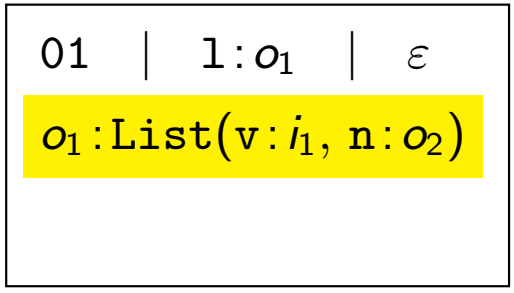

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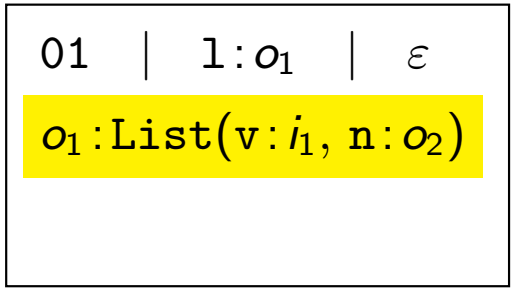

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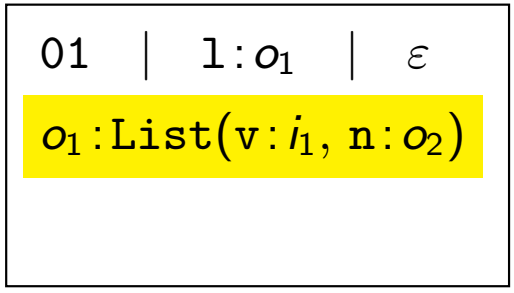

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information about the heap

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- o_1 and o_2 do not share or alias and point to tree-shaped objects

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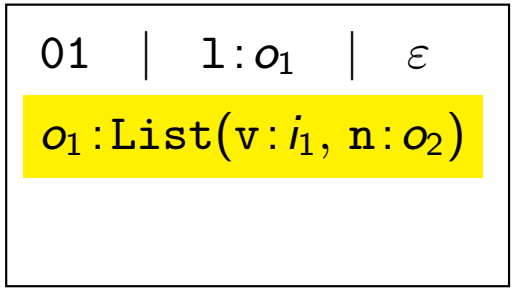

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$01 \mid \mathtt{l}:o_1 \mid \varepsilon$

$o_1 : \mathtt{List}(\mathtt{v}:i_1, \mathtt{n}:o_2)$

$o_1 \not\perp o_2$

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$$o_1 =^{?} o_2$$

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$$o_1 =^{?} o_2, \quad o_1!$$

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3. values on the operand stack

information about the heap

- object at o_1 is `List`, `v`-field has value i_1, `n`-field has value o_2
- o_1 and o_2 may alias and o_1 may point to a non-tree-shaped object

explicit sharing information

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```

A

01	l: o_1	ε

State A:

- o_1 is tree-shaped `List` or `null`

```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

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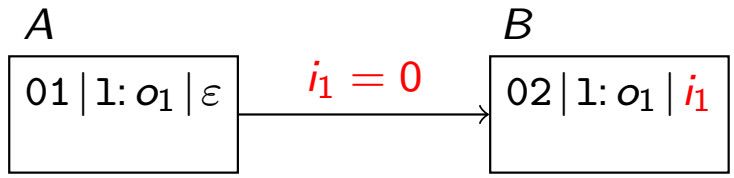

State B:

- load constant 0 on opstack
- *evaluation edge* from A to B
- labeled by condition $i_1 = 0$

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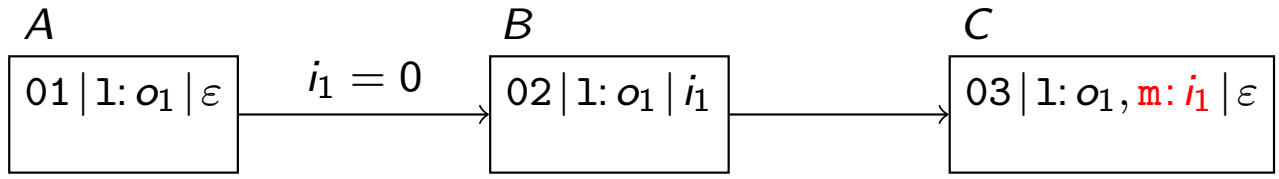
17: ireturn
```

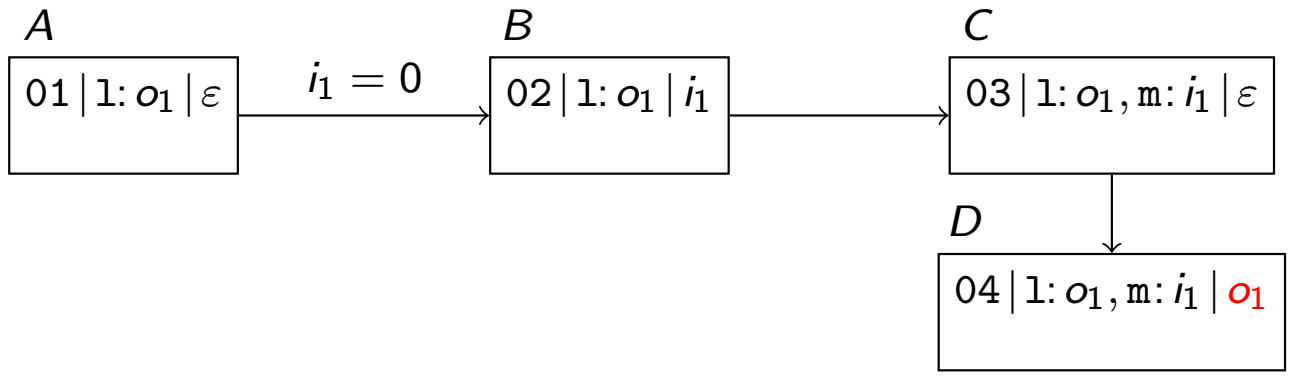


State C:

- store i_1 in program variable m

```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
12: aload_0
13: getfield n
14: astore_0
15: goto 3
16: iload_1
17: ireturn
```

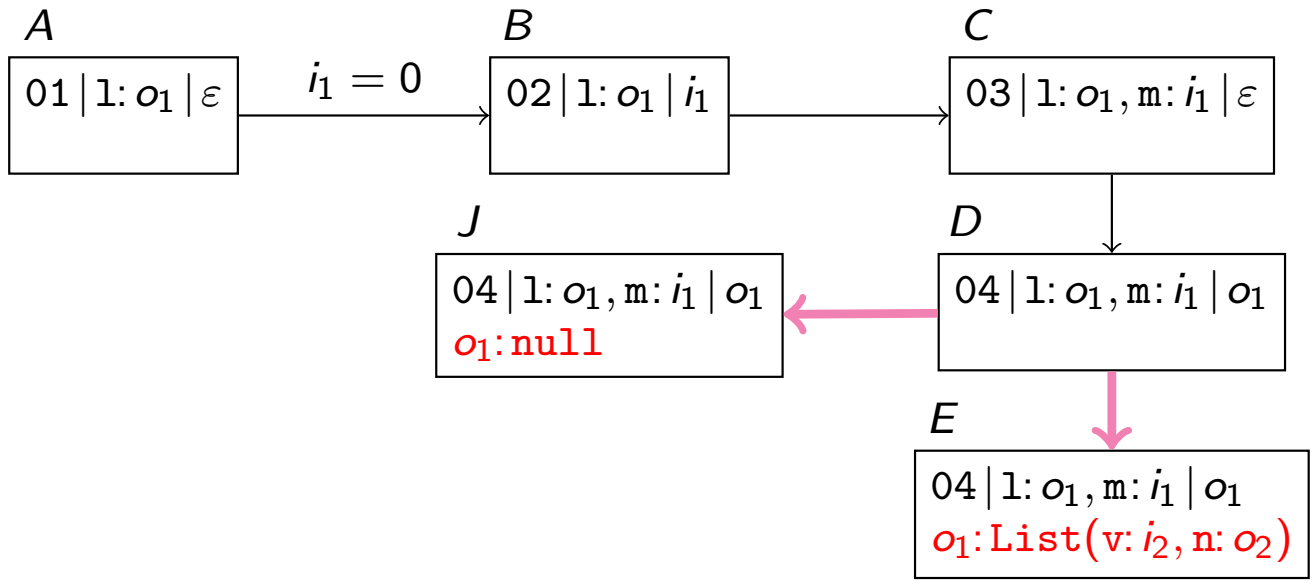


State *D*:

- load o_1 on opstack

```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
12: aload_0
13: getfield n
14: astore_0
15: goto 3
16: iload_1
17: ireturn
```

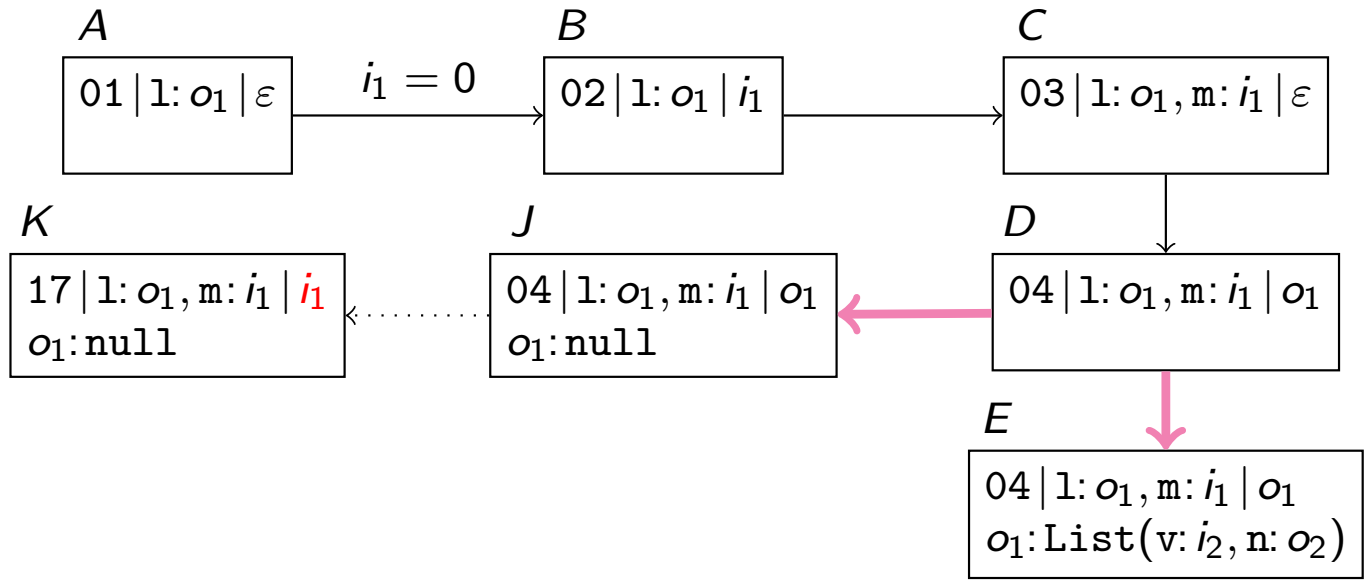


States E and J:

- need to know whether o_1 is `null`
- *refine* information about heap (refinement edges)

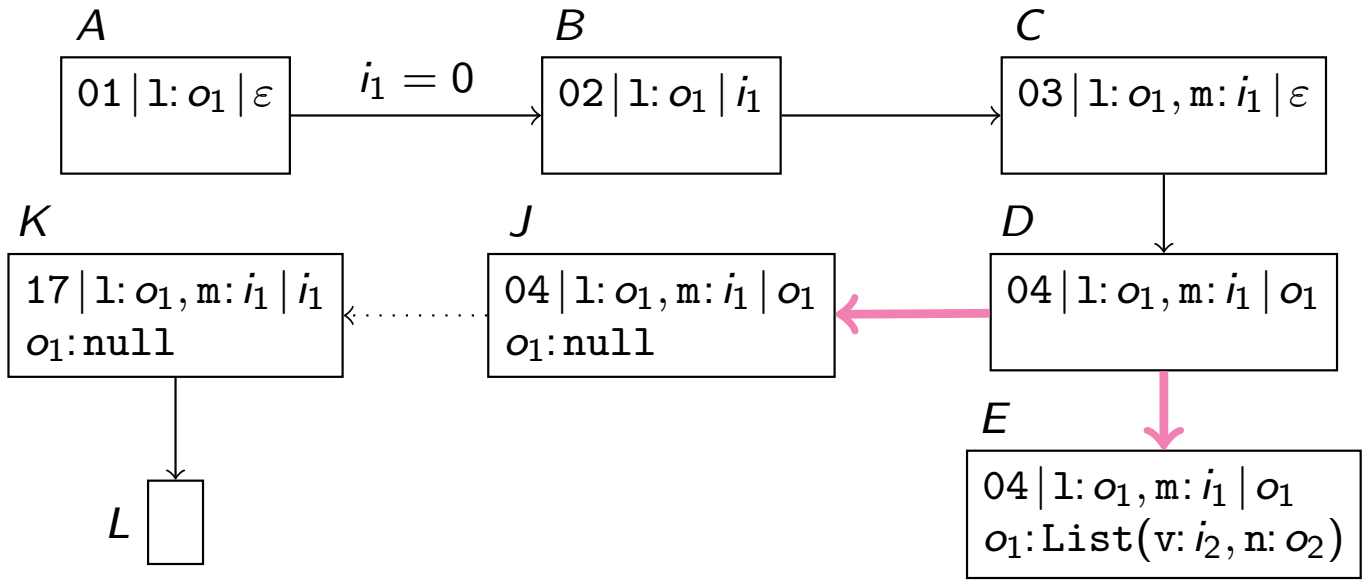
```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
12: aload_0
13: getfield n
14: astore_0
15: goto 3
16: iload_1
17: ireturn
```



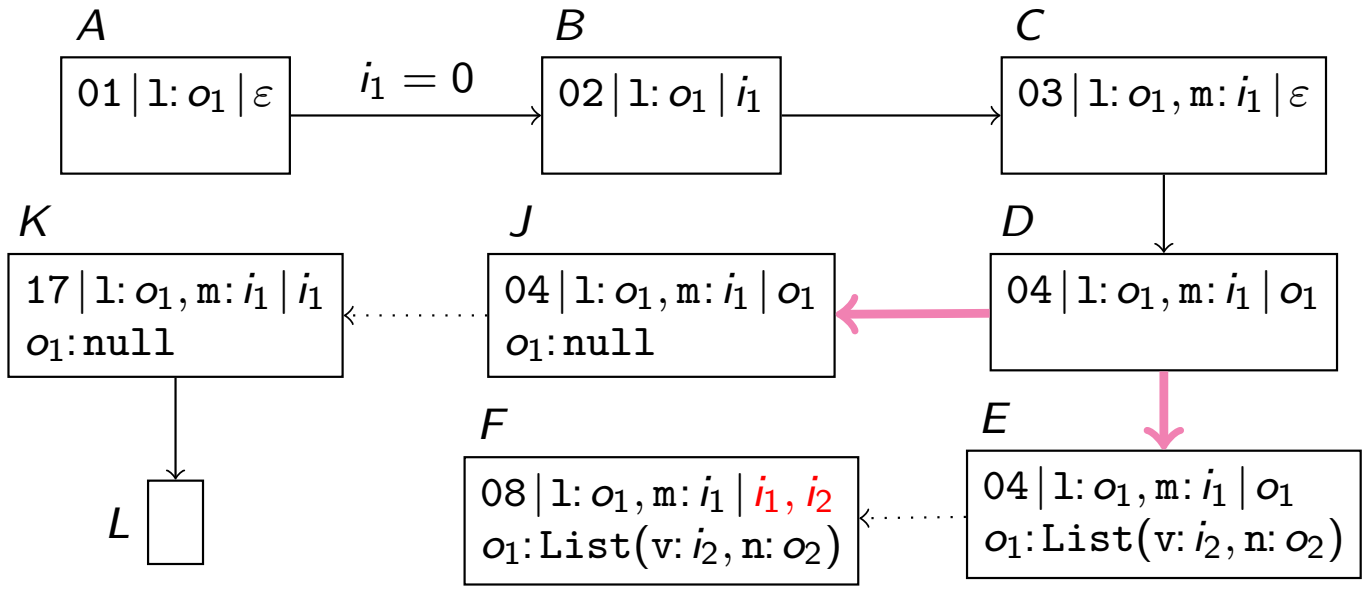

```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
12: aload_0
13: getfield n
14: astore_0
15: goto 3
16: iload_1
17: ireturn
```




```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
12: aload_0
13: getfield n
14: astore_0
15: goto 3
16: iload_1
17: ireturn
```

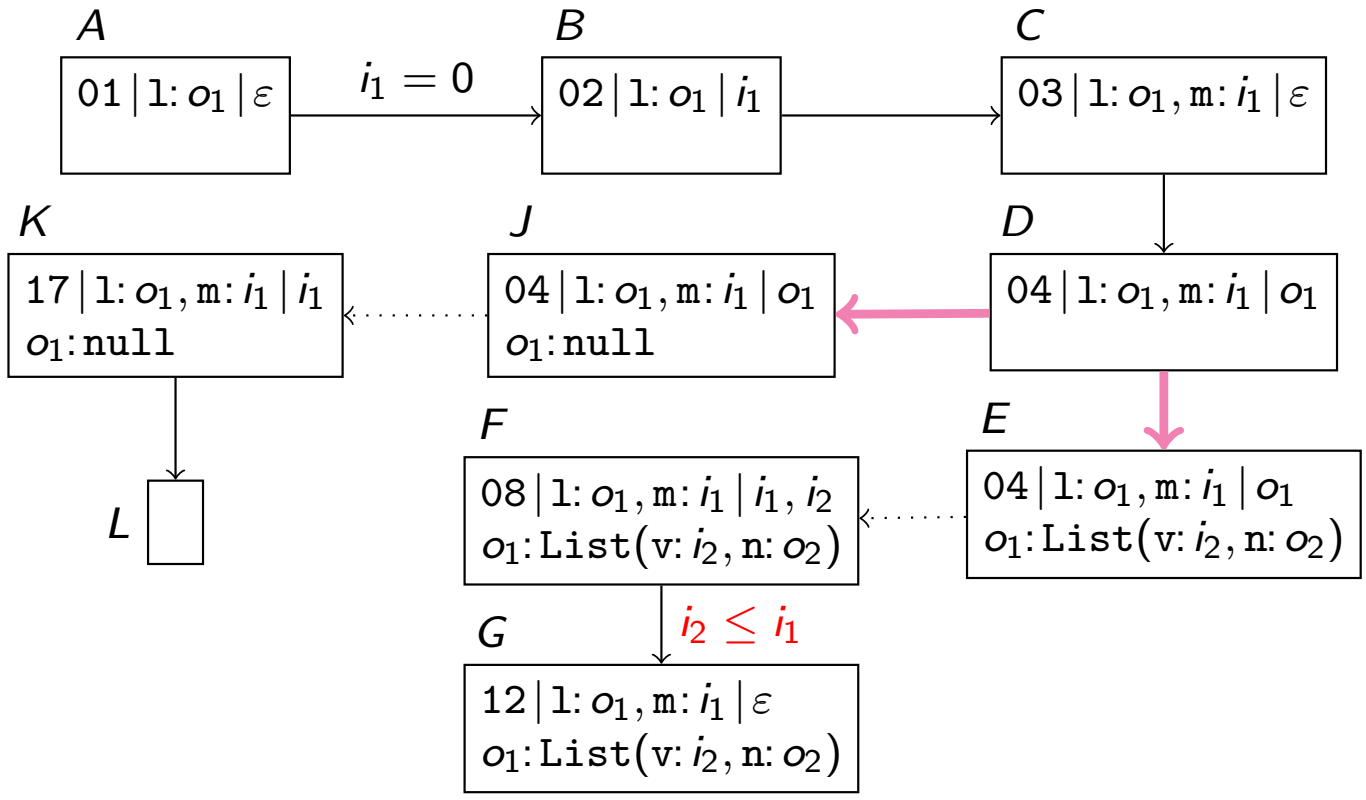


State F:

- load `l.v` (i_2) and `m` (i_1) on opstack

```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
12: aload_0
13: getfield n
14: astore_0
15: goto 3
16: iload_1
17: ireturn
```

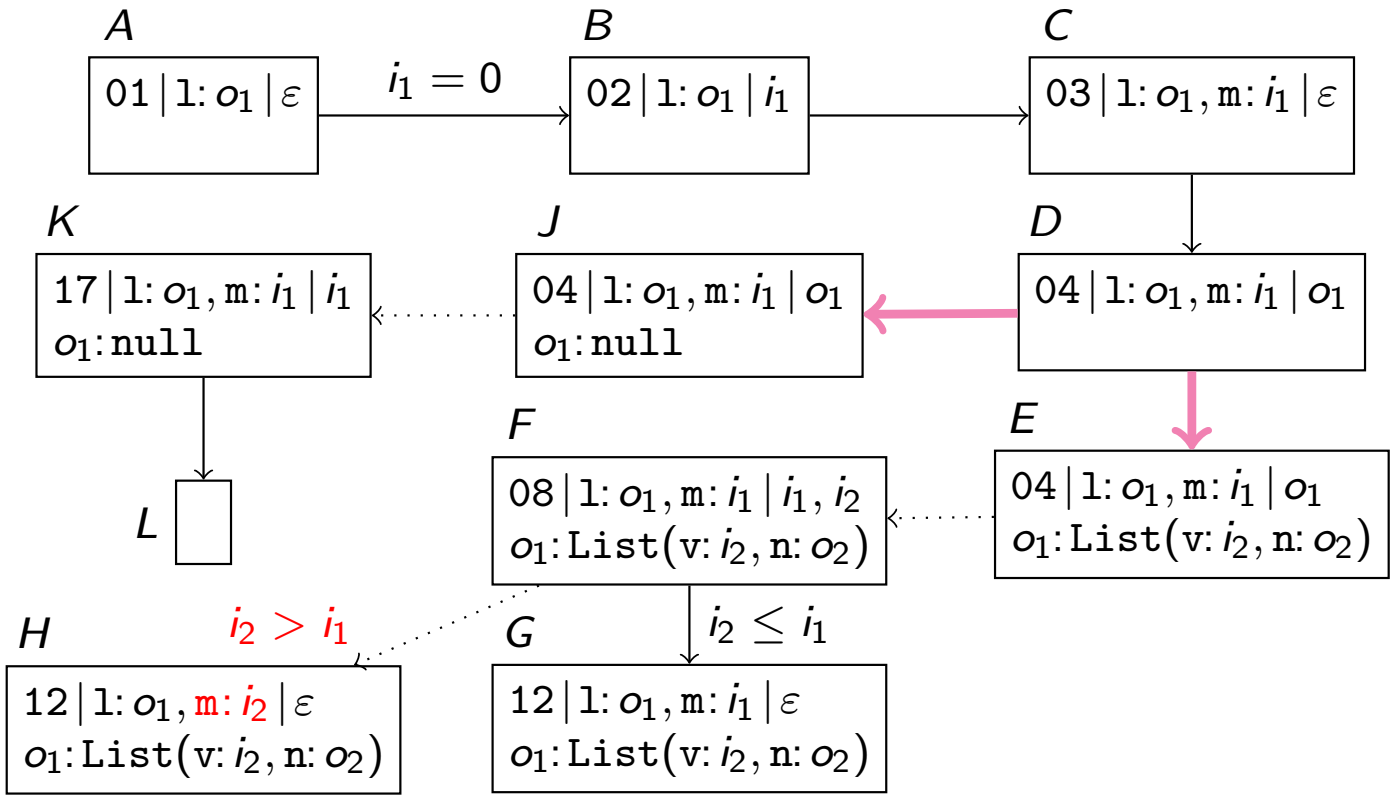



```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

State G:

- goto 12 if $\texttt{l.v} \leq \texttt{m}$
- *evaluation edge* labeled by condition $i_2 \leq i_1$

```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
12: aload_0
13: getfield n
14: astore_0
15: goto 3
16: iload_1
17: ireturn
```

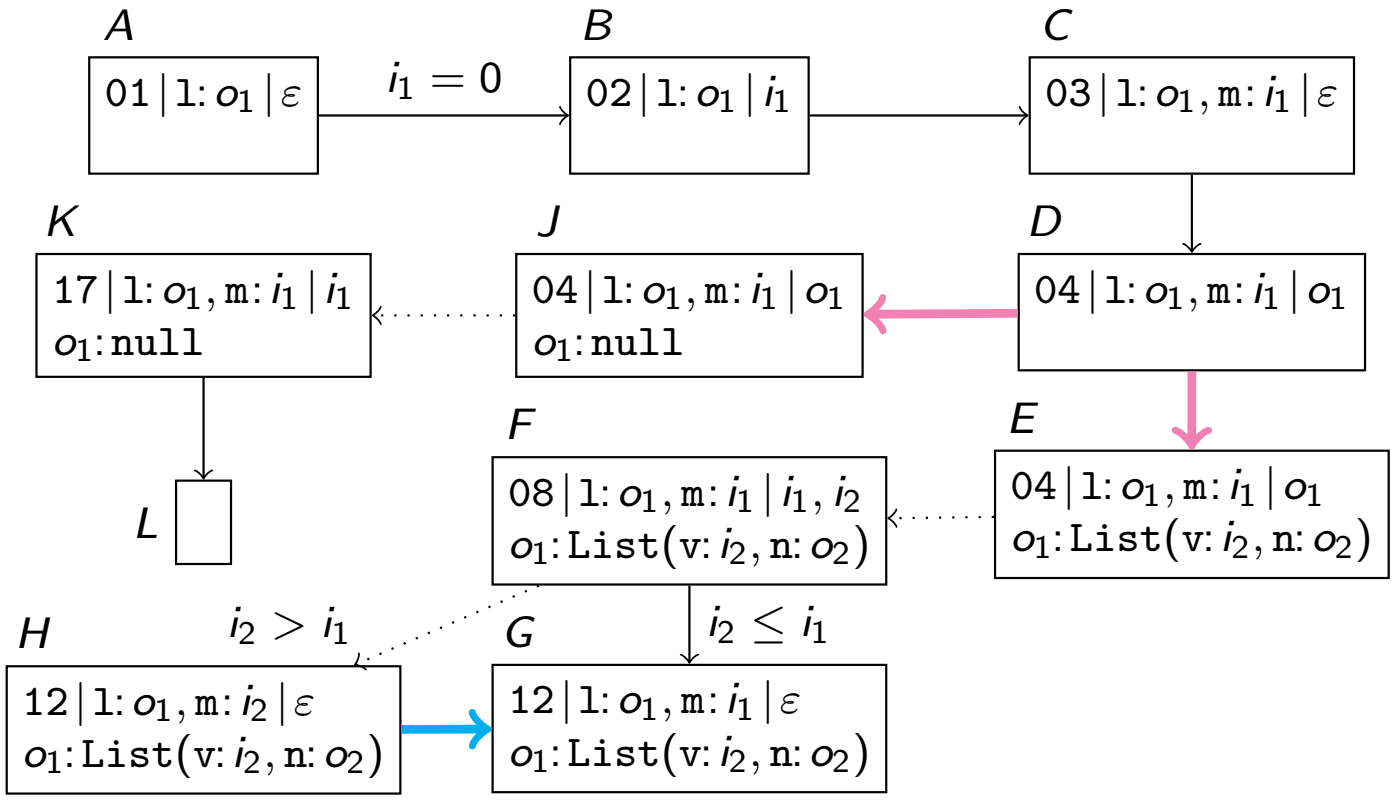



```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

State *H*:

- labeled by condition $i_2 > i_1$
- store `l.v` (i_2) in program variable `m`

```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
12: aload_0
13: getfield n
14: astore_0
15: goto 3
16: iload_1
17: ireturn
```

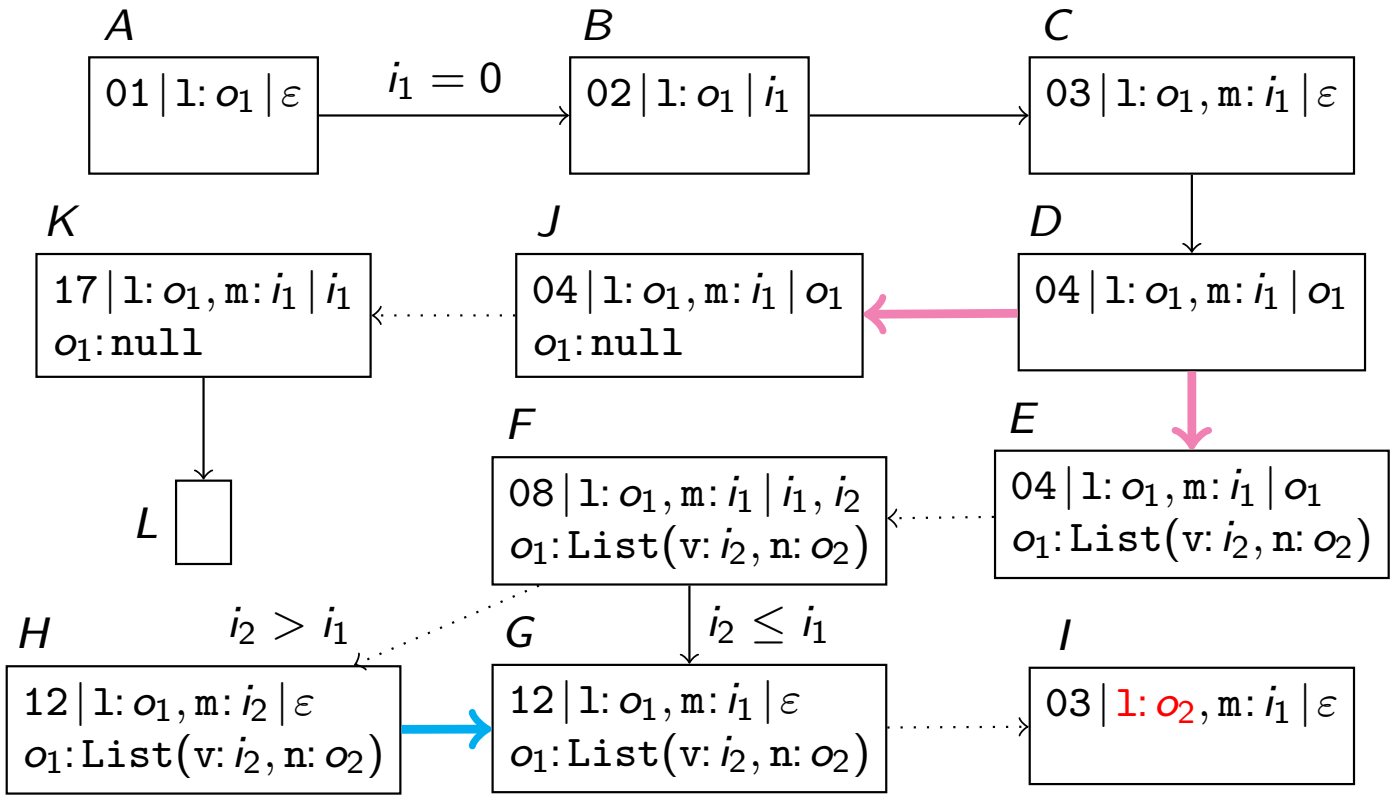



```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

State *H*:

- labeled by condition $i_2 > i_1$
- store `l.v` (i_2) in program variable `m`
- *G* more general than *H* (generalization edge)

```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
12: aload_0
13: getfield n
14: astore_0
15: goto 3
16: iload_1
17: ireturn
```

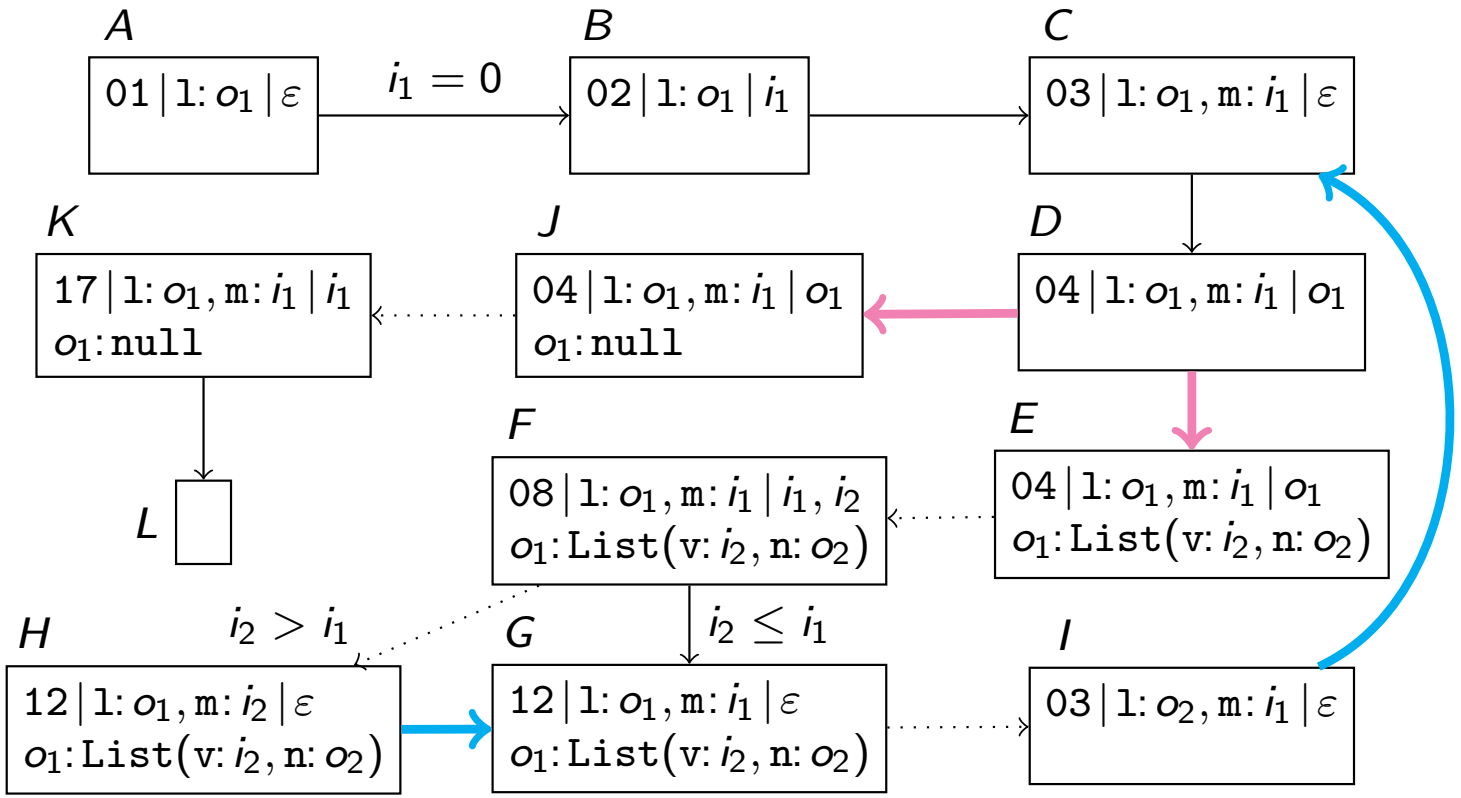



```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

State *l*:

- store `l.n` (o_2) in program variable `l`
- goto Line 3

```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
12: aload_0
13: getfield n
14: astore_0
15: goto 3
16: iload_1
17: ireturn
```

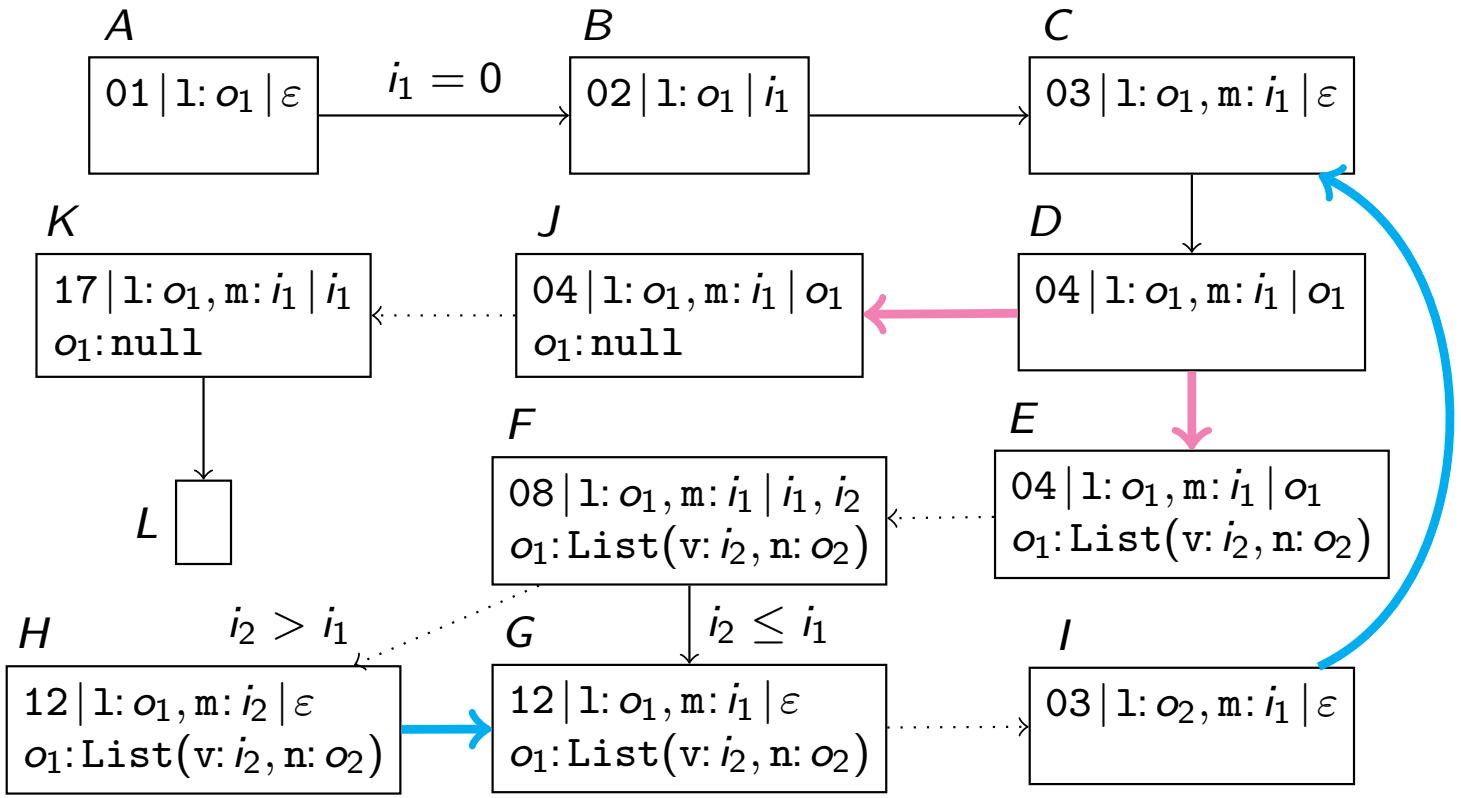



```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

State *I*:

- store `l.n` (o_2) in program variable `l`
- goto Line 3
- *I* corresponds to *C* (generalization edge)

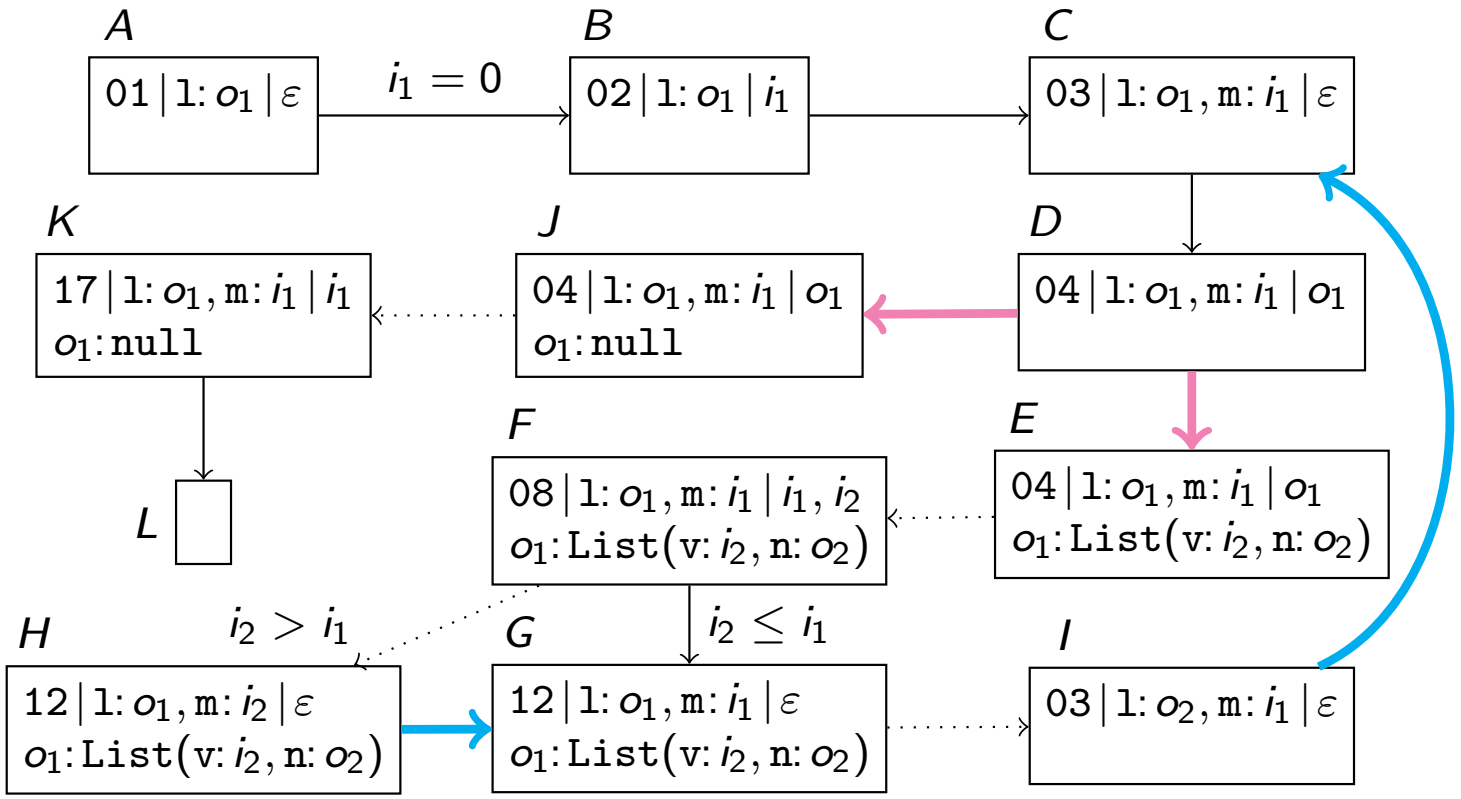
```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
12: aload_0
13: getfield n
14: astore_0
15: goto 3
16: iload_1
17: ireturn
```



Symbolic Execution Graph

- expand nodes until all leaves correspond to program ends

```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
12: aload_0
13: getfield n
14: astore_0
15: goto 3
16: iload_1
17: ireturn
```



Symbolic Execution Graph

- expand nodes until all leaves correspond to program ends
- by appropriate generalization steps,
  one always reaches a *finite* symbolic execution graph

Complexity of Java Methods

Concrete State (c, τ)

- state c with full information on symbolic references o

$\texttt{01} \mid \texttt{l}:o_1 \mid \varepsilon$

$o_1 : \texttt{List}(\texttt{v}: i_1, \texttt{n}: o_2)$

Complexity of Java Methods

Concrete State (c, τ)

- state c with full information on symbolic references o

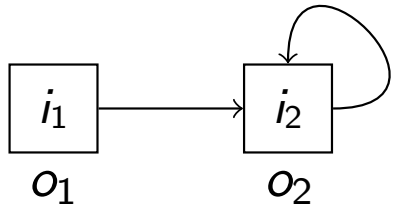
$$\texttt{01} \mid \texttt{l}{:}o_1 \mid \varepsilon$$
$$o_1 : \texttt{List}(\texttt{v}{:}i_1, \texttt{n}{:}o_2)$$
$$o_2 : \texttt{List}(\texttt{v}{:}i_2, \texttt{n}{:}o_2)$$

Complexity of Java Methods

Concrete State (c, τ)

- state c with full information on symbolic references o

$$\texttt{01} \mid \texttt{l}:o_1 \mid \varepsilon$$
$$o_1 : \texttt{List}(\texttt{v}: i_1, \texttt{n}: o_2)$$
$$o_2 : \texttt{List}(\texttt{v}: i_2, \texttt{n}: o_2)$$

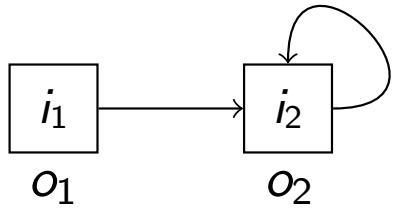


Complexity of Java Methods

Concrete State (c, τ)

- state c with full information on symbolic references o
- τ maps symb. integers i to $\mathbb{Z}$

$$\texttt{01} \mid \texttt{l}:o_1 \mid \varepsilon$$
$$o_1 : \texttt{List}(\texttt{v}: i_1, \texttt{n}: o_2)$$
$$o_2 : \texttt{List}(\texttt{v}: i_2, \texttt{n}: o_2)$$

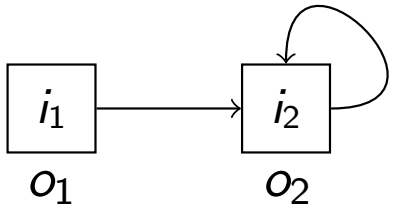


Complexity of Java Methods

Concrete State (c, τ)

- state c with full information on symbolic references o
- τ maps symb. integers i to $\mathbb{Z}$: $\tau(i_1) = -3, \quad \tau(i_2) = 5$

$$
\begin{array}{l}
\texttt{01} \mid \texttt{l}{:}o_1 \mid \varepsilon \\
o_1 : \texttt{List}(\texttt{v}{:}i_1, \texttt{n}{:}o_2) \\
o_2 : \texttt{List}(\texttt{v}{:}i_2, \texttt{n}{:}o_2)
\end{array}
$$



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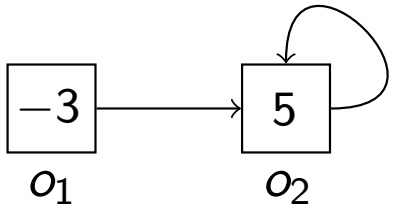
- state c with full information on symbolic references o
- τ maps symb. integers i to $\mathbb{Z}$: $\tau(i_1) = -3, \quad \tau(i_2) = 5$

$$\texttt{01} \mid \texttt{l}:o_1 \mid \varepsilon$$
$$o_1 : \texttt{List}(\texttt{v}: i_1, \texttt{n}: o_2)$$
$$o_2 : \texttt{List}(\texttt{v}: i_2, \texttt{n}: o_2)$$

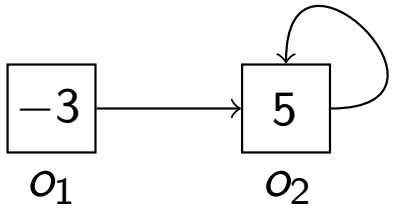


Complexity of Java Methods

Concrete State (c, τ)

- state c with full information on symbolic references o
- τ maps symb. integers i to $\mathbb{Z}$: $\tau(i_1) = -3, \quad \tau(i_2) = 5$
- abstract state s *represents* (c, τ)
  if s is more general than c

$$\texttt{01} \mid \texttt{l}:o_1 \mid \varepsilon$$
$$o_1 : \texttt{List}(\texttt{v}: i_1, \texttt{n}: o_2)$$
$$o_2 : \texttt{List}(\texttt{v}: i_2, \texttt{n}: o_2)$$

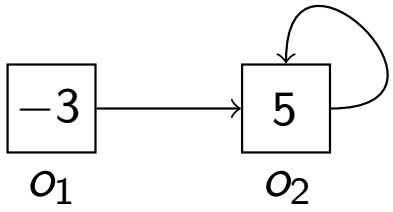


Complexity of Java Methods

Concrete State (c, τ)

- state c with full information on symbolic references o
- τ maps symb. integers i to $\mathbb{Z}$: $\tau(i_1) = -3, \quad \tau(i_2) = 5$
- abstract state s *represents* (c, τ)
  if s is more general than c
- every concrete Java evaluation can be *embedded*
  into the symbolic execution graph

$$\texttt{01} \mid \texttt{l}{:}o_1 \mid \varepsilon$$
$$o_1{:}\texttt{List}(\texttt{v}{:}i_1, \texttt{n}{:}o_2)$$
$$o_2{:}\texttt{List}(\texttt{v}{:}i_2, \texttt{n}{:}o_2)$$



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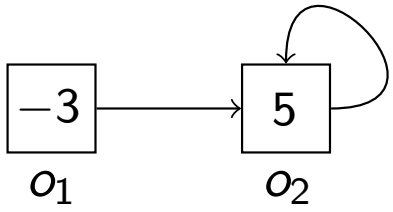
- state c with full information on symbolic references o
- τ maps symb. integers i to $\mathbb{Z}$: $\tau(i_1) = -3, \quad \tau(i_2) = 5$
- abstract state s *represents* (c, τ)
  if s is more general than c

$$\texttt{01} \mid \texttt{l}:o_1 \mid \varepsilon$$
$$o_1 : \texttt{List}(\texttt{v}: i_1, \texttt{n}: o_2)$$
$$o_2 : \texttt{List}(\texttt{v}: i_2, \texttt{n}: o_2)$$



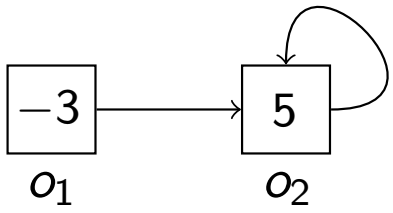
Size of object at reference o in concrete state (c, τ)

Complexity of Java Methods

Concrete State (c, τ)

- state c with full information on symbolic references o
- τ maps symb. integers i to $\mathbb{Z}$: $\tau(i_1) = -3, \quad \tau(i_2) = 5$
- abstract state s *represents* (c, τ)
  if s is more general than c

$$\texttt{01} \mid \texttt{l}:o_1 \mid \varepsilon$$
$$o_1 : \texttt{List}(\texttt{v}: i_1, \texttt{n}: o_2)$$
$$o_2 : \texttt{List}(\texttt{v}: i_2, \texttt{n}: o_2)$$



Size of object at reference o in concrete state (c, τ)

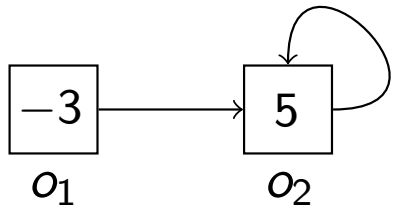
$\|o\|$ = number of objects reachable from o + absolute values of integers in their fields

Complexity of Java Methods

Concrete State (c, τ)

- state c with full information on symbolic references o
- τ maps symb. integers i to $\mathbb{Z}$: $\tau(i_1) = -3, \quad \tau(i_2) = 5$
- abstract state s *represents* (c, τ) if s is more general than c

$$\texttt{01} \mid \texttt{l}:o_1 \mid \varepsilon$$
$$o_1 : \texttt{List}(\texttt{v}: i_1, \texttt{n}: o_2)$$
$$o_2 : \texttt{List}(\texttt{v}: i_2, \texttt{n}: o_2)$$



Size of object at reference o in concrete state (c, τ)

$\|o\|$ = number of objects reachable from o + absolute values of integers in their fields

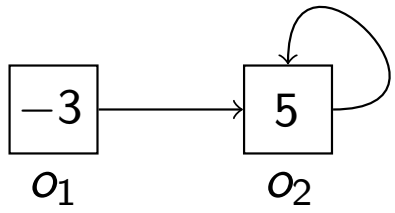
$\|o_2\| = 1 + |5| = 6$

Complexity of Java Methods

Concrete State (c, τ)

- state c with full information on symbolic references o
- τ maps symb. integers i to $\mathbb{Z}$: $\tau(i_1) = -3, \quad \tau(i_2) = 5$
- abstract state s *represents* (c, τ) if s is more general than c

$$\texttt{01} \mid \texttt{l}:o_1 \mid \varepsilon$$
$$o_1 : \texttt{List}(\texttt{v}: i_1, \texttt{n}: o_2)$$
$$o_2 : \texttt{List}(\texttt{v}: i_2, \texttt{n}: o_2)$$



Size of object at reference o in concrete state (c, τ)

$\|o\|$ = number of objects reachable from o + absolute values of integers in their fields

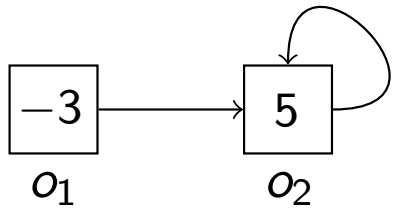
$$\|o_2\| = 1 + |5| = 6$$
$$\|o_1\| = 2 + |-3| + |5| = 10$$

Complexity of Java Methods

Concrete State (c, τ)

- state c with full information on symbolic references o
- τ maps symb. integers i to $\mathbb{Z}$: $\tau(i_1) = -3, \quad \tau(i_2) = 5$
- abstract state s *represents* (c, τ) if s is more general than c

$$\texttt{01} \mid \texttt{l}{:}o_1 \mid \varepsilon$$
$$o_1{:}\texttt{List}(\texttt{v}{:}i_1, \texttt{n}{:}o_2)$$
$$o_2{:}\texttt{List}(\texttt{v}{:}i_2, \texttt{n}{:}o_2)$$



Size of object at reference o in concrete state (c, τ)

$\|o\|$ = number of objects reachable from o + absolute values of integers in their fields

$$\|o_2\| = 1 + |5| = 6$$
$$\|o_1\| = 2 + |-3| + |5| = 10$$

Complexity Bound b for abstract state s

Complexity of Java Methods

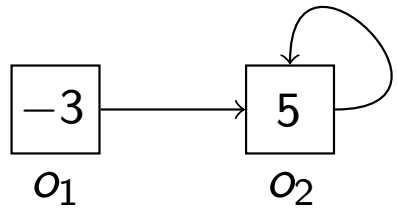
Concrete State (c, τ)

- state c with full information on symbolic references o
- τ maps symb. integers i to $\mathbb{Z}$: $\tau(i_1) = -3, \quad \tau(i_2) = 5$
- abstract state s *represents* (c, τ) if s is more general than c

o_1 | 1 : o_1 | ε

o_1 : List(v : i_1, n : o_2)

o_2 : List(v : i_2, n : o_2)



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Complexity Bound b for abstract state s

b is arithmetic term with

$b \geq$ length of any Java evaluation starting with corresponding concrete state represented by s

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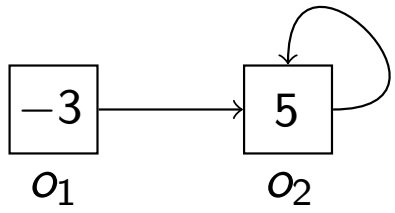
  if s is more general than c

$$\texttt{01} \mid \texttt{l}:o_1 \mid \varepsilon$$
$$o_1 : \texttt{List}(\texttt{v}: i_1, \texttt{n}: o_2)$$
$$o_2 : \texttt{List}(\texttt{v}: i_2, \texttt{n}: o_2)$$



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Complexity Bound b for abstract state s
with symb. integers $i_1, \ldots, i_n$ and symb. references $o_1, \ldots, o_m$

b is arithmetic term with

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Complexity of Java Methods

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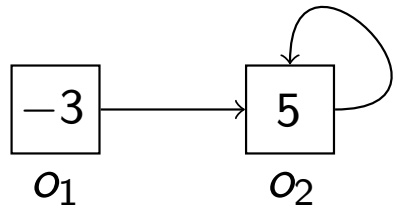

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Complexity Bound b for abstract state s
with symb. integers $i_1, \ldots, i_n$ and symb. references $o_1, \ldots, o_m$

b is arithmetic term with variables $i_1, \ldots, i_n, \|o_1\|, \ldots, \|o_m\|$ such that:

$b \geq$ length of any Java evaluation
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Complexity of Java Methods

Concrete State (c, τ)

- state c with full information on symbolic references o
- τ maps symb. integers i to $\mathbb{Z}$: $\tau(i_1) = -3, \quad \tau(i_2) = 5$
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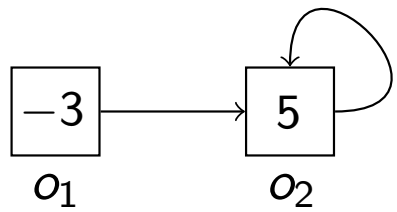


Size of object at reference o in concrete state (c, τ)

$\|o\|$ = number of objects reachable from o + absolute values of integers in their fields

$$\|o_2\| = 1 + |5| = 6$$
$$\|o_1\| = 2 + |-3| + |5| = 10$$

Complexity Bound b for abstract state s with symb. integers $i_1, \ldots, i_n$ and symb. references $o_1, \ldots, o_m$

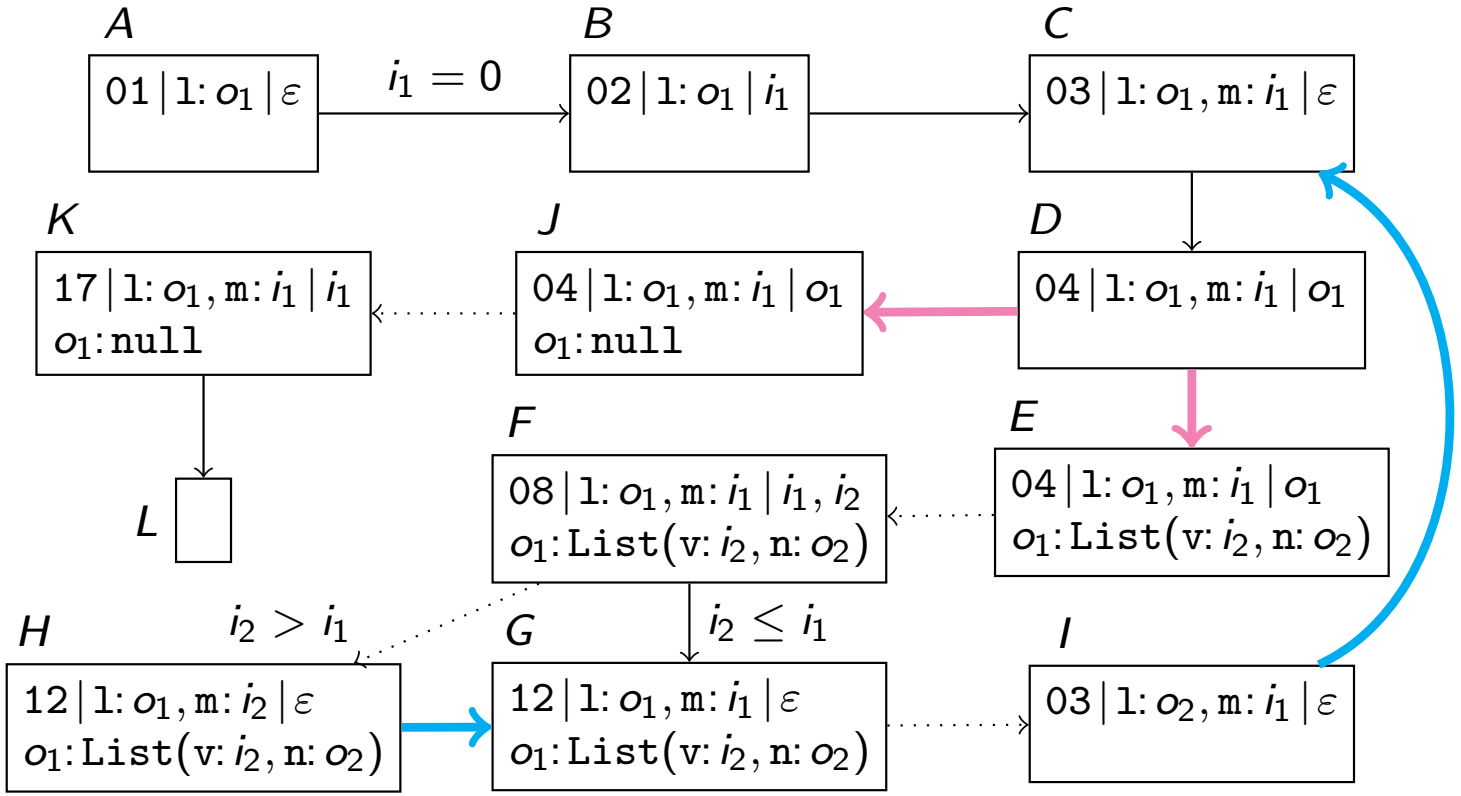
b is arithmetic term with variables $i_1, \ldots, i_n, \|o_1\|, \ldots, \|o_m\|$ such that:

$b \geq$ length of any Java evaluation starting with corresponding concrete state represented by s

for all instantiations of $i_1, \ldots, i_n, \|o_1\|, \ldots, \|o_m\|$

Complexity of Java Methods

```
01: iconst_0
02: istore_1
03: aload_0
04: ifnull 16
05: aload_0
06: getfield v
07: iload_1
08: if_icmple 12
09: aload_0
10: getfield v
11: istore_1
      ...
```



Complexity Bound b for abstract state s
with symb. integers $i_1, \ldots, i_n$ and symb. references $o_1, \ldots, o_m$

b is arithmetic term with variables $i_1, \ldots, i_n, \|o_1\|, \ldots, \|o_m\|$ such that:

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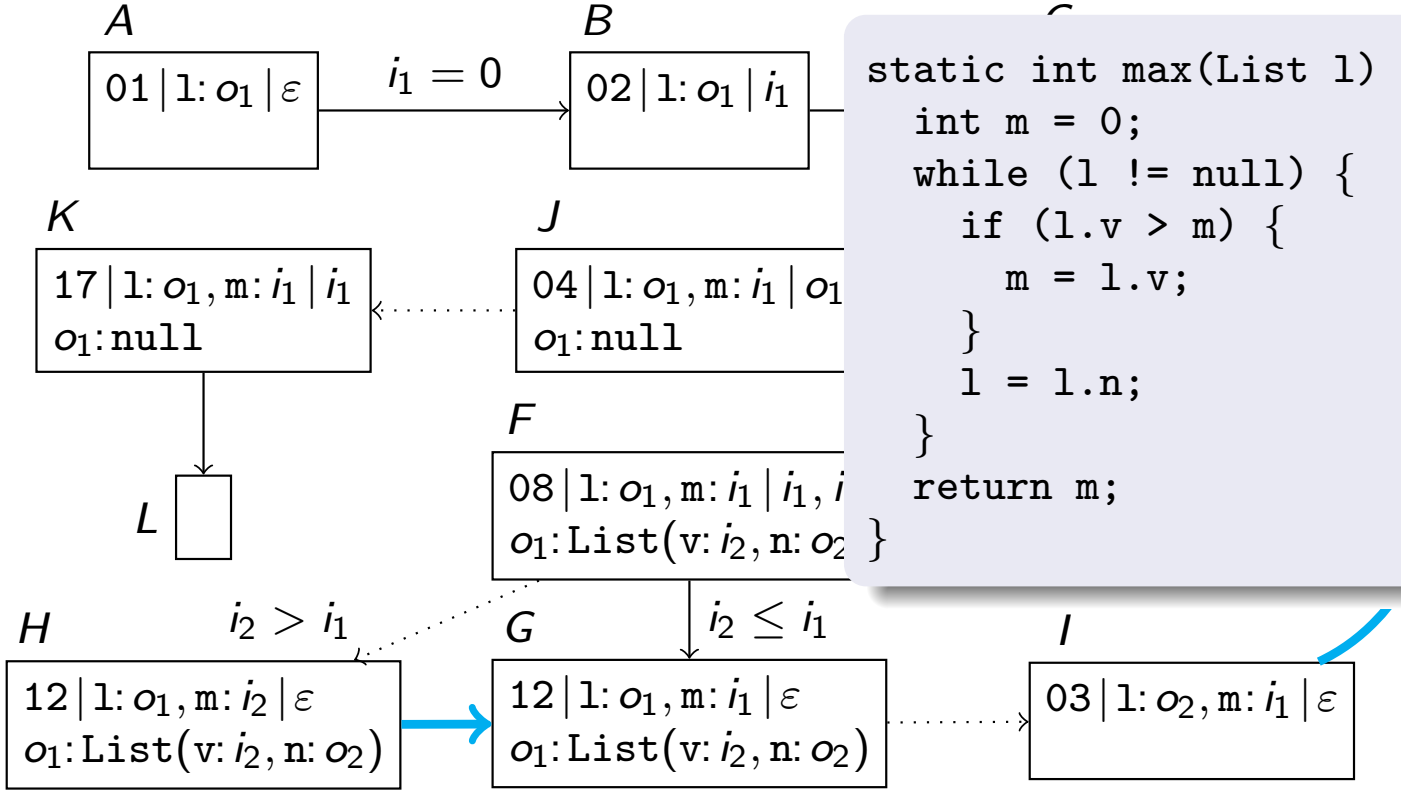
      ...
```



Complexity Bound b for abstract state s
with symb. integers $i_1, \ldots, i_n$ and symb. references $o_1, \ldots, o_m$

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Complexity of Java Methods

```
01: iconst_0
      ...
```

A

$$01 \mid \mathtt{l}: o_1 \mid \varepsilon$$

```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

Complexity Bound b for abstract state s
with symb. integers $i_1, \ldots, i_n$ and symb. references $o_1, \ldots, o_m$

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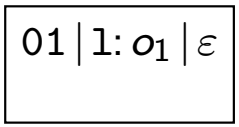
Complexity of Java Methods

```
01: iconst_0
    ...
```

A




```
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    if (l.v > m) {
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    }
    l = l.n;
  }
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}
```

$13 \cdot \|o_1\| + 6$ is complexity bound for State A

Complexity Bound b for abstract state s
with symb. integers $i_1, \ldots, i_n$ and symb. references $o_1, \ldots, o_m$

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Complexity of Java Methods

```
01: iconst_0
      ...
```

A

$01 \mid l{:}\, o_1 \mid \varepsilon$

```
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      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

$13 \cdot \|o_1\| + 6$ is complexity bound for State A

- for every concrete state at begin of method `max`

Complexity Bound b for abstract state s
with symb. integers $i_1, \ldots, i_n$ and symb. references $o_1, \ldots, o_m$

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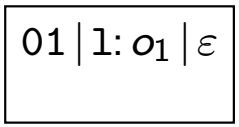
for all instantiations of $i_1, \ldots, i_n, \|o_1\|, \ldots, \|o_m\|$

Complexity of Java Methods

```
01: iconst_0
    ...
```

A

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    }
    l = l.n;
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  return m;
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```

$13 \cdot \|o_1\| + 6$ is complexity bound for State A

- for every concrete state at begin of method `max` where o_1 is a `List` of size k,

Complexity Bound b for abstract state s
with symb. integers $i_1, \ldots, i_n$ and symb. references $o_1, \ldots, o_m$

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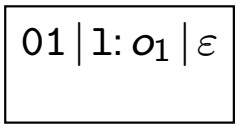
Complexity of Java Methods

```
01: iconst_0
    ...
```

A

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$13 \cdot \|o_1\| + 6$ is complexity bound for State A

- for every concrete state at begin of method `max` where o_1 is a `List` of size k,
  Java evaluation has at most length $13 \cdot k + 6$

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with symb. integers $i_1, \ldots, i_n$ and symb. references $o_1, \ldots, o_m$

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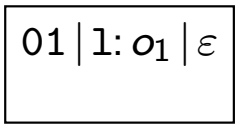
$b \geq$ length of any Java evaluation
starting with corresponding concrete state represented by s

for all instantiations of $i_1, \ldots, i_n, \|o_1\|, \ldots, \|o_m\|$

Complexity of Java Methods

```
01: iconst_0
    ...
```

A




```
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  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

$13 \cdot \|o_1\| + 6$ is complexity bound for State A

- for every concrete state at begin of method `max` where o_1 is a `List` of size k,
  Java evaluation has at most length $13 \cdot k + 6$
- $13 \cdot \|\texttt{l}\| + 6$ is upper bound for runtime of `max`

Complexity Bound b for abstract state s
with symb. integers $i_1, \ldots, i_n$ and symb. references $o_1, \ldots, o_m$

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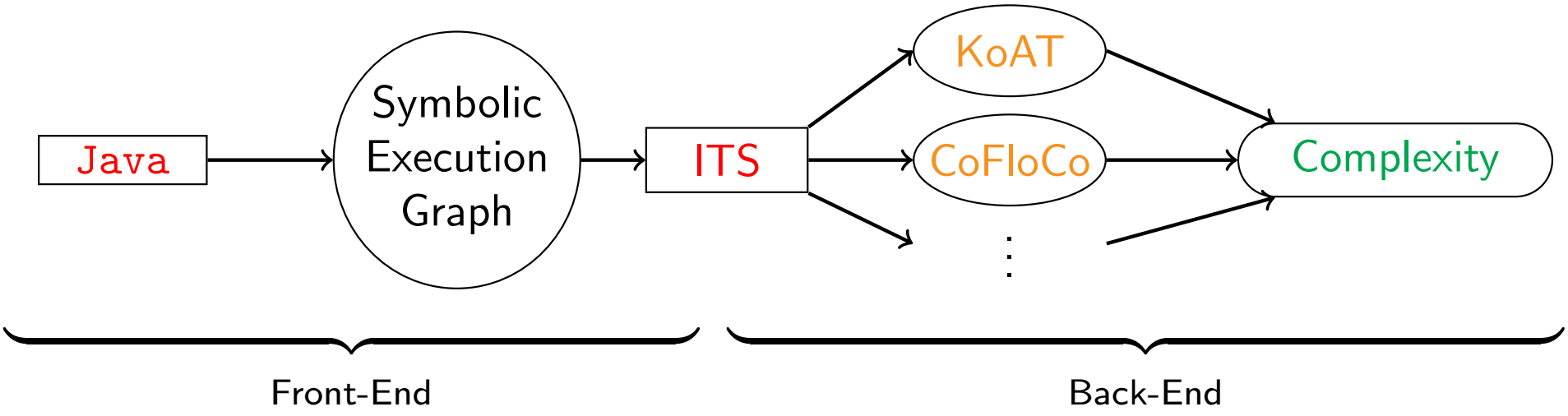
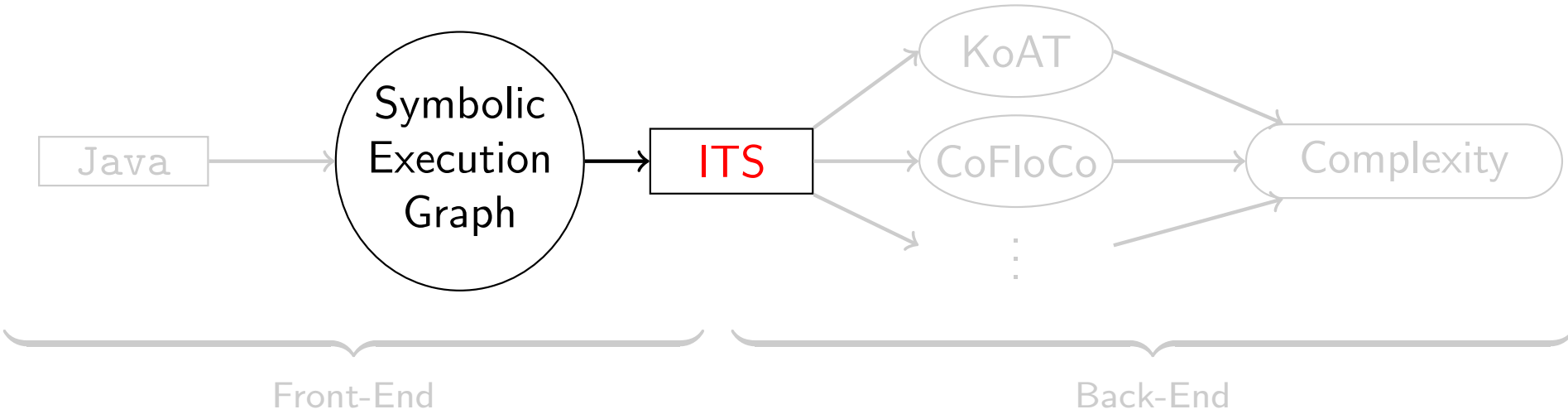
for all instantiations of $i_1, \ldots, i_n, \|o_1\|, \ldots, \|o_m\|$

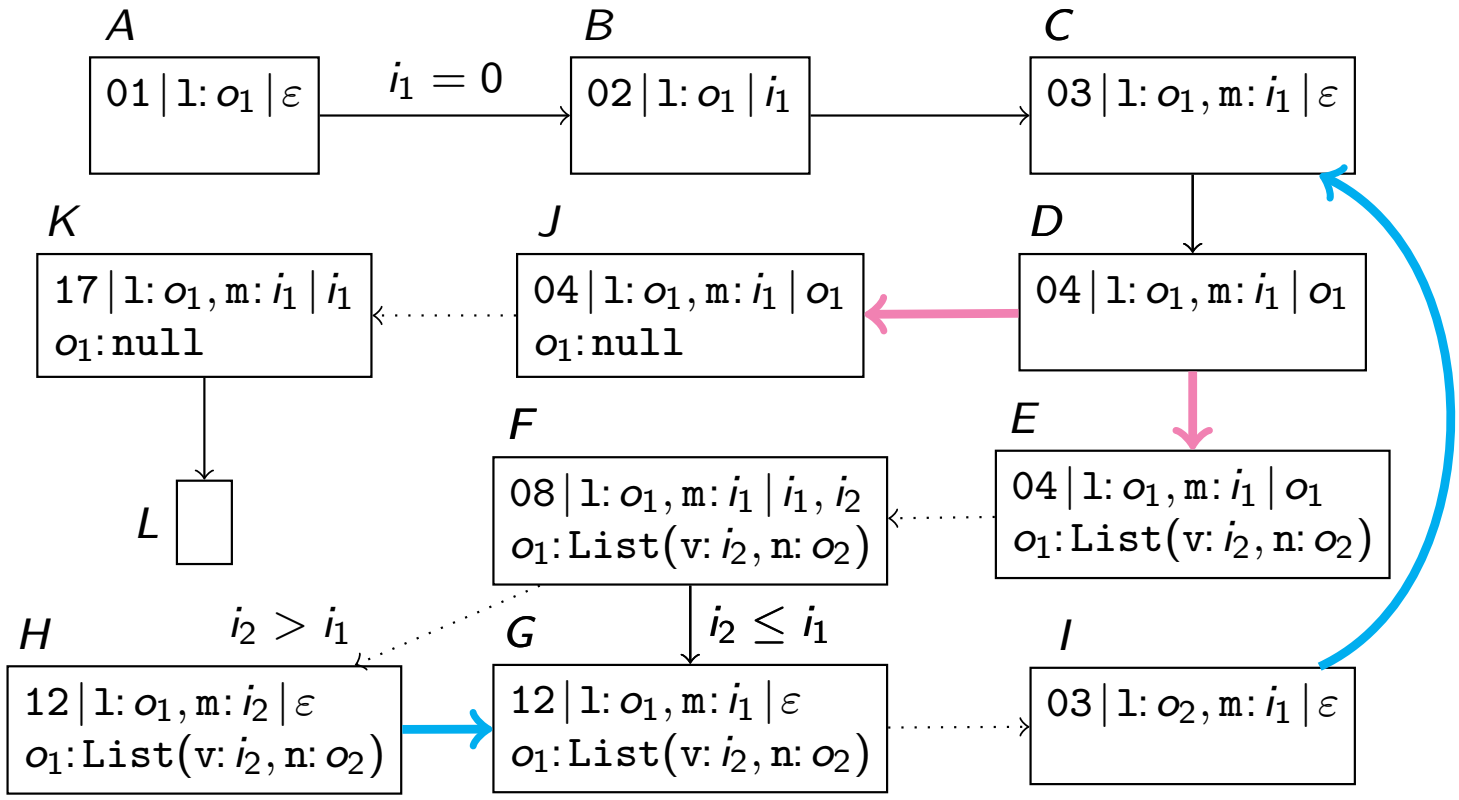
AProVE for Complexity Analysis of Java



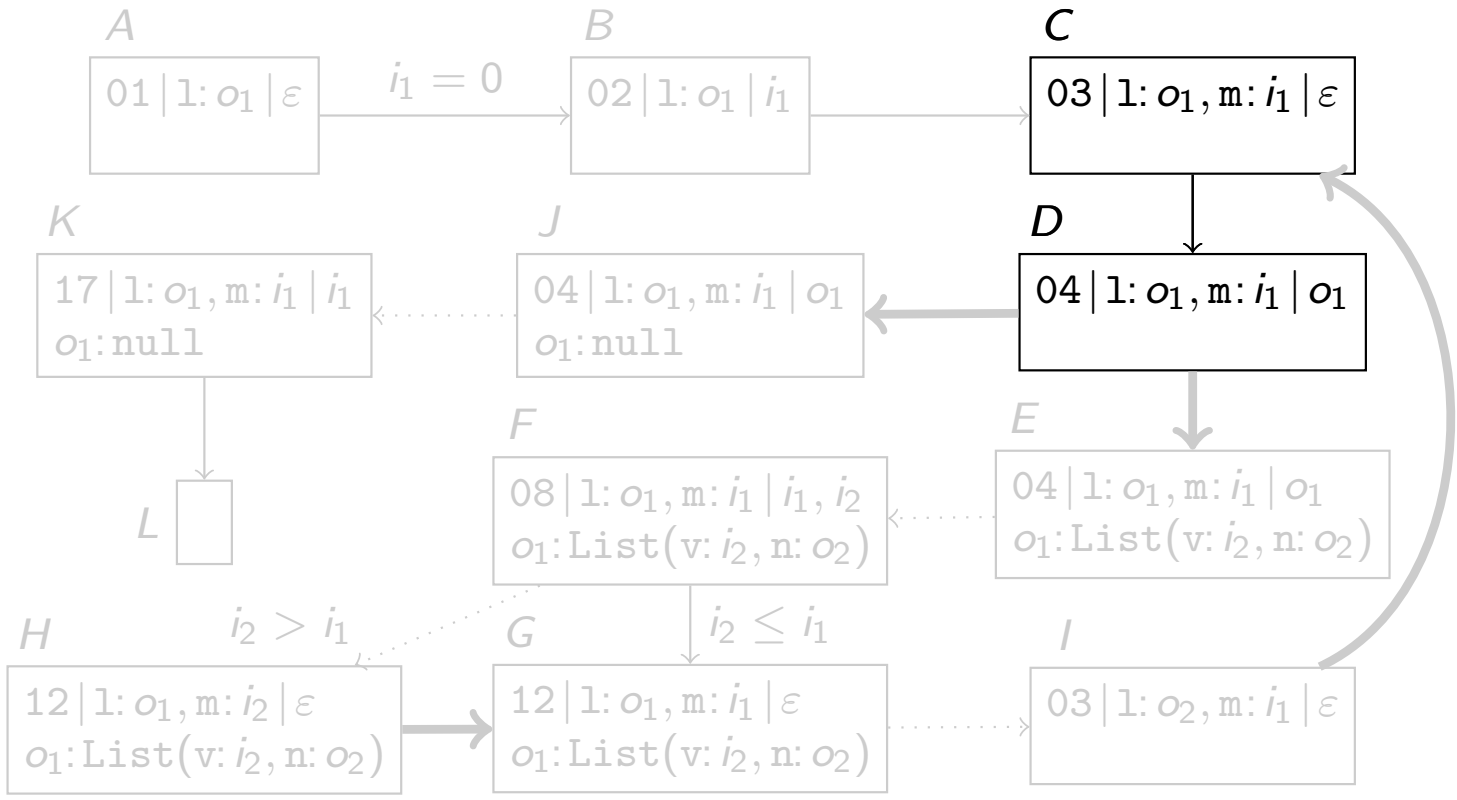
AProVE for Complexity Analysis of Java



Transform Evaluation Edges



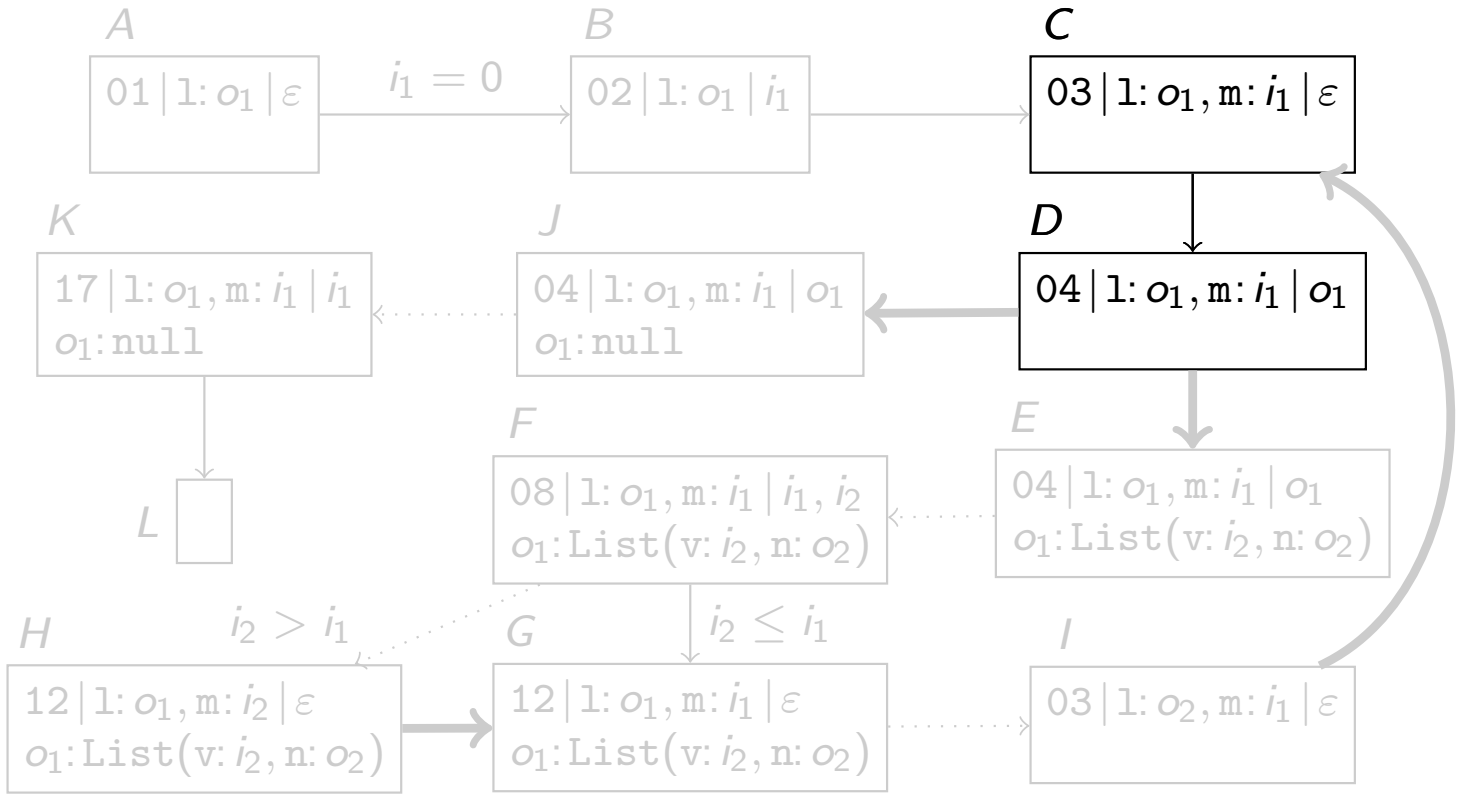
Transform Evaluation Edges



Transform Evaluation Edges

ITS over variables

i_1, $\|o_1\|$

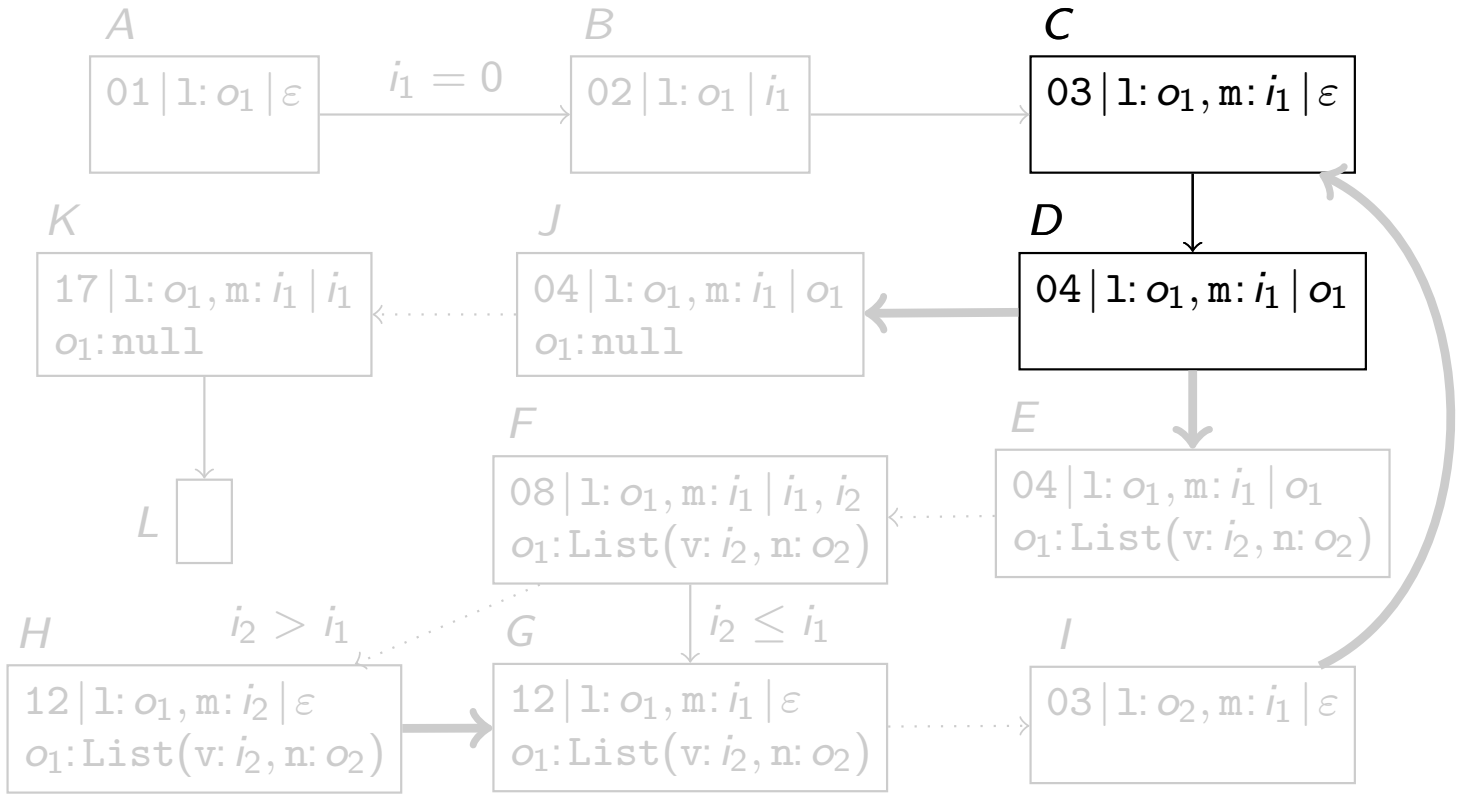


Transform Evaluation Edges

ITS over variables

i_1, $\|o_1\|$

i_1', $\|o_1\|'$

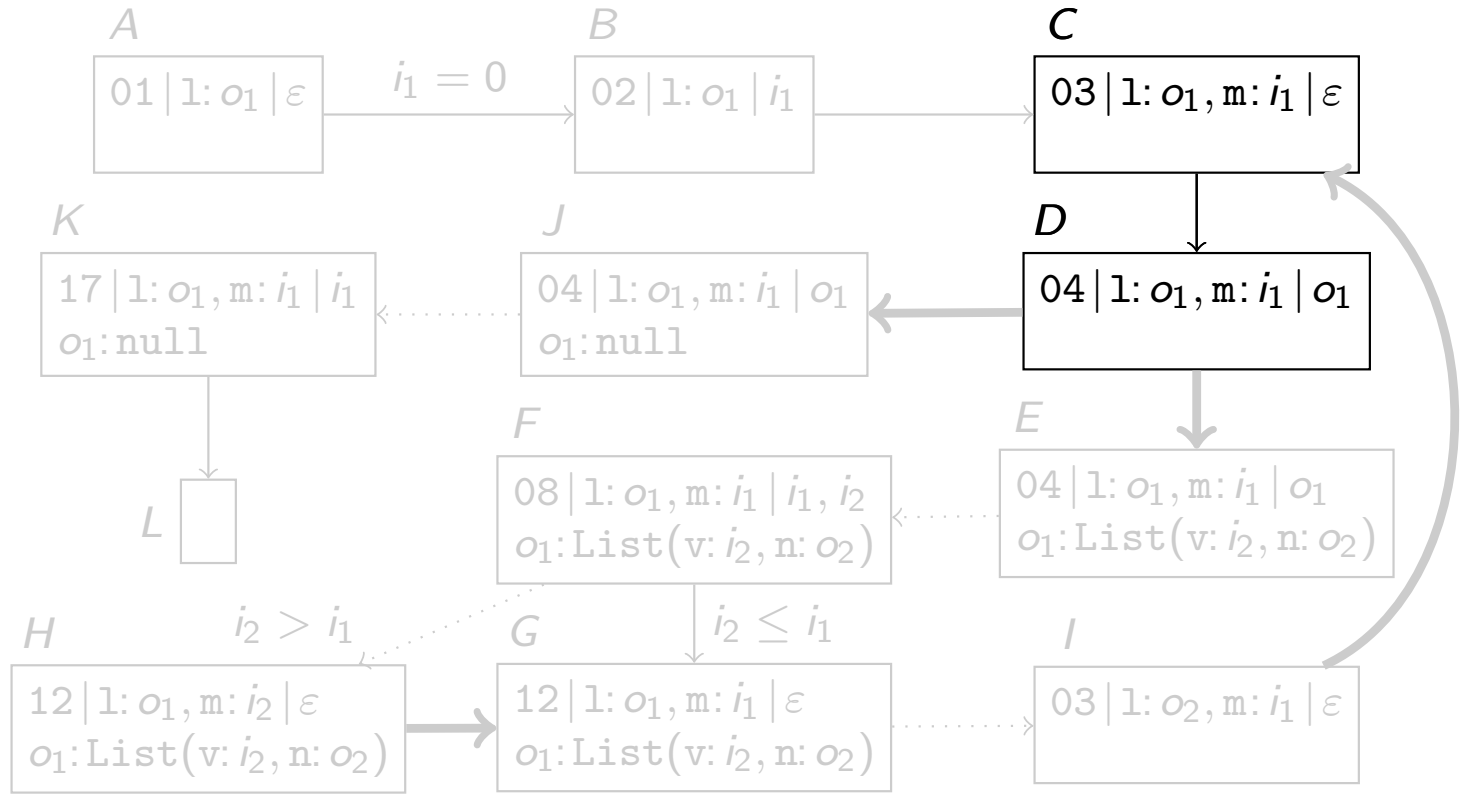


Transform Evaluation Edges

ITS over variables

i_1, $\|o_1\|$

i_1', $\|o_1\|'$



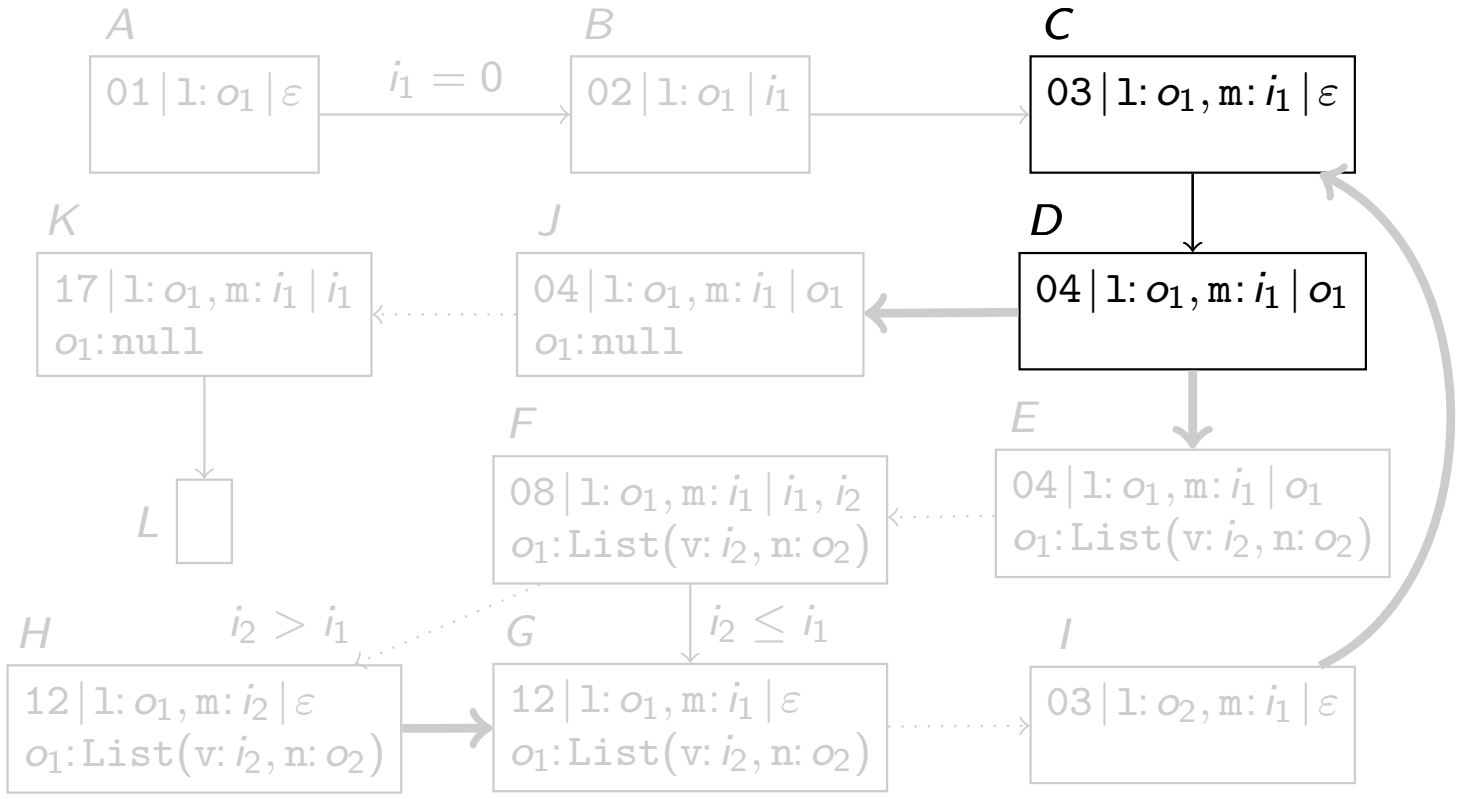
$C \longrightarrow D$

Transform Evaluation Edges

ITS over variables

i_1, $\|o_1\|$

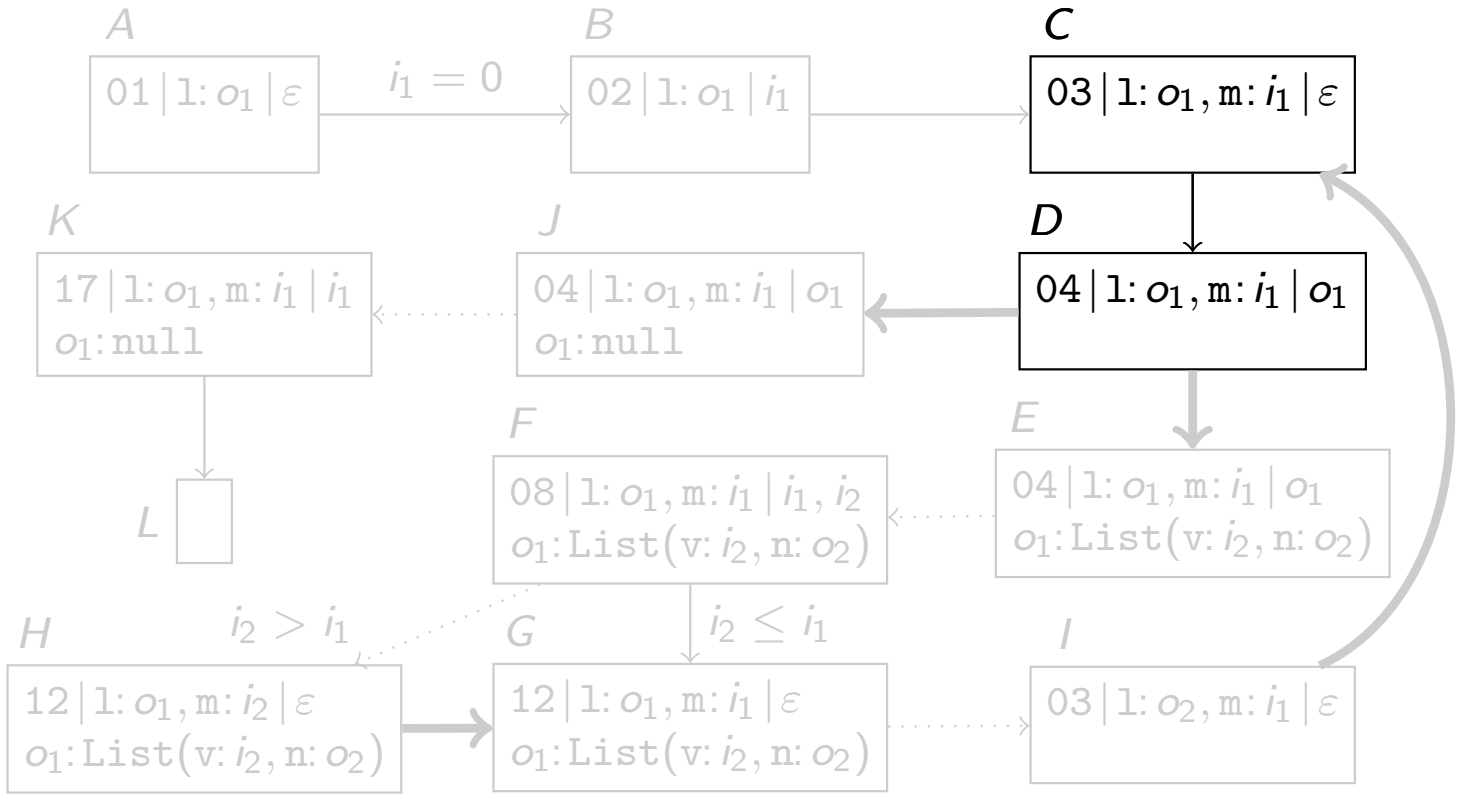
i_1', $\|o_1\|'$



$C \xrightarrow{1} D$

weight: 1

Transform Evaluation Edges



ITS over variables

i_1, $\|o_1\|$

i_1', $\|o_1\|'$

$C \xrightarrow{1} D$

weight: 1

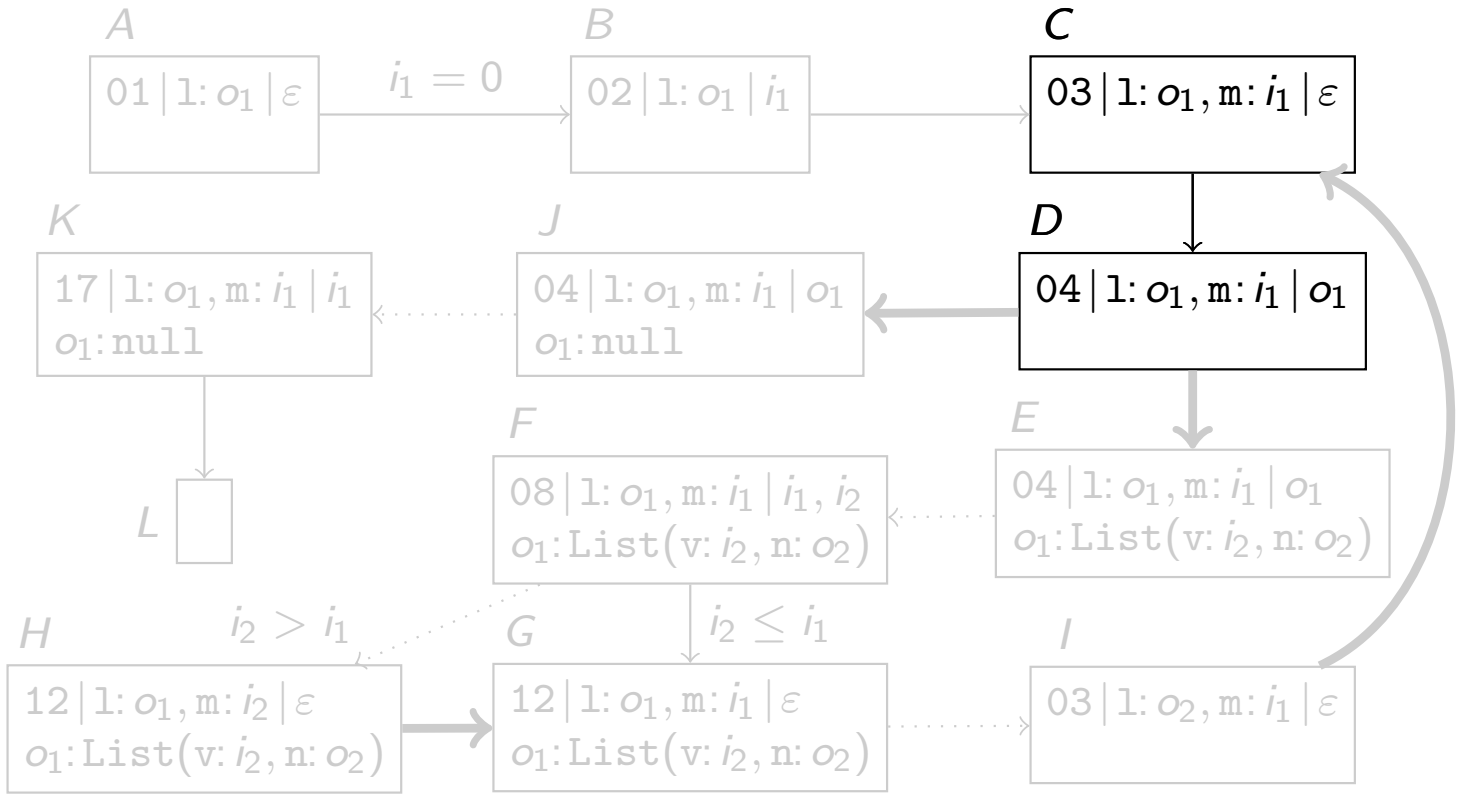
condition: $\|o\| = 0$ if $o : \texttt{null}$

Transform Evaluation Edges

ITS over variables

i_1, $\|o_1\|$

i_1', $\|o_1\|'$



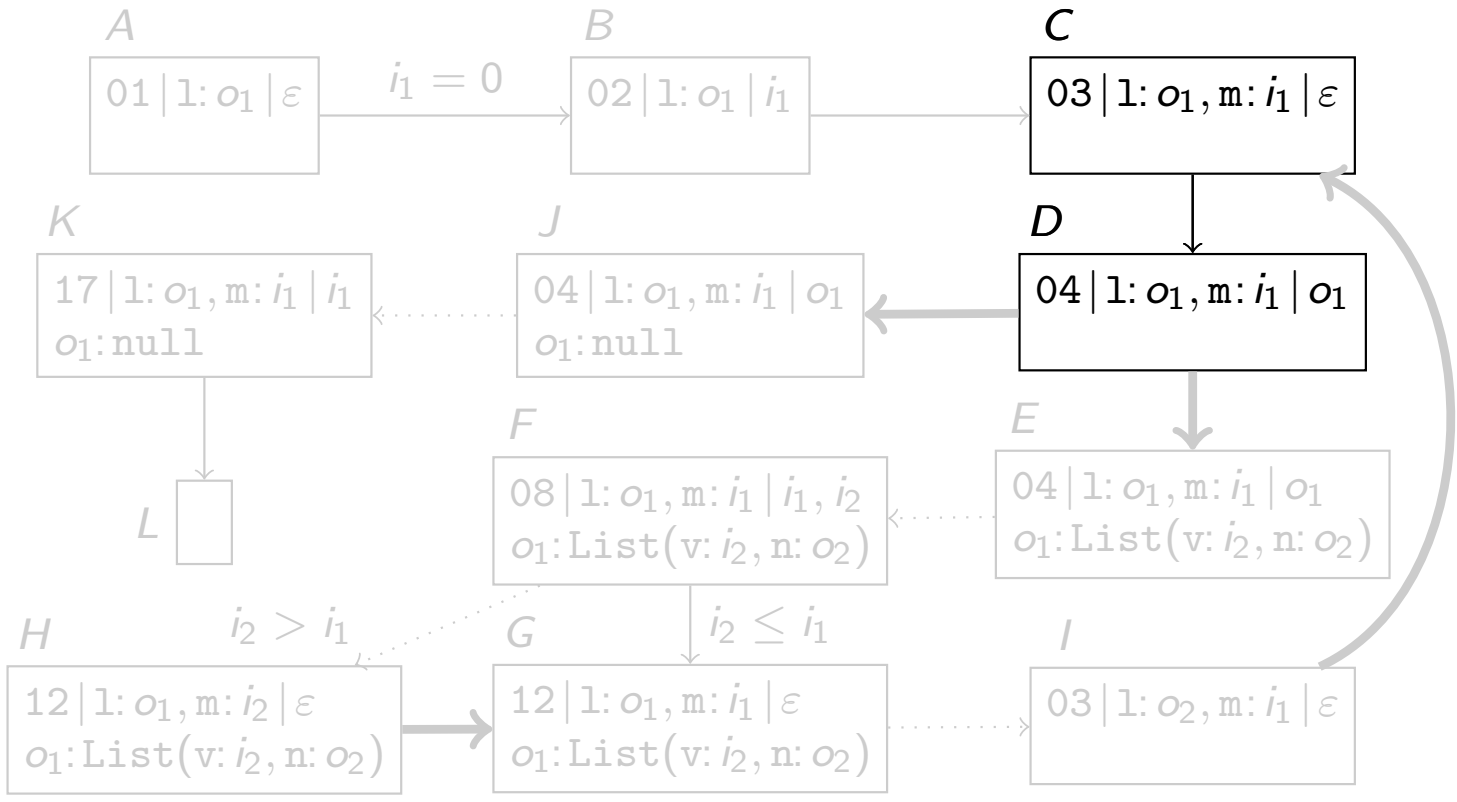
$C \xrightarrow{1} D$

weight: 1

condition: $\|o\| = 0$ if $o : \texttt{null}$

$\|o\| \geq 1$ if $o : \texttt{List}(\ldots)$

Transform Evaluation Edges



ITS over variables

i_1, $\|o_1\|$

i_1', $\|o_1\|'$

$C \xrightarrow{1} D$ **if** $\|o_1\| \geq 0$

weight: 1

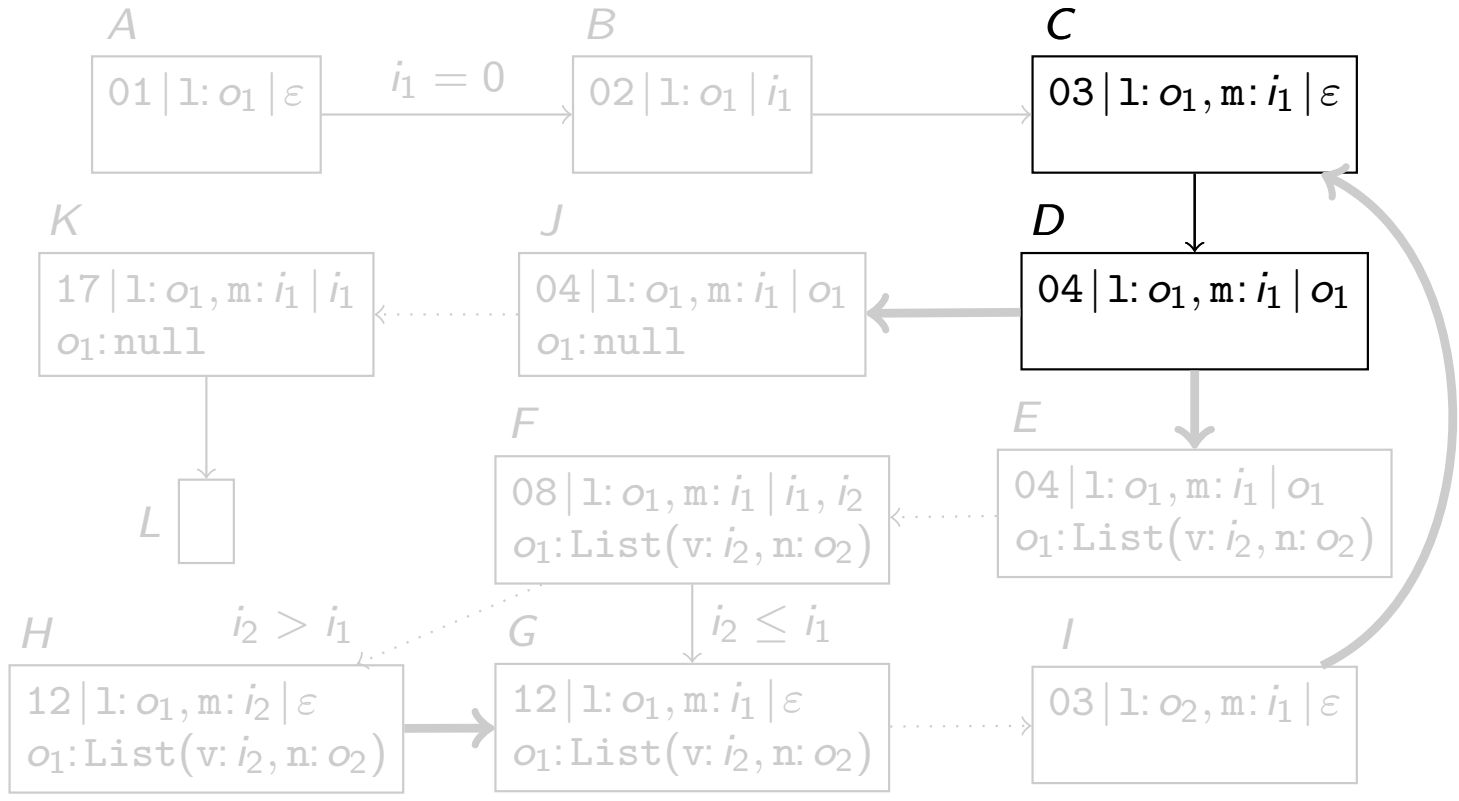
condition:

$\|o\| = 0$ if $o : \texttt{null}$

$\|o\| \geq 1$ if $o : \texttt{List}(\ldots)$

$\|o\| \geq 0$ otherwise

Transform Evaluation Edges



ITS over variables

i_1, $\|o_1\|$

i_1', $\|o_1\|'$

$C \xrightarrow{1} D$ **if** $\|o_1\| \geq 0 \wedge \|o_1\|' = \|o_1\| \wedge i_1' = i_1$

weight: 1

condition:

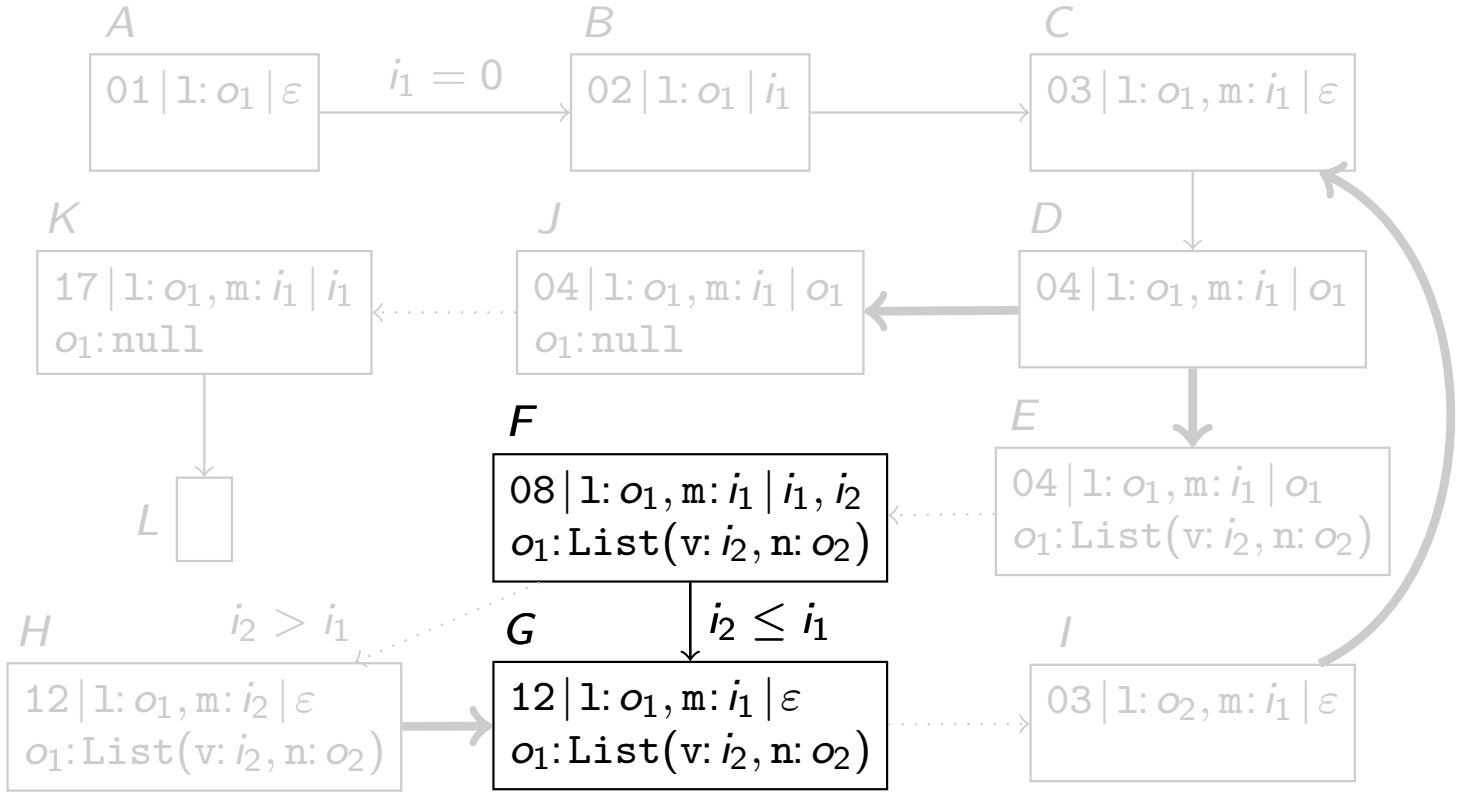
$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Evaluation Edges

ITS over variables

i_1, $\|o_1\|$

i_1', $\|o_1\|'$



$C \xrightarrow{1} D$ **if** $\|o_1\| \geq 0 \wedge \|o_1\|' = \|o_1\| \wedge i_1' = i_1$

weight: 1

condition:

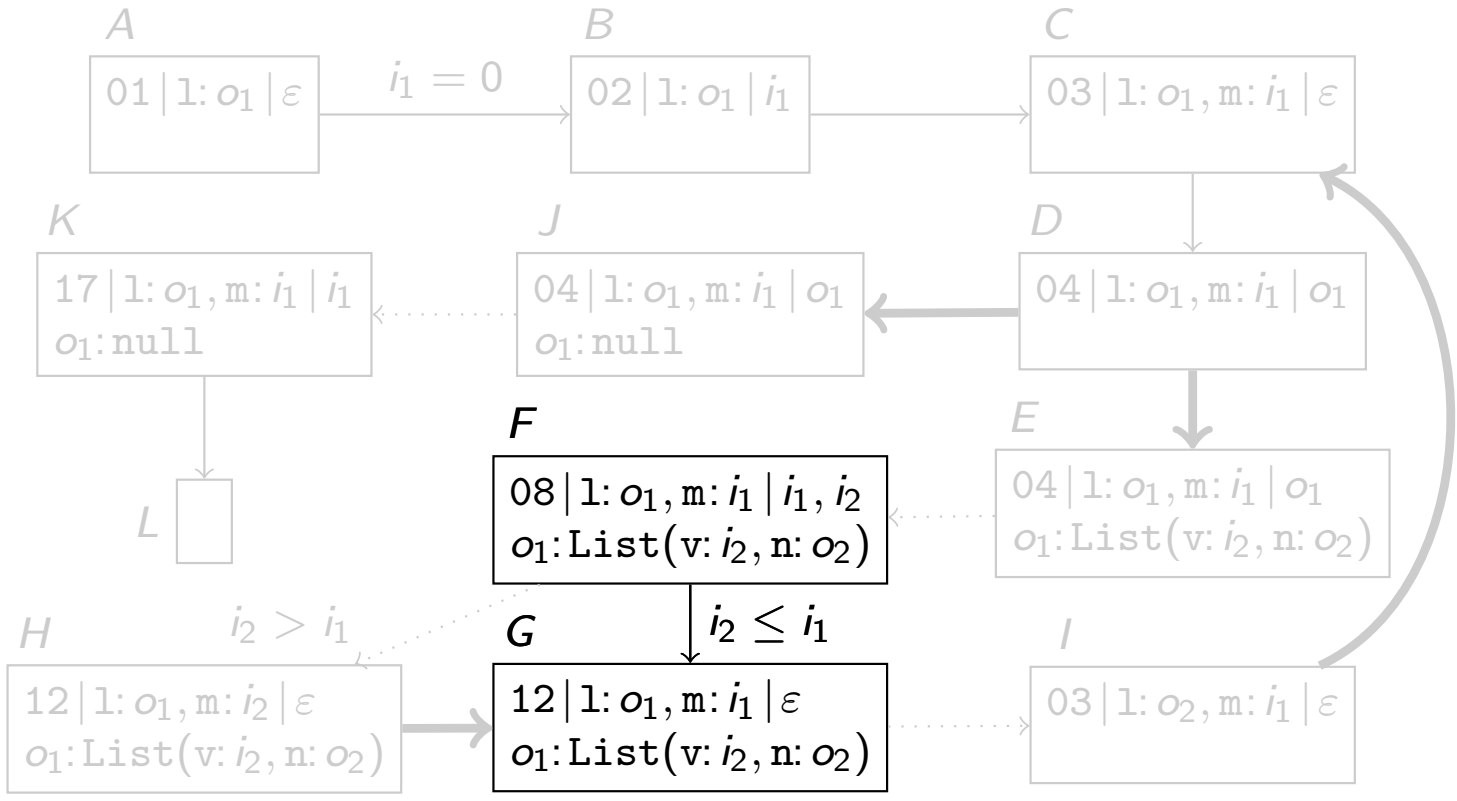
$\|o\| = 0$	if	o : `null`
$\|o\| \geq 1$	if	o : `List(...)`
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Evaluation Edges

ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$



$C \xrightarrow{1} D$ **if** $\|o_1\| \geq 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$

weight: 1

condition:

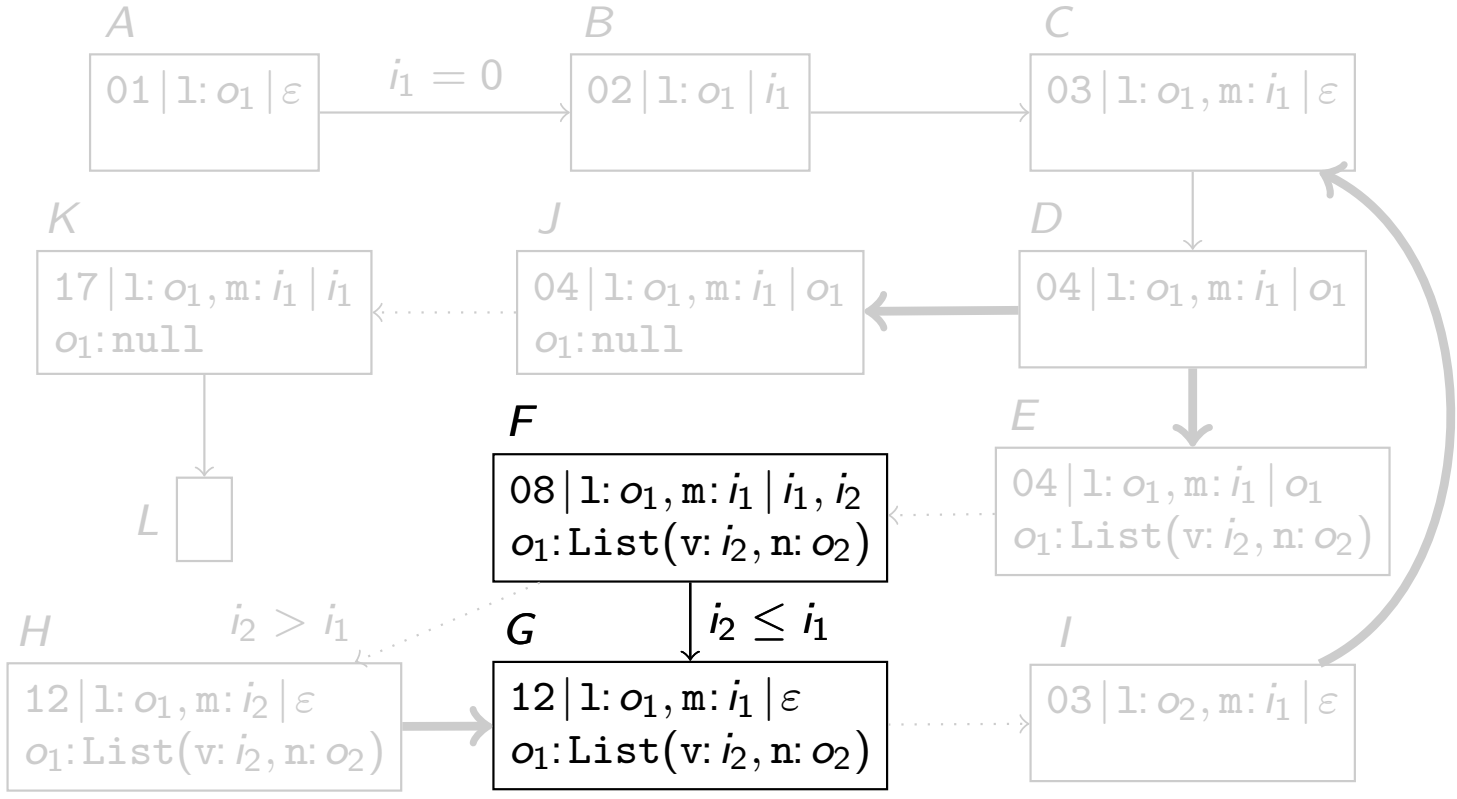
$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Evaluation Edges

ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$



$C \xrightarrow{1} D$ **if** $\|o_1\| \geq 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$

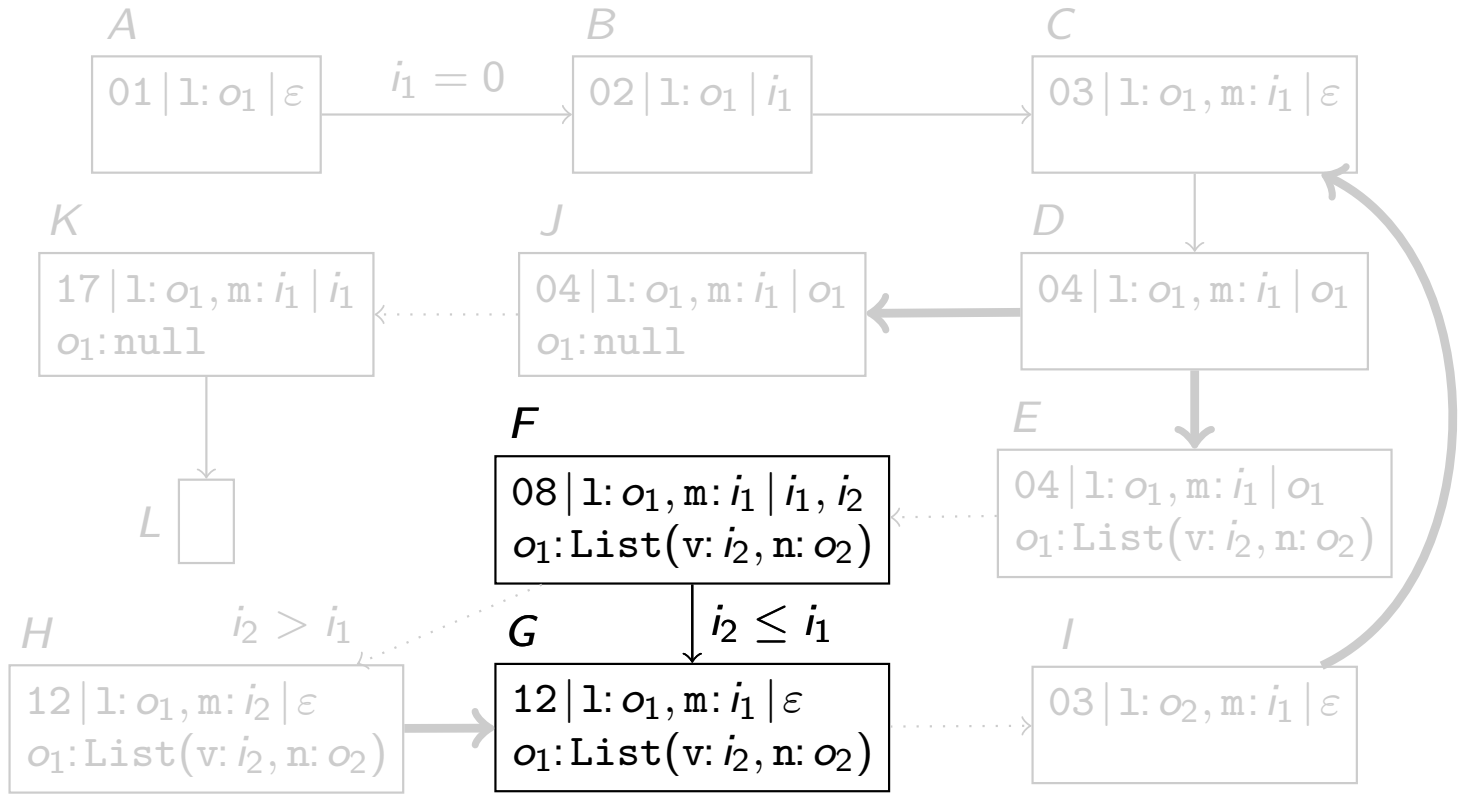
$F \xrightarrow{1} G$

weight: 1

condition:

$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Evaluation Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$C \xrightarrow{1} D$ **if** $\|o_1\| \geq 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$

$F \xrightarrow{1} G$ **if** $i_2 \leq i_1$

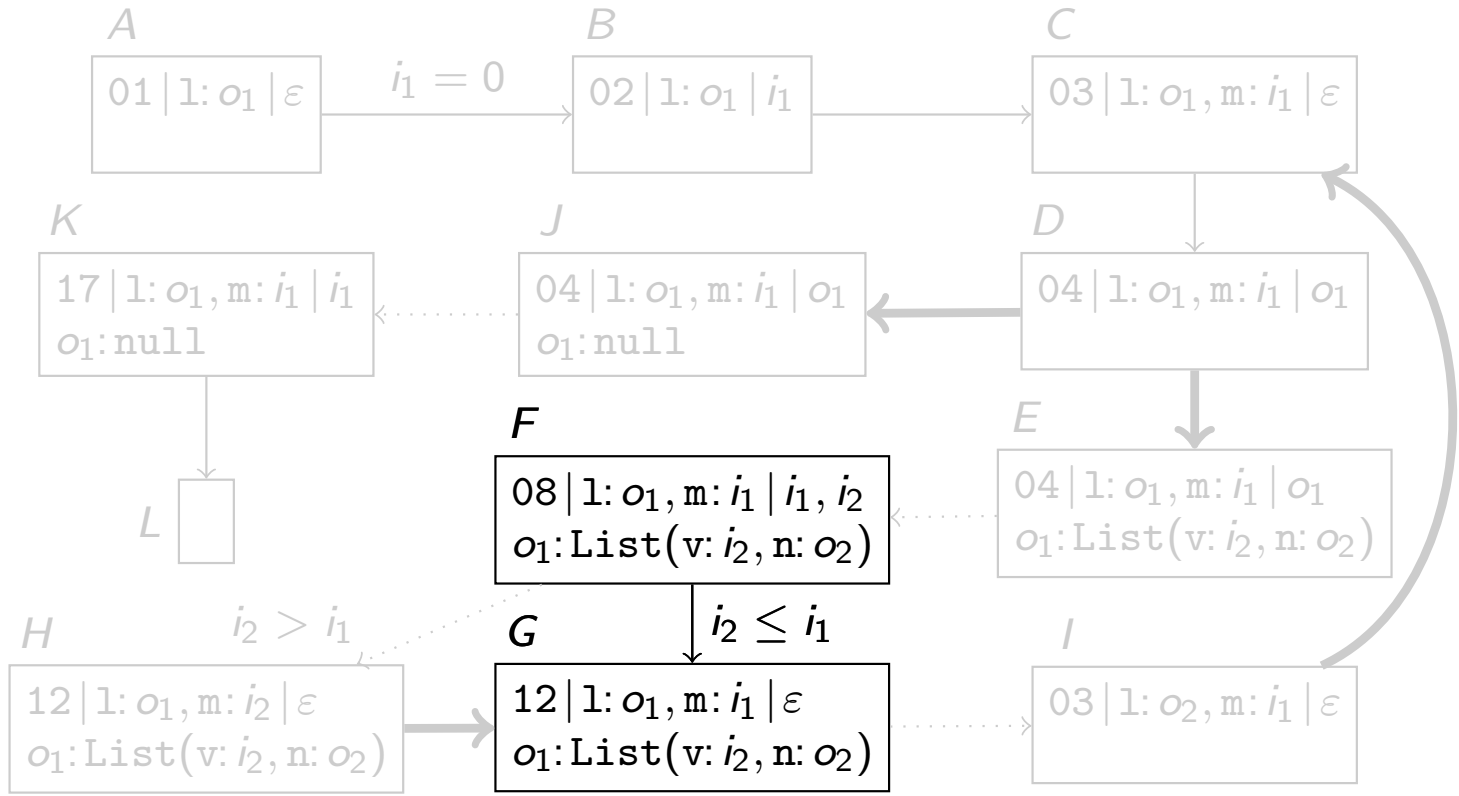
weight: 1

condition:

$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

condition of the edge in the symbolic execution graph

Transform Evaluation Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$$C \xrightarrow{1} D \quad \textbf{if} \quad \|o_1\| \geq 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$$

$$F \xrightarrow{1} G \quad \textbf{if} \quad i_2 \leq i_1 \ \wedge\ \|o_1\| \geq 1 \ \wedge\ \|o_2\| \geq 0$$

weight: 1

condition:

$\|o\| = 0$ if $o : \texttt{null}$

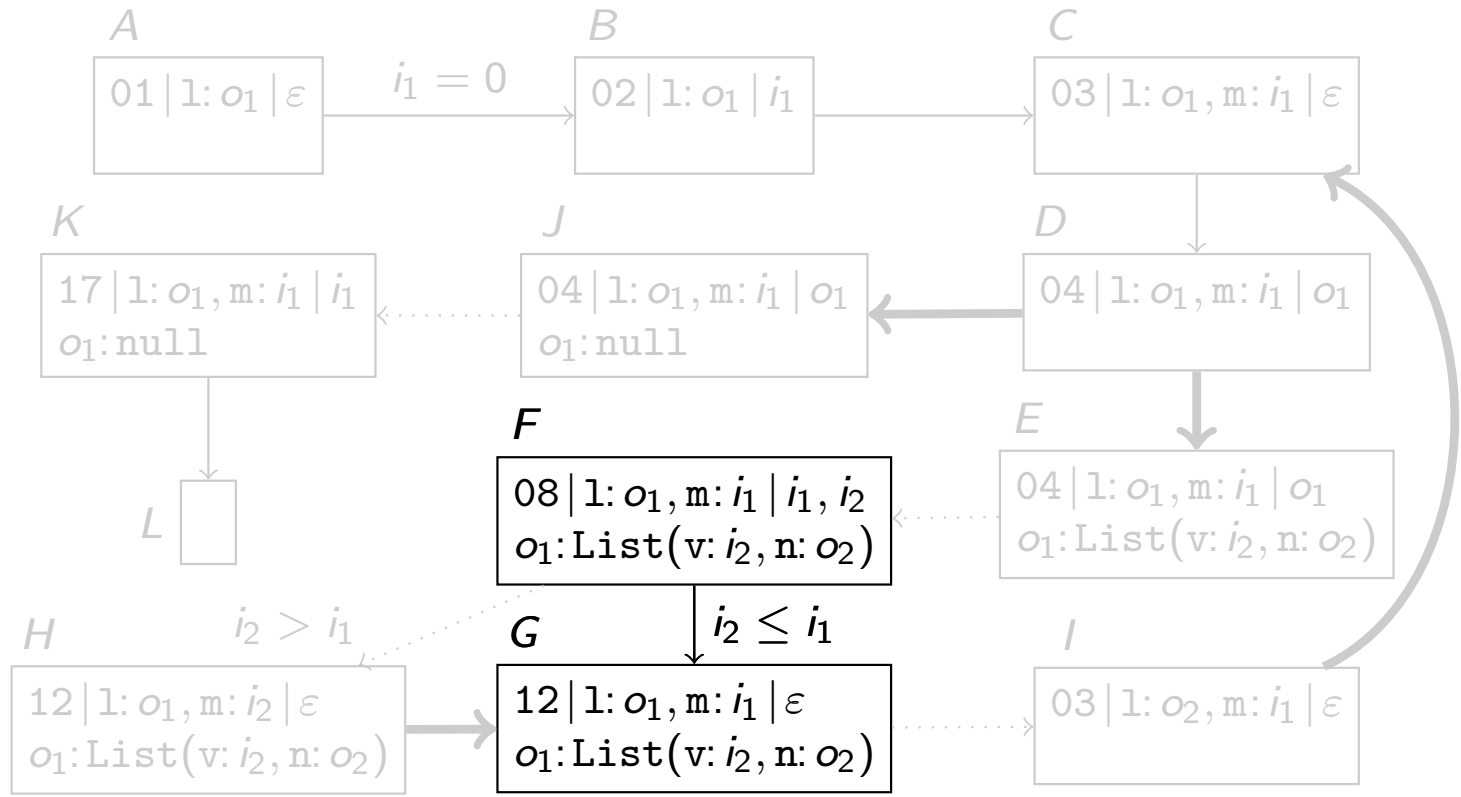
$\|o\| \geq 1$ if $o : \texttt{List}(\ldots)$

$\|o\| \geq 0$ otherwise

$x' = x$ for all variables x

condition of the edge in the symbolic execution graph

Transform Evaluation Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$$C \xrightarrow{1} D \quad \textbf{if} \quad \|o_1\| \geq 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$$

$$F \xrightarrow{1} G \quad \textbf{if} \quad i_2 \leq i_1 \ \wedge\ \|o_1\| \geq 1 \ \wedge\ \|o_2\| \geq 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ \|o_2\|' = \|o_2\| \ldots$$

weight: 1

condition:

$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

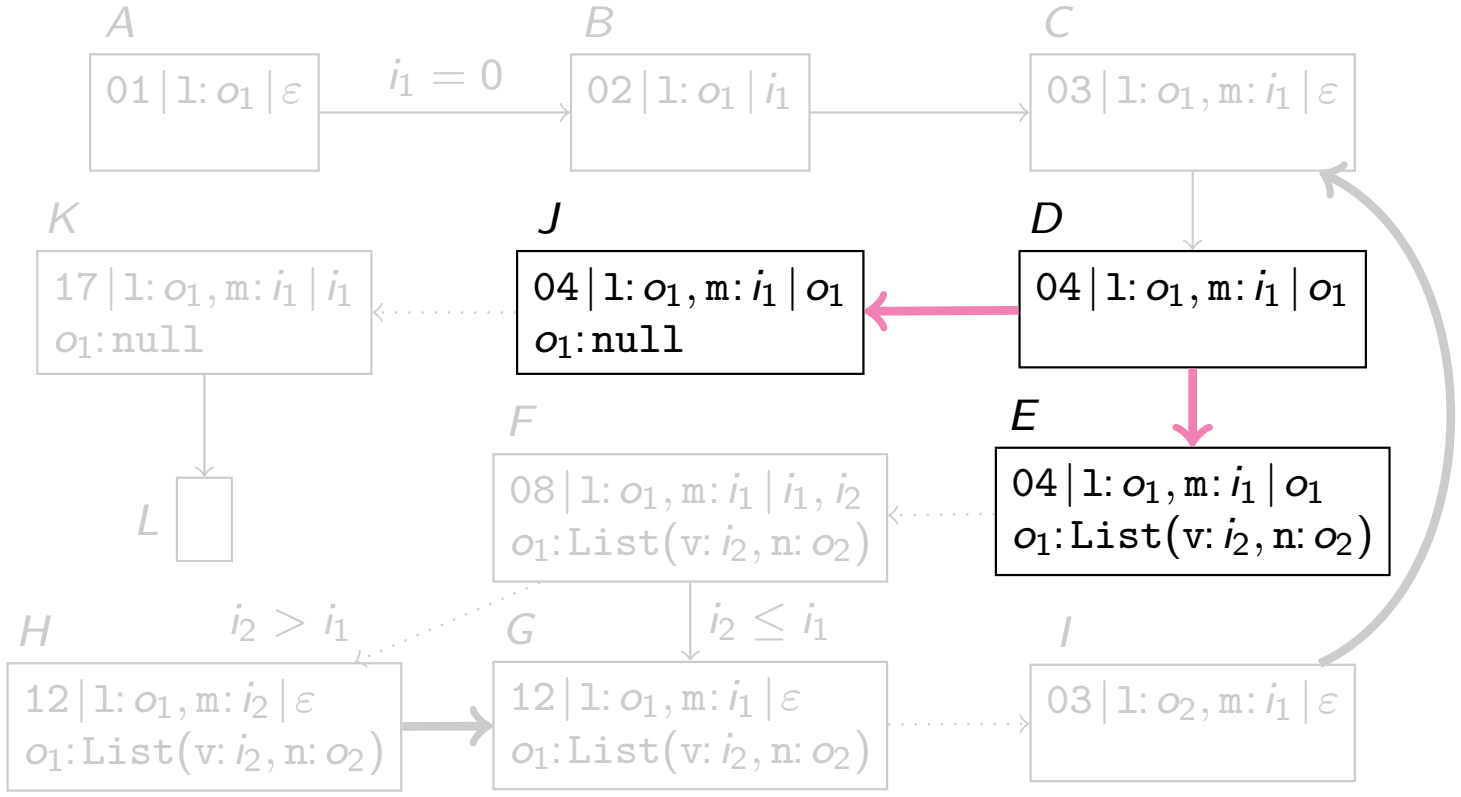
condition of the edge in the symbolic execution graph

Transform Refinement Edges

ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$



weight: 1

condition:

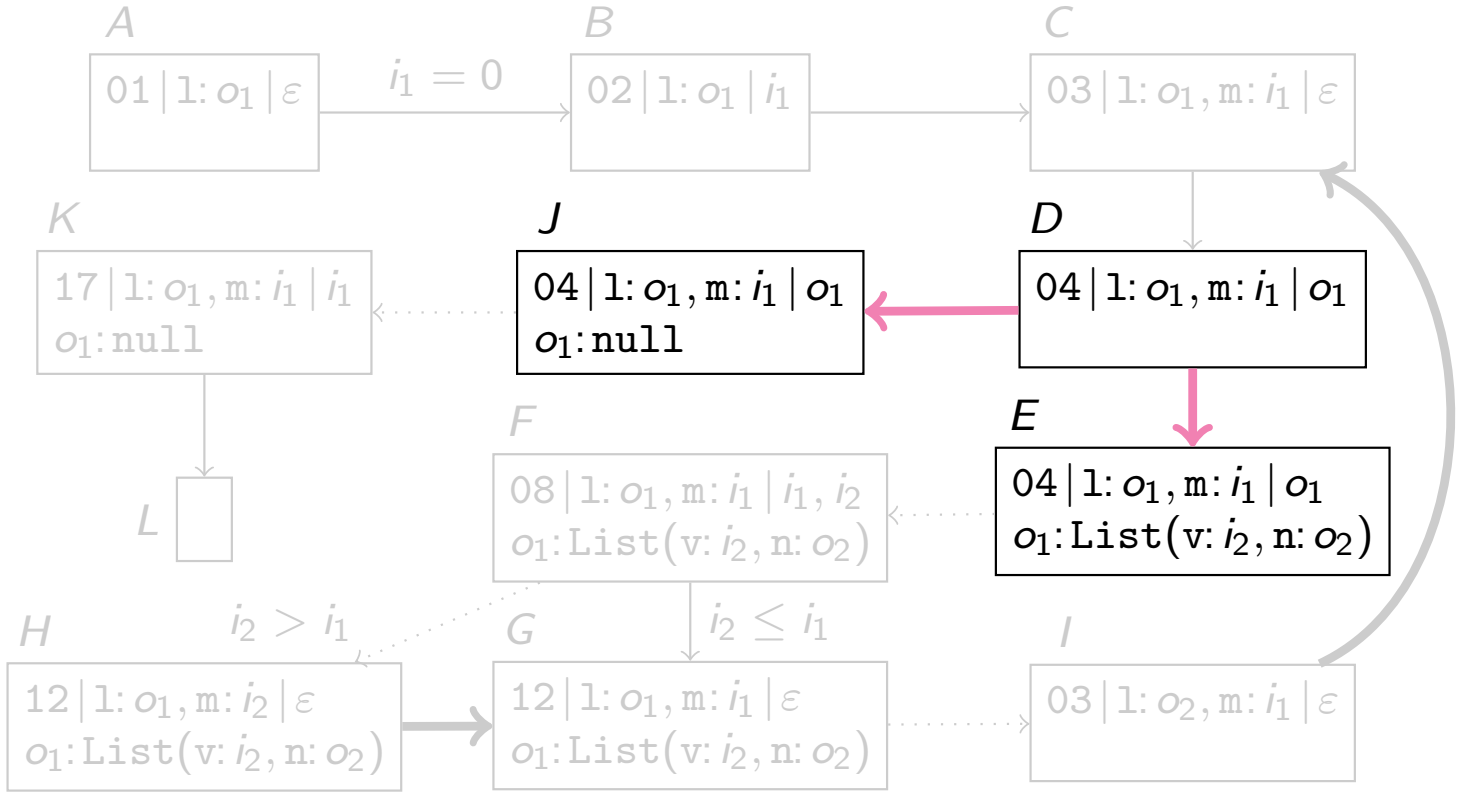
$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Refinement Edges

ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$



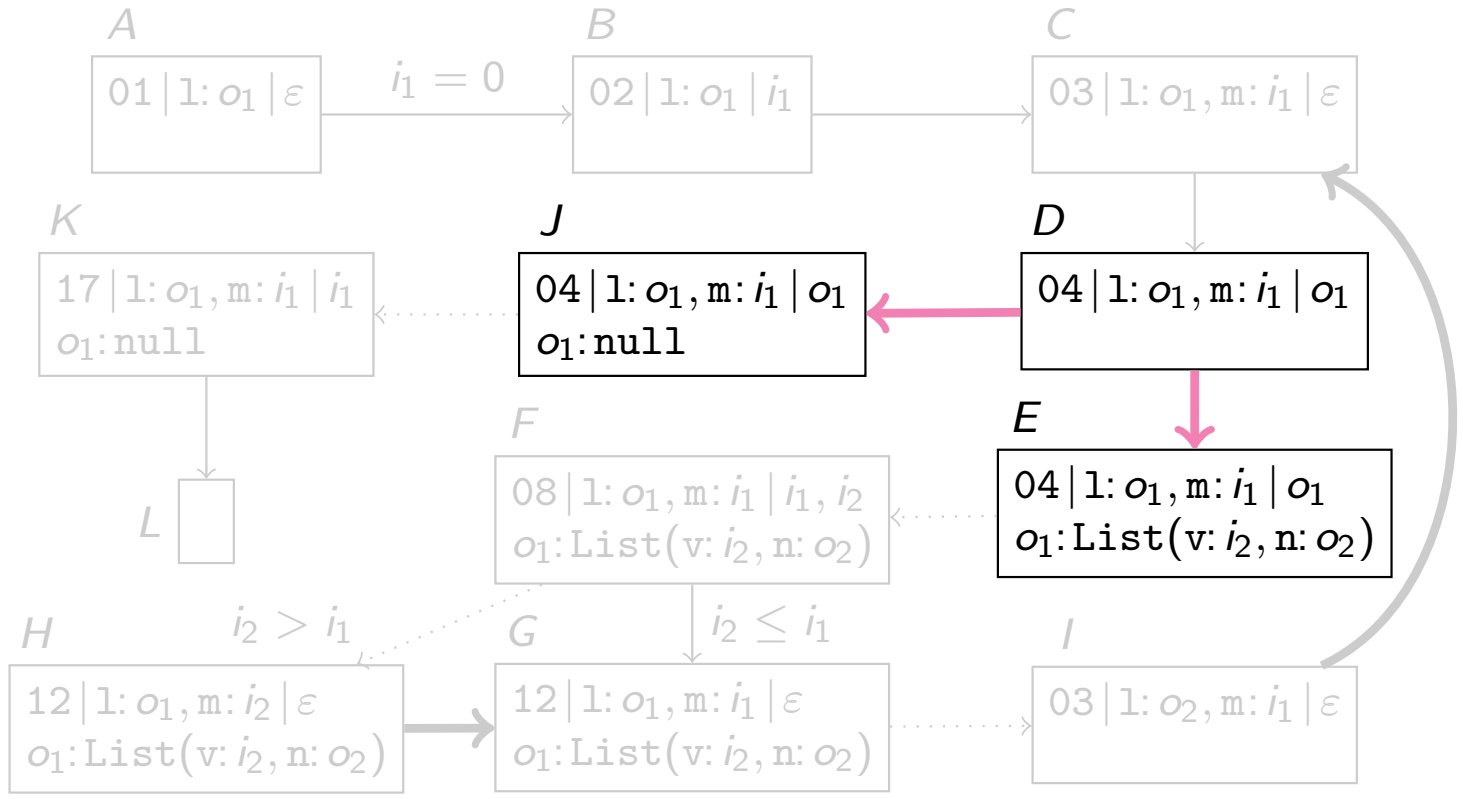
$D \xrightarrow{0} J$

weight: 0

condition:

$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Refinement Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$D \xrightarrow{0} J$ **if** $\|o_1\| = 0$

weight: 0

condition:

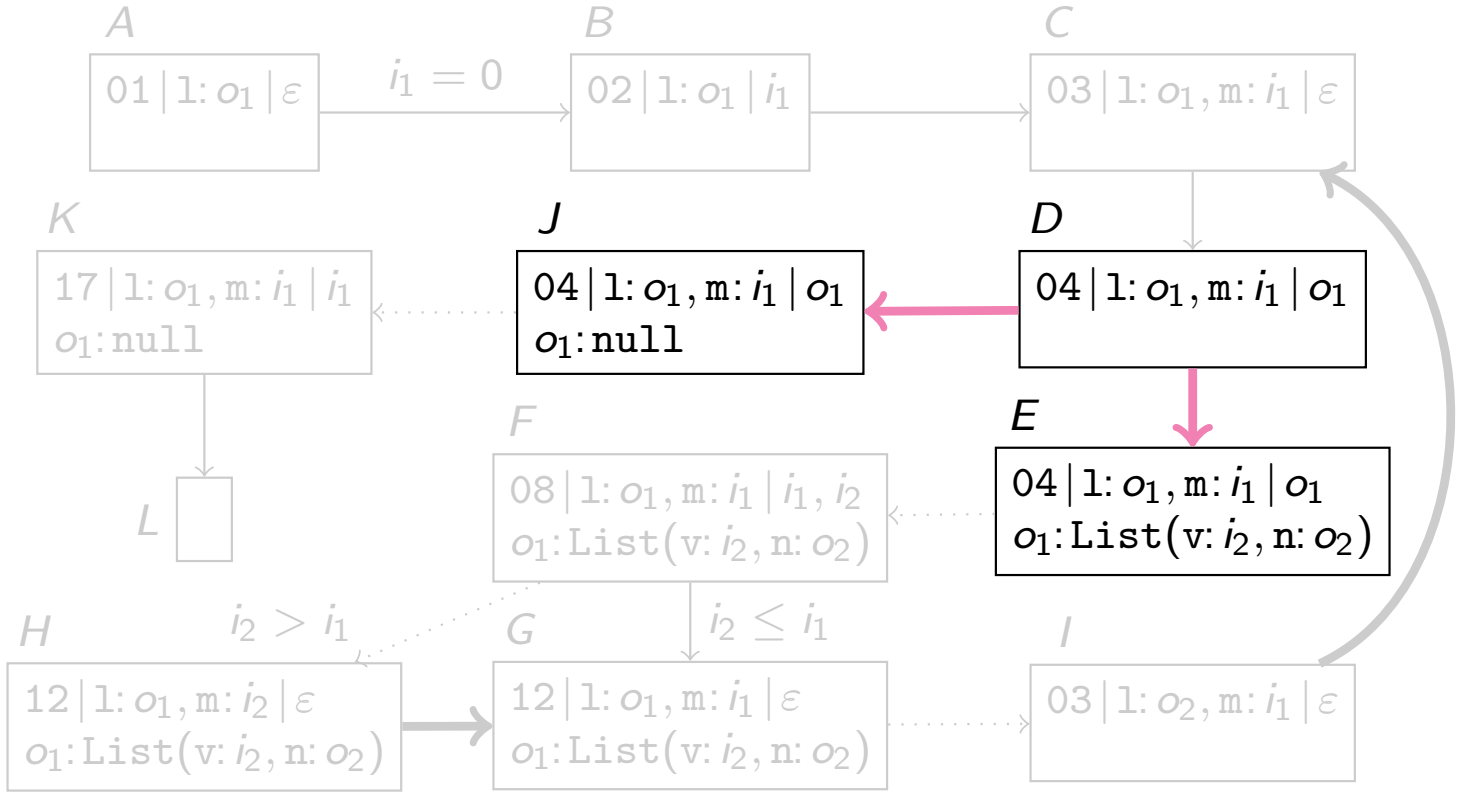
$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Refinement Edges

ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$



$D \xrightarrow{0} J$ **if** $\|o_1\| = 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$

weight: 0

condition:

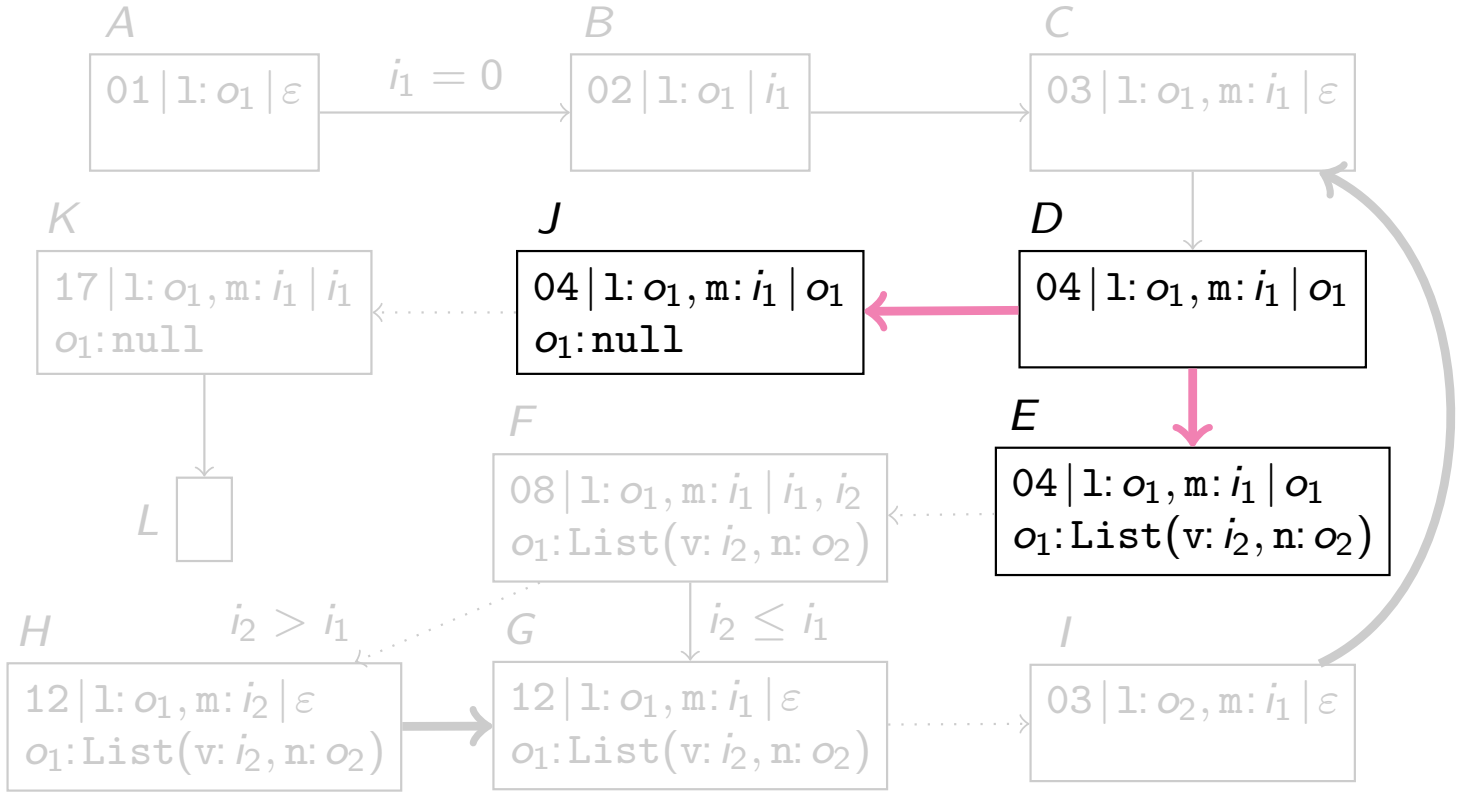
$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Refinement Edges

ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$



$D \xrightarrow{0} J$ **if** $\|o_1\| = 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$

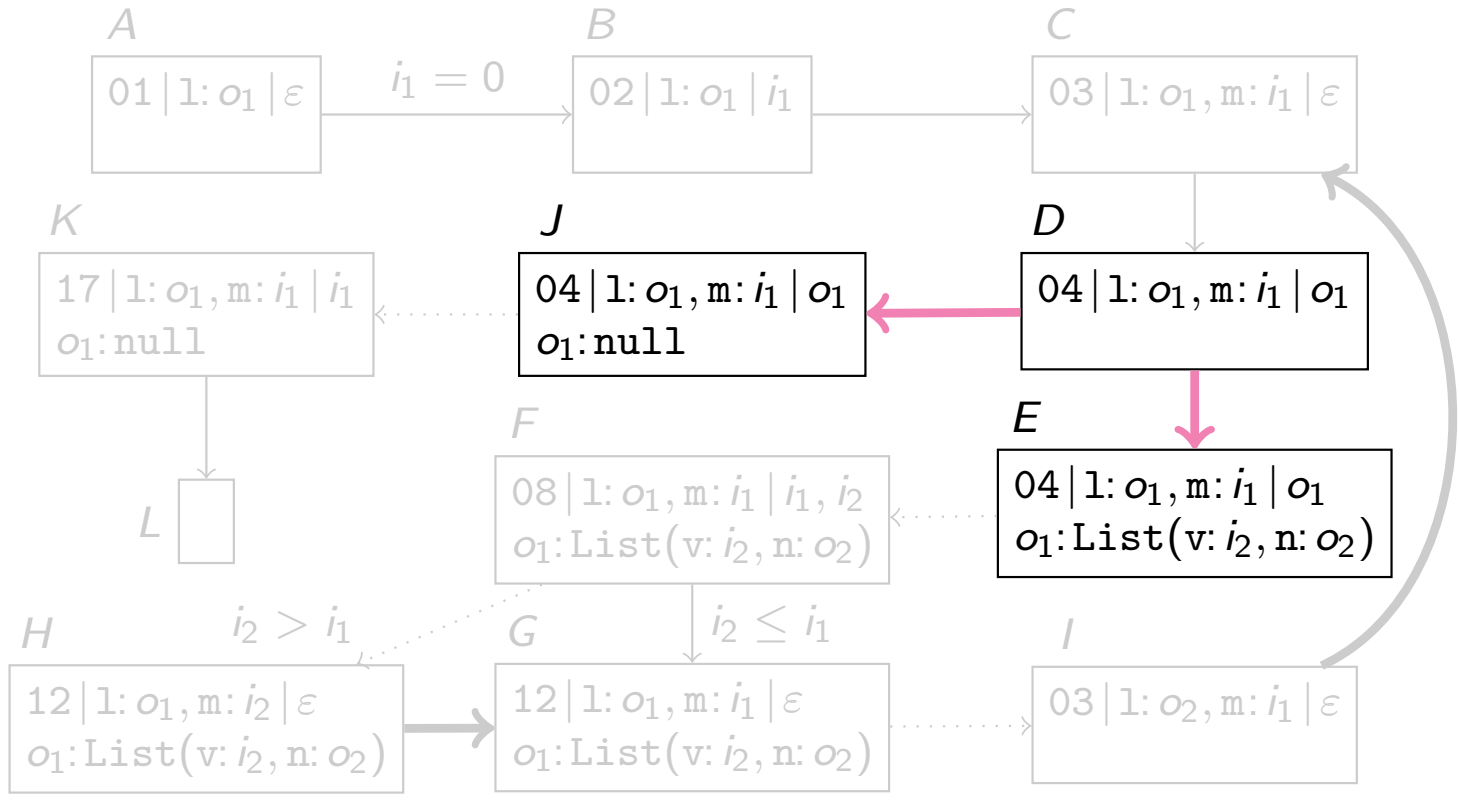
$D \xrightarrow{0} E$

weight: 0

condition:

$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \ge 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \ge 0$	otherwise	
$x' = x$	for all variables x	

Transform Refinement Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$$D \xrightarrow{0} J \quad \textbf{if} \quad \|o_1\| = 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$$

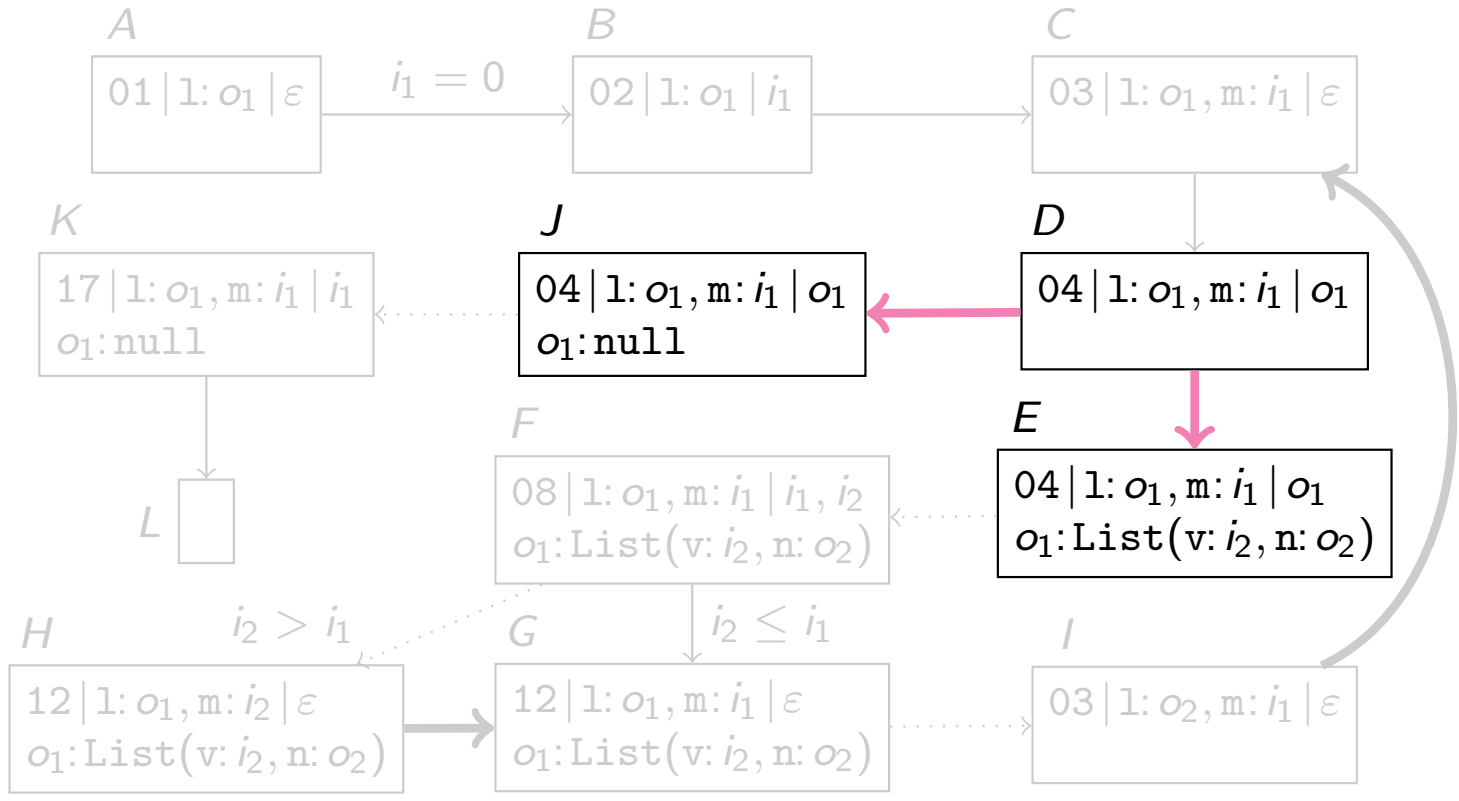
$$D \xrightarrow{0} E \quad \textbf{if} \quad \|o_1\| \geq 1 \ \wedge\ \|o_2\|' \geq 0$$

weight: 0

condition:

$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Refinement Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$$D \xrightarrow{0} J \quad \textbf{if} \quad \|o_1\| = 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$$

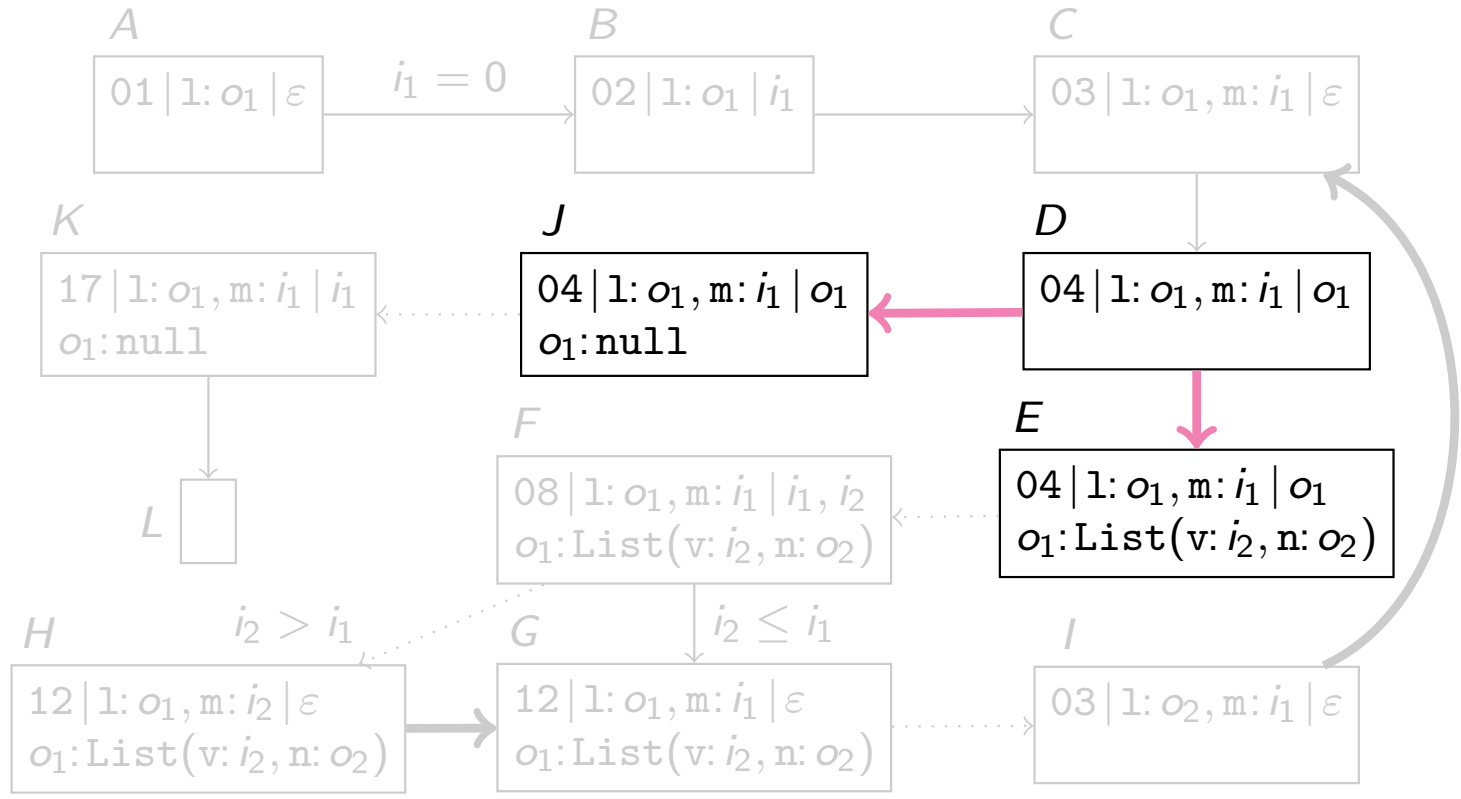
$$D \xrightarrow{0} E \quad \textbf{if} \quad \|o_1\| \geq 1 \ \wedge\ \|o_2\|' \geq 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$$

weight: 0

condition:

$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Refinement Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$$D \xrightarrow{0} J \quad \textbf{if} \quad \|o_1\| = 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$$

$$D \xrightarrow{0} E \quad \textbf{if} \quad \|o_1\| \geq 1 \ \wedge\ \|o_2\|' \geq 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$$

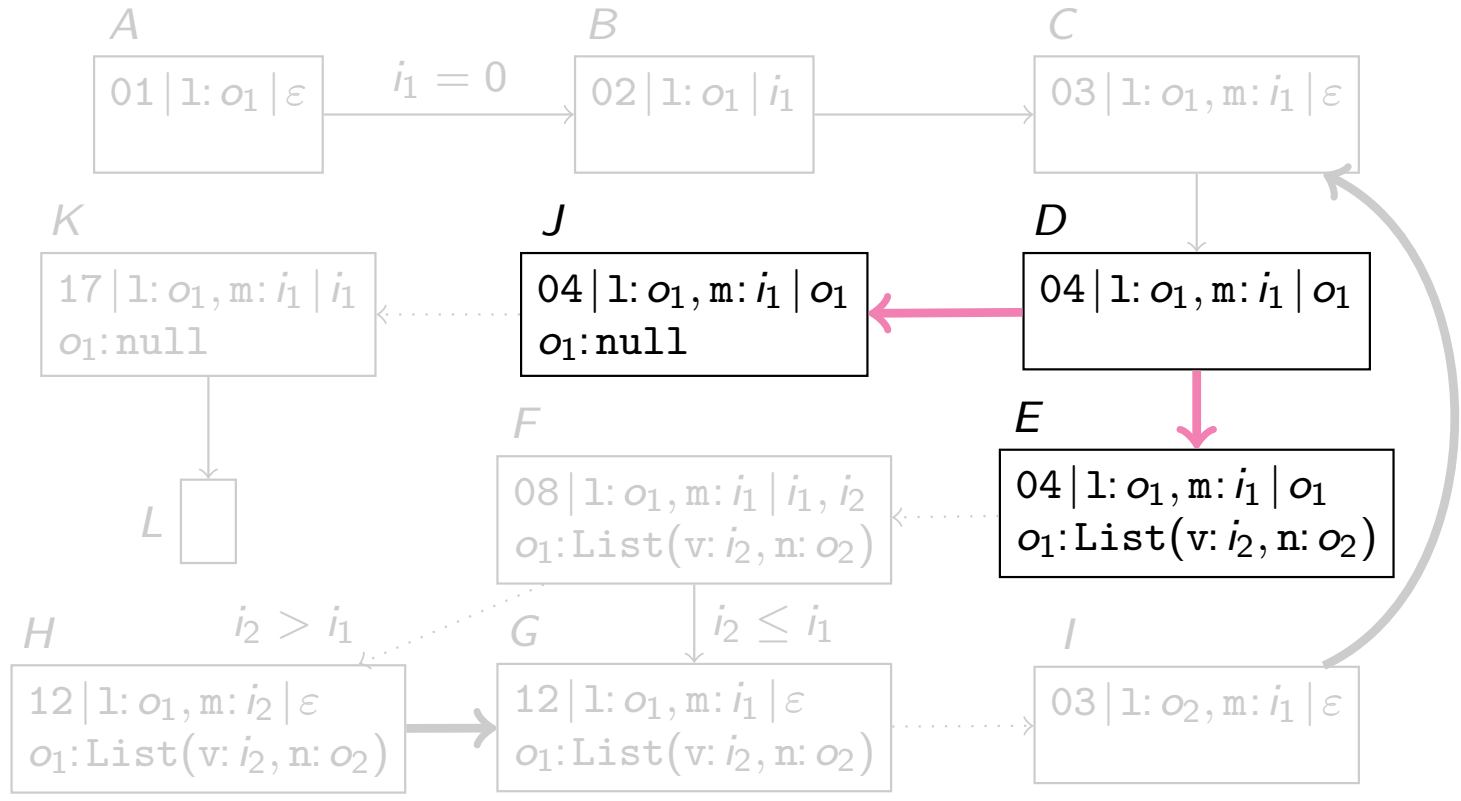
weight: 0

condition:

$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

$\|o_1\| \geq \|o_2\|'$ if o_1 is refined and o_1 reaches o_2

Transform Refinement Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$$D \xrightarrow{0} J \quad \textbf{if} \quad \|o_1\| = 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$$

$$D \xrightarrow{0} E \quad \textbf{if} \quad \|o_1\| \geq 1 \ \wedge\ \|o_2\|' \geq 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$$

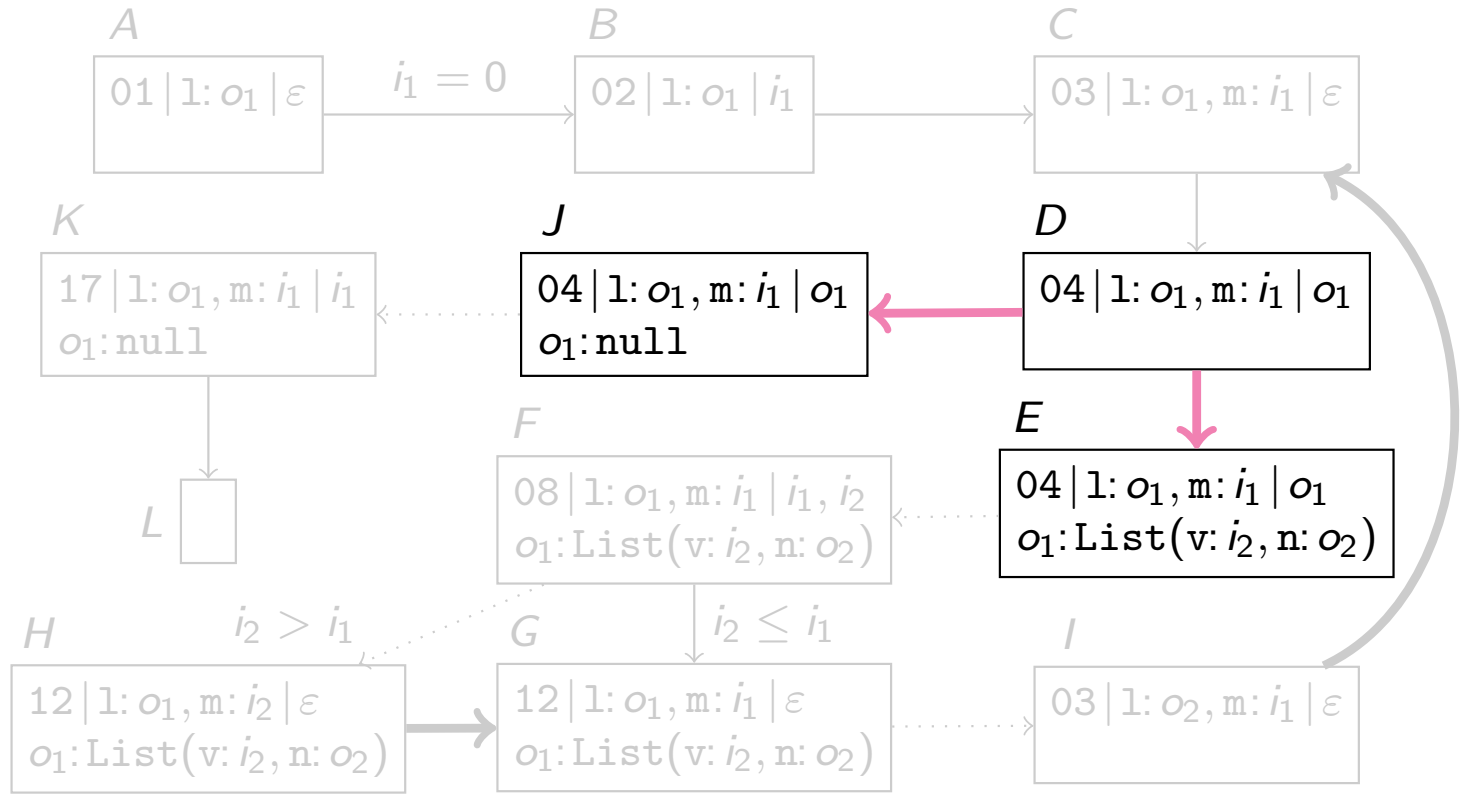
weight: 0

condition:

$\|o\| = 0$	if	o : `null`
$\|o\| \geq 1$	if	o : `List(...)`
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

$\|o_1\| > \|o_2\|'$ if o_1 is refined and o_1 reaches o_2 and o_1! not in refined state

Transform Refinement Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$D \xrightarrow{0} J$ **if** $\|o_1\| = 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$

$D \xrightarrow{0} E$ **if** $\|o_1\| \geq 1 \ \wedge\ \|o_2\|' \geq 0 \ \wedge \ldots \wedge \|o_1\| > \|o_2\|'$

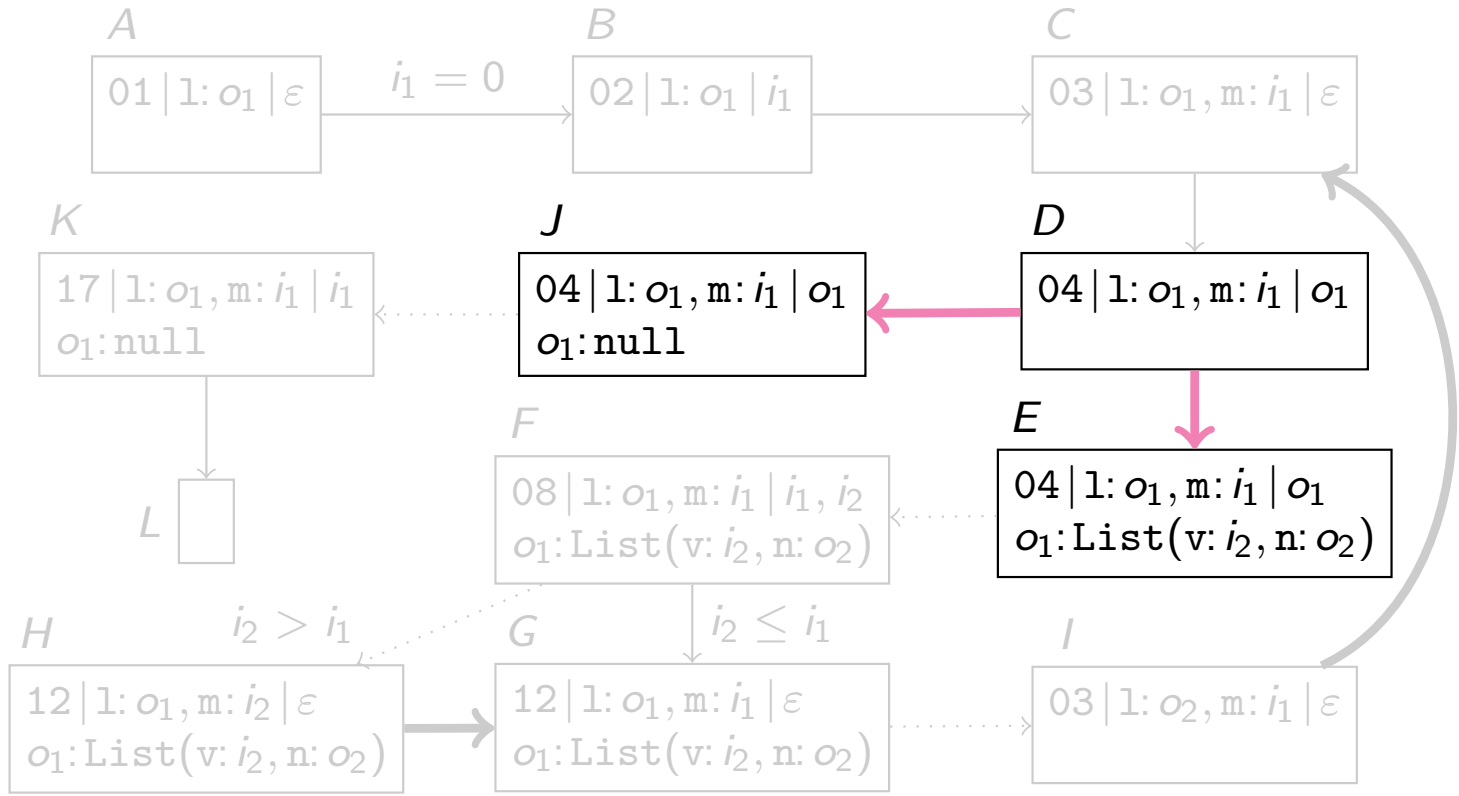
weight: 0

condition:

$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

$\|o_1\| > \|o_2\|'$ if o_1 is refined and o_1 reaches o_2 and o_1! not in refined state

Transform Refinement Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$$D \xrightarrow{0} J \quad \textbf{if} \quad \|o_1\| = 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$$

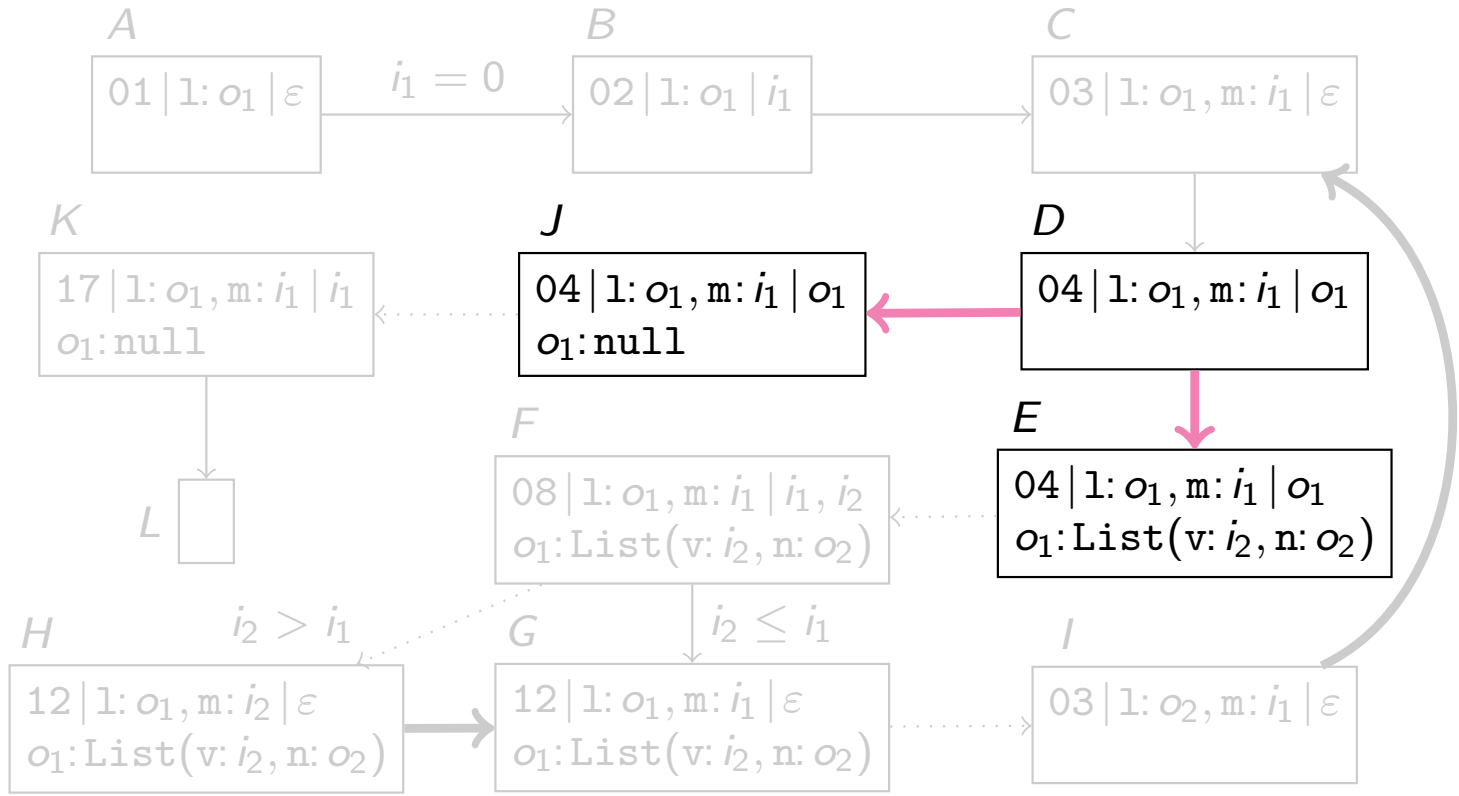
$$D \xrightarrow{0} E \quad \textbf{if} \quad \|o_1\| \geq 1 \ \wedge\ \|o_2\|' \geq 0 \ \wedge \ldots \wedge \|o_1\| > \|o_2\|'$$

weight: 0

condition:

$\|o\| = 0$	if o : null	$\|o_1\| > \|o_2\|'$	if o_1 is refined and o_1 reaches o_2 and o_1! not in refined state
$\|o\| \geq 1$	if o : List(...)		
$\|o\| \geq 0$	otherwise		
$x' = x$	for all variables x	$\|o_1\| > \|i'\|$	if o_1 is refined and o_1 reaches i

Transform Refinement Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$D \xrightarrow{0} J$ **if** $\|o_1\| = 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$

$D \xrightarrow{0} E$ **if** $\|o_1\| \geq 1 \ \wedge\ \|o_2\|' \geq 0 \ \wedge \ldots \wedge \|o_1\| > \|o_2\|'$

weight: 0

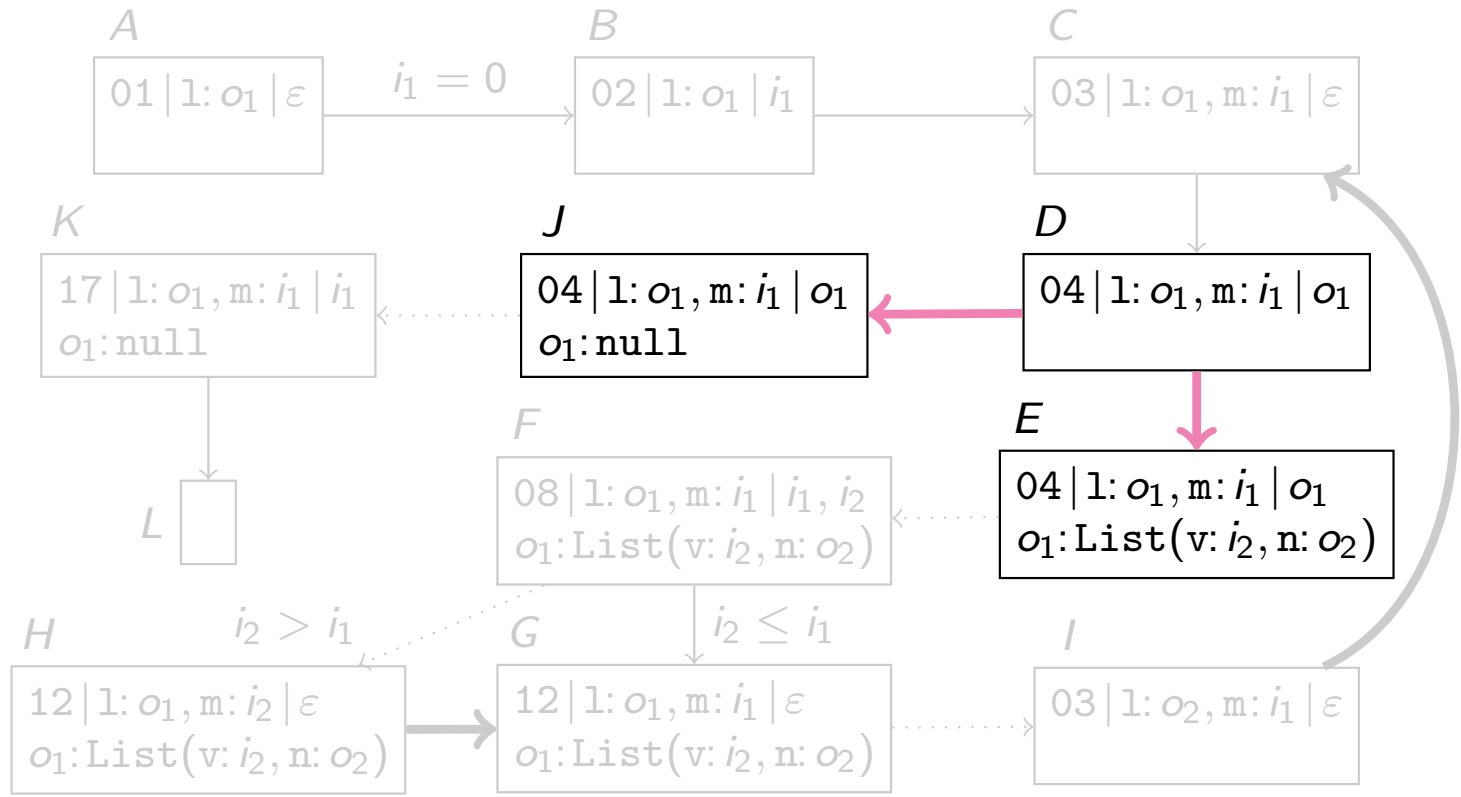
condition:

$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

$\|o_1\| > \|o_2\|'$ if o_1 is refined and o_1 reaches o_2 and o_1! not in refined state

$\|o_1\| > i' > -\|o_1\|$ if o_1 is refined and o_1 reaches i

Transform Refinement Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$D \xrightarrow{0} J$ **if** $\|o_1\| = 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_1$

$D \xrightarrow{0} E$ **if** $\|o_1\| \geq 1 \ \wedge\ \|o_2\|' \geq 0 \ \wedge \ldots \wedge \|o_1\| > \|o_2\|' \ \wedge\ \|o_1\| > i_2' > -\|o_1\|$

weight: 0

condition:

$\|o\| = 0$	if	o : `null`
$\|o\| \geq 1$	if	o : `List(...)`
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

$\|o_1\| > \|o_2\|'$ if o_1 is refined and o_1 reaches o_2 and o_1! not in refined state

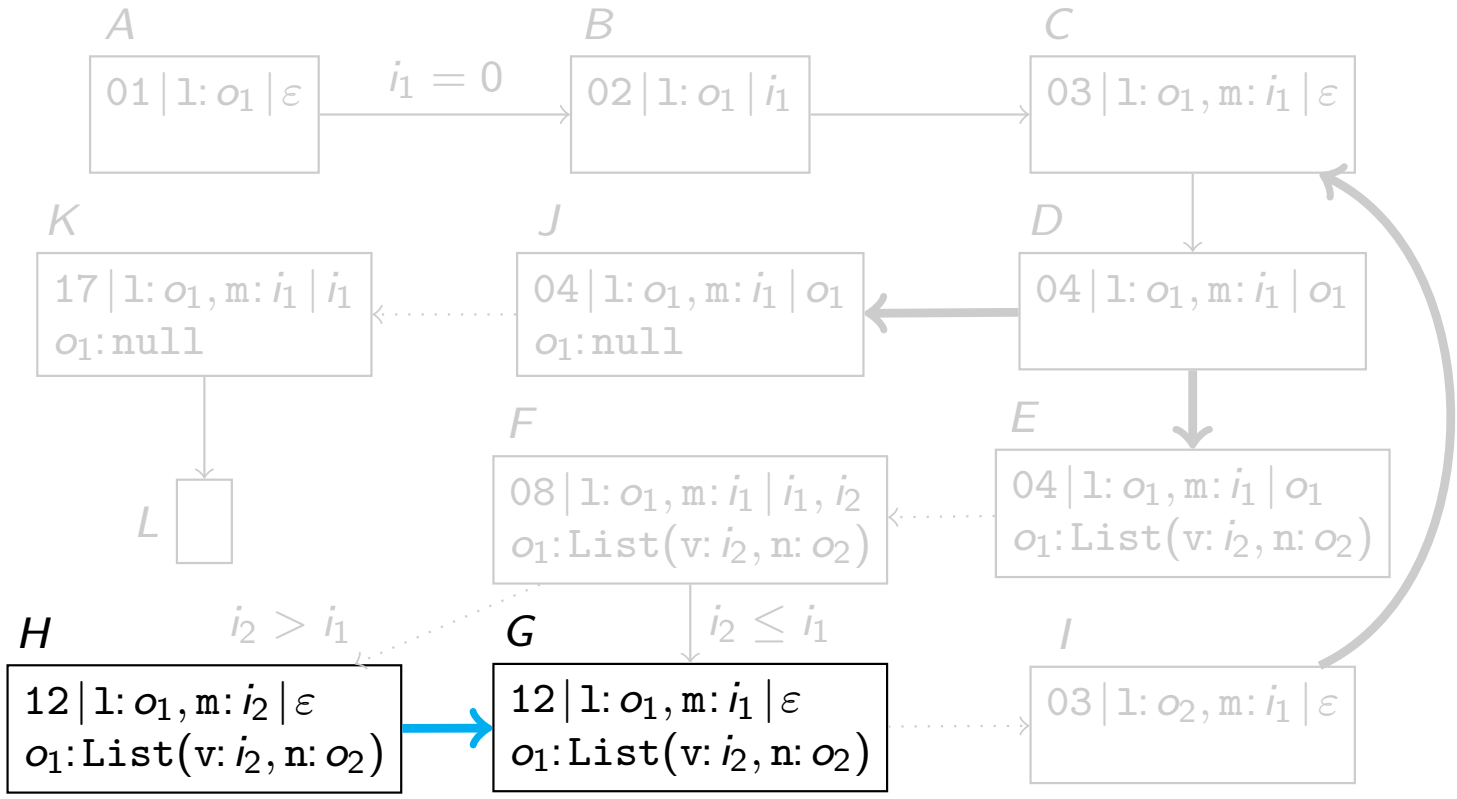
$\|o_1\| > i' > -\|o_1\|$ if o_1 is refined and o_1 reaches i

Transform Generalization Edges

ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$



weight: 0

condition:

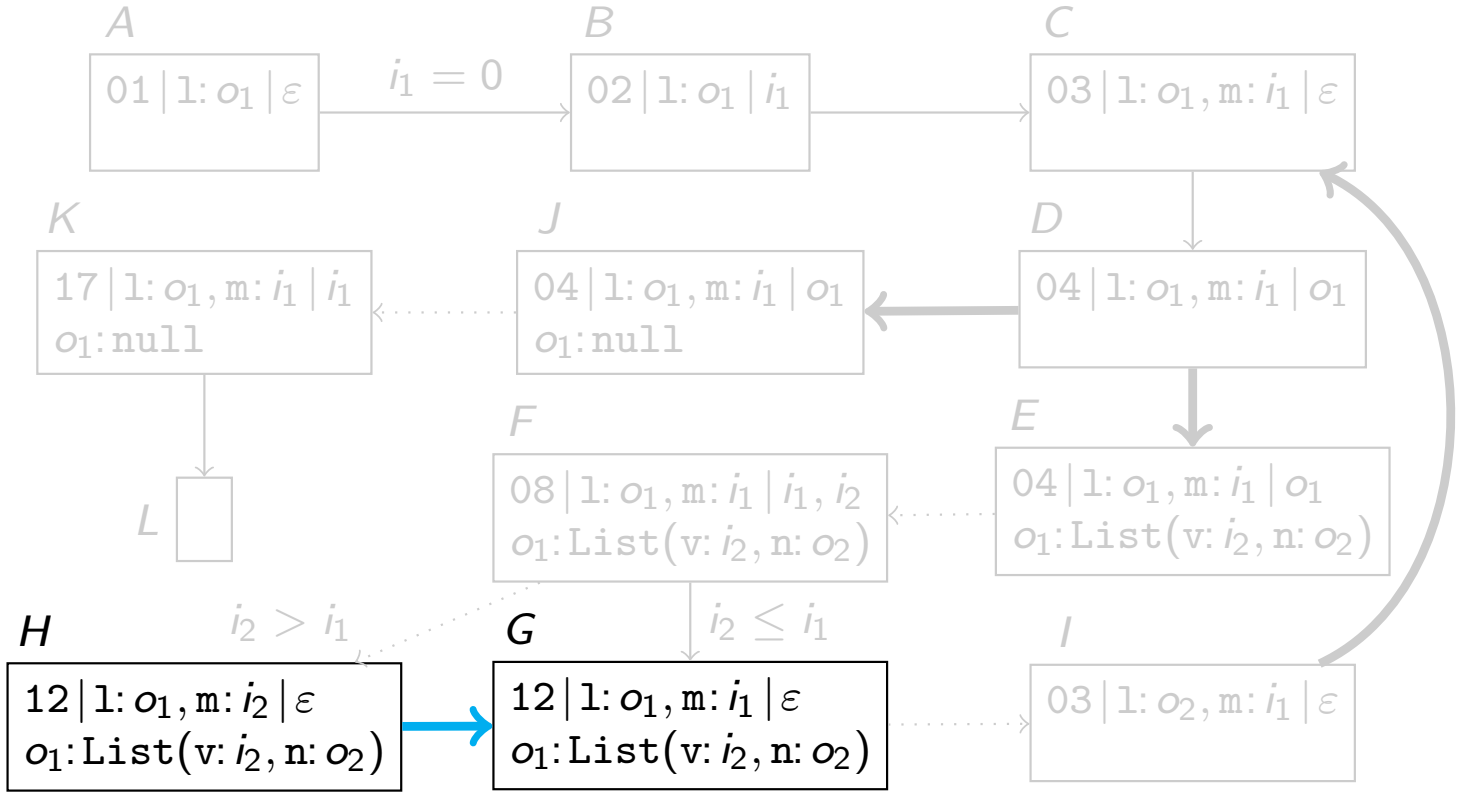
$\|o\| = 0$	if	o : null
$\|o\| \geq 1$	if	o : List(...)
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Generalization Edges

ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$



$H \xrightarrow{0} G$

weight: 0

condition:

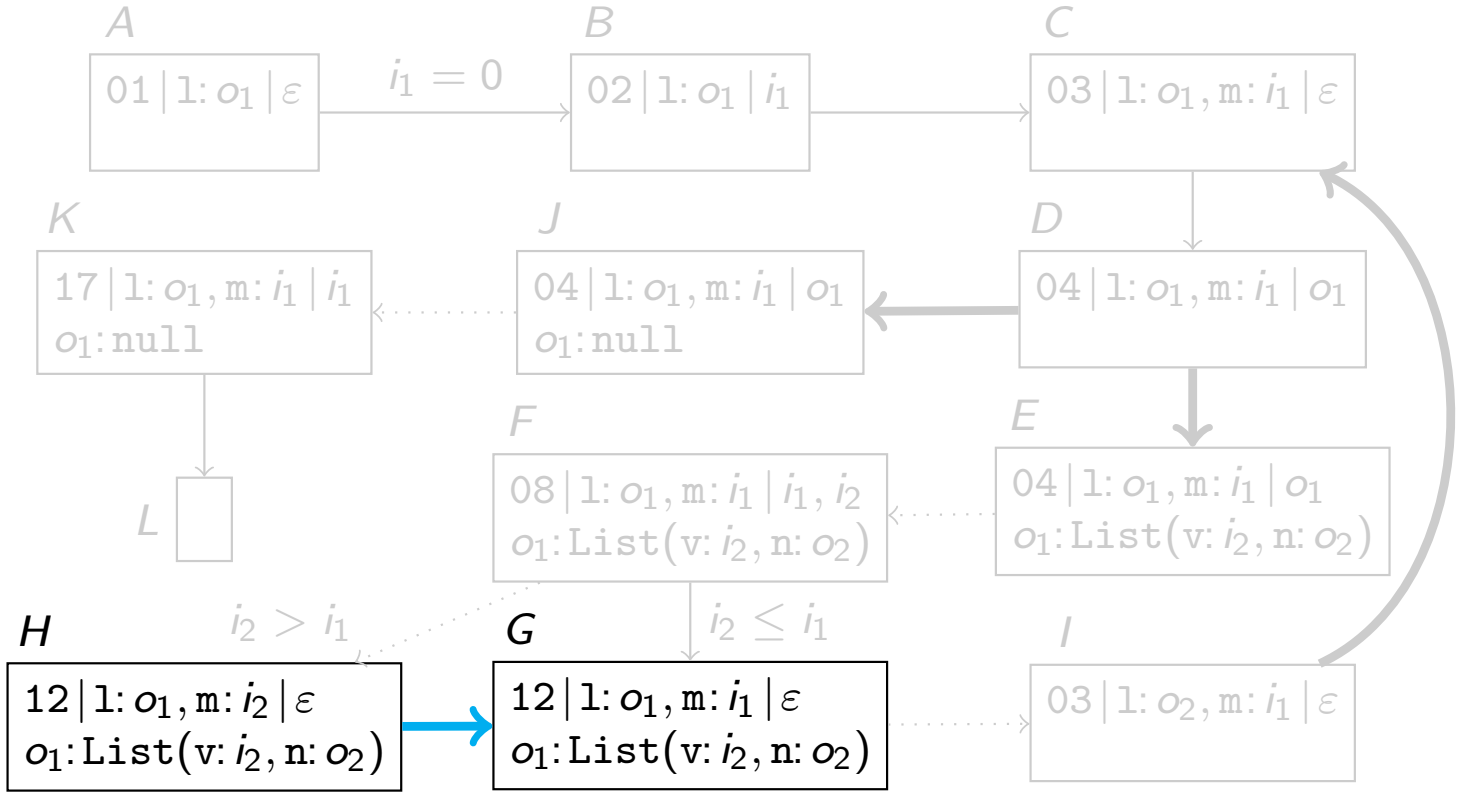
$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Generalization Edges

ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$



$H \xrightarrow{0} G$ **if** $\|o_1\| \geq 1 \ \wedge\ \|o_2\| \geq 0$

weight: 0

condition:

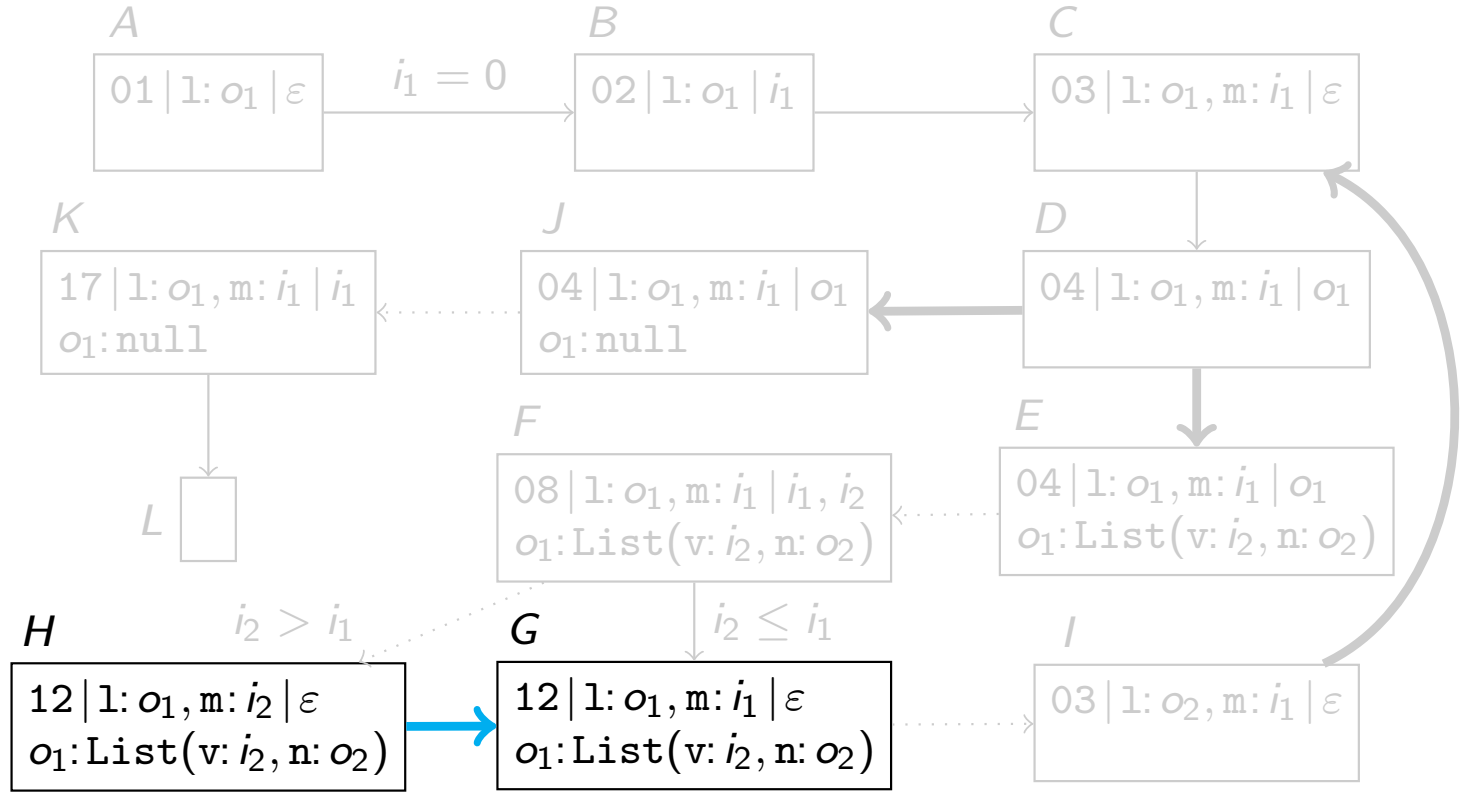
$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = x$	for all variables x	

Transform Generalization Edges

ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$



$H \xrightarrow{0} G$ **if** $\|o_1\| \geq 1 \ \wedge\ \|o_2\| \geq 0$

weight: 0

condition:

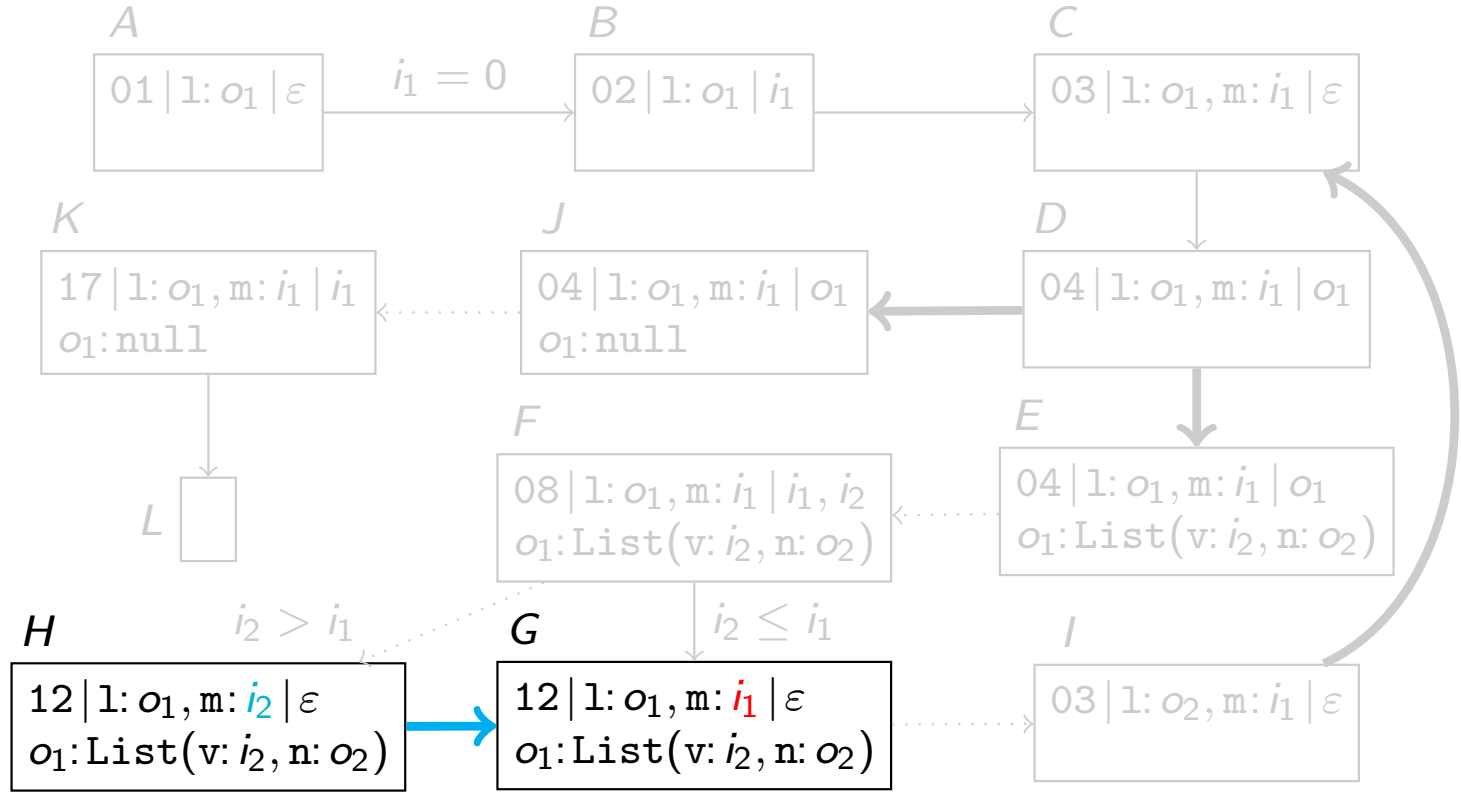
$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = y$	if x in general state corresponds to y in special state	

Transform Generalization Edges

ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$



$H \xrightarrow{0} G$ **if** $\|o_1\| \geq 1 \ \wedge \ \|o_2\| \geq 0$

weight: 0

condition:

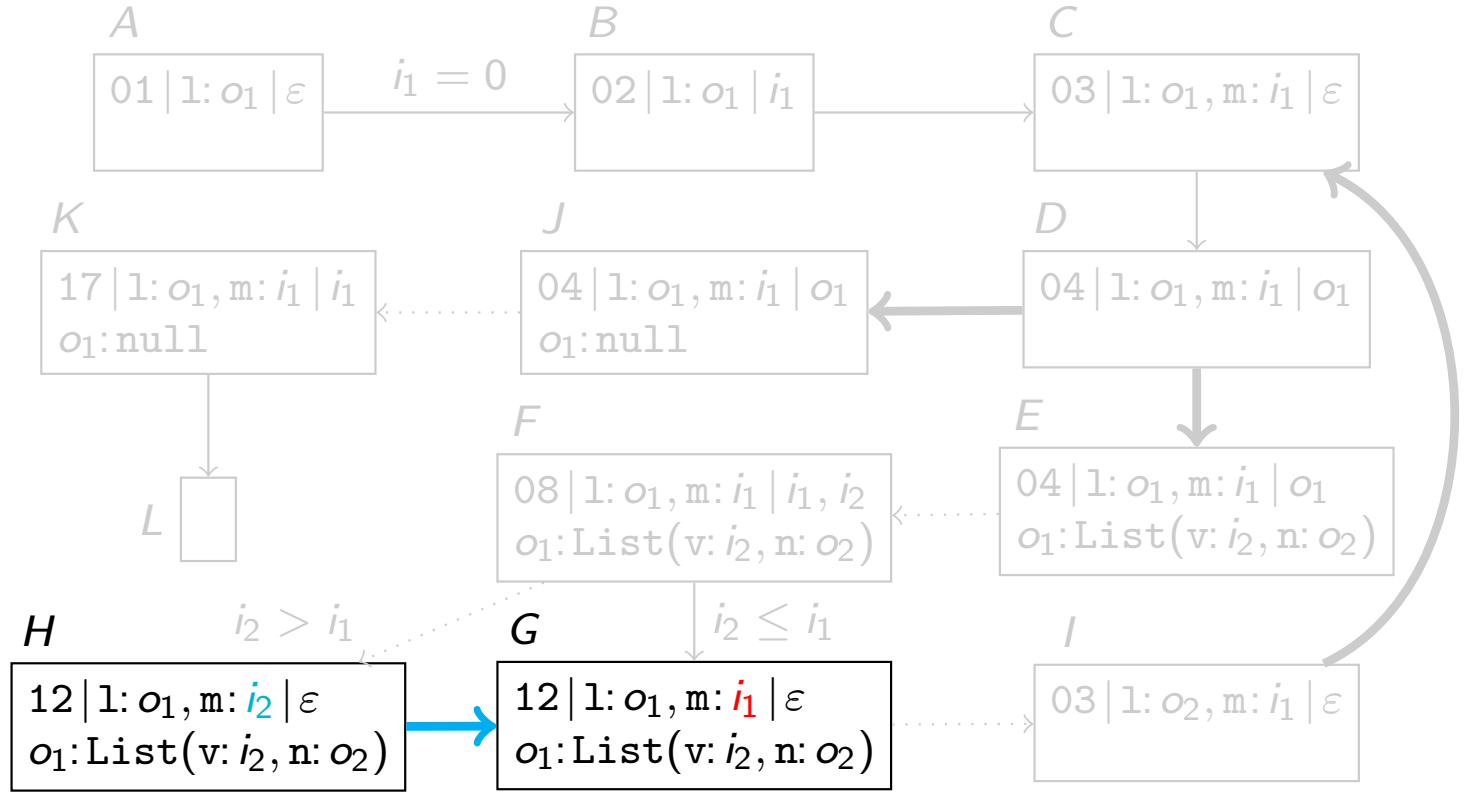
$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = y$	if x in general state corresponds to y in special state	

Transform Generalization Edges

ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$



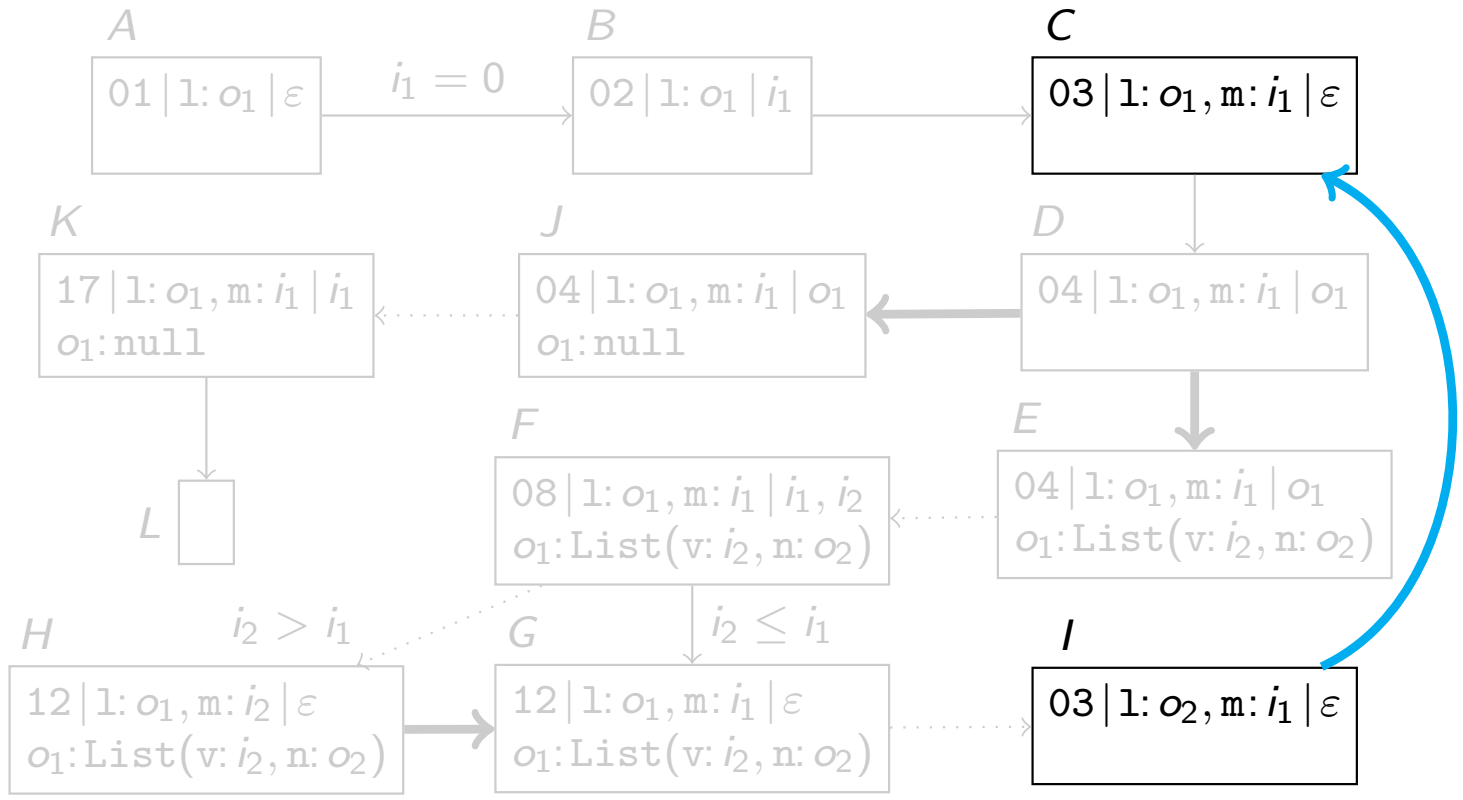
$H \xrightarrow{0} G$ **if** $\|o_1\| \geq 1 \ \wedge\ \|o_2\| \geq 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_2 \ \wedge \ldots$

weight: 0

condition:

$\|o\| = 0$	if $o : \texttt{null}$
$\|o\| \geq 1$	if $o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise
$x' = y$	if x in general state corresponds to y in special state

Transform Generalization Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

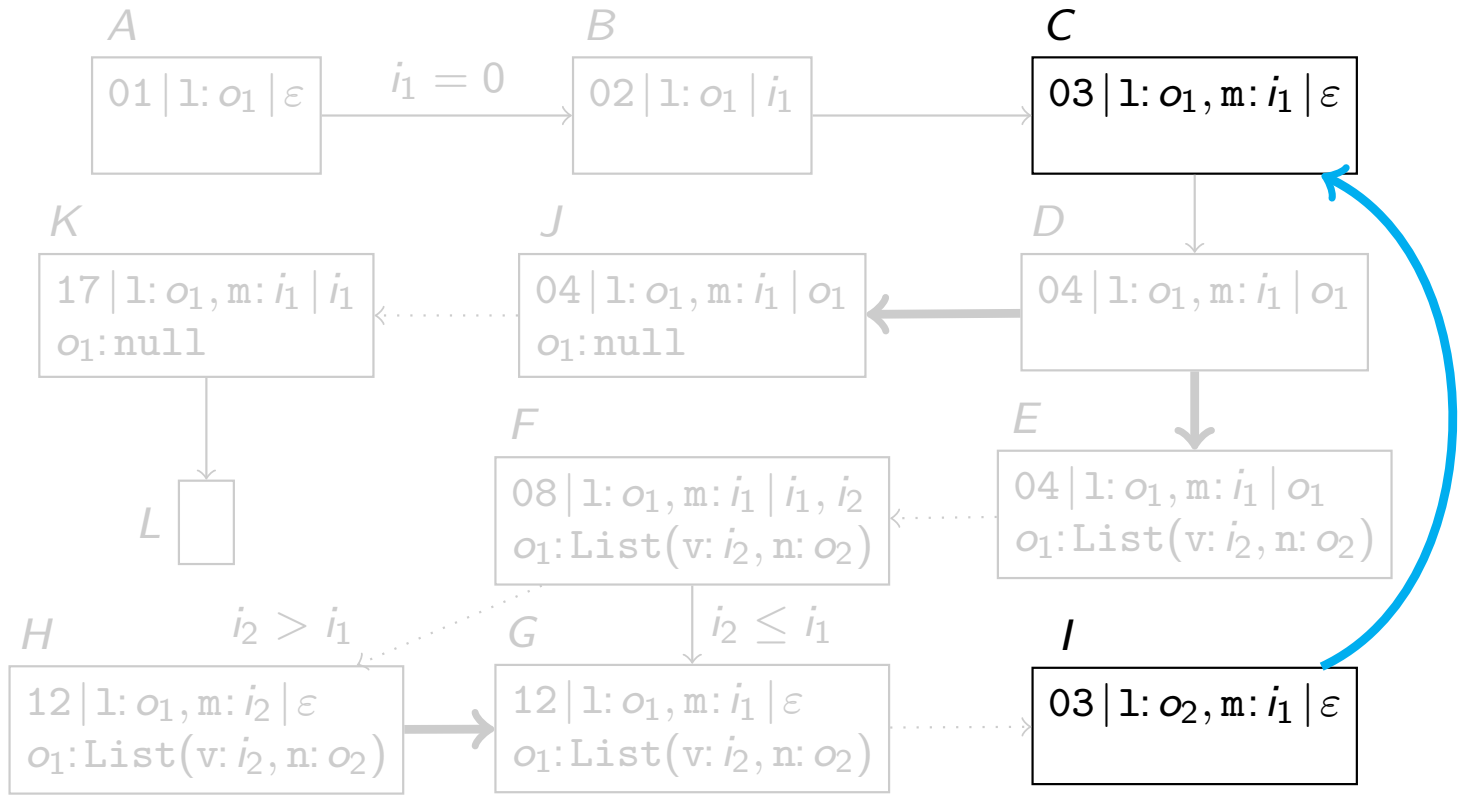
$$H \xrightarrow{0} G \quad \textbf{if} \quad \|o_1\| \geq 1 \ \wedge\ \|o_2\| \geq 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_2 \ \wedge \ldots$$

weight: 0

condition:

$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
$x' = y$	if x in general state corresponds to y in special state	

Transform Generalization Edges



ITS over variables

$i_1,\ i_2,\ \|o_1\|\ ,\ \|o_2\|$

$i_1',\ i_2',\ \|o_1\|',\ \|o_2\|'$

$H \xrightarrow{0} G$ **if** $\|o_1\| \geq 1 \ \wedge\ \|o_2\| \geq 0 \ \wedge\ \|o_1\|' = \|o_1\| \ \wedge\ i_1' = i_2 \ \wedge \ldots$

$I \xrightarrow{0} C$

weight: 0

condition:

$\|o\| = 0$	if	$o : \texttt{null}$
$\|o\| \geq 1$	if	$o : \texttt{List}(\ldots)$
$\|o\| \geq 0$	otherwise	
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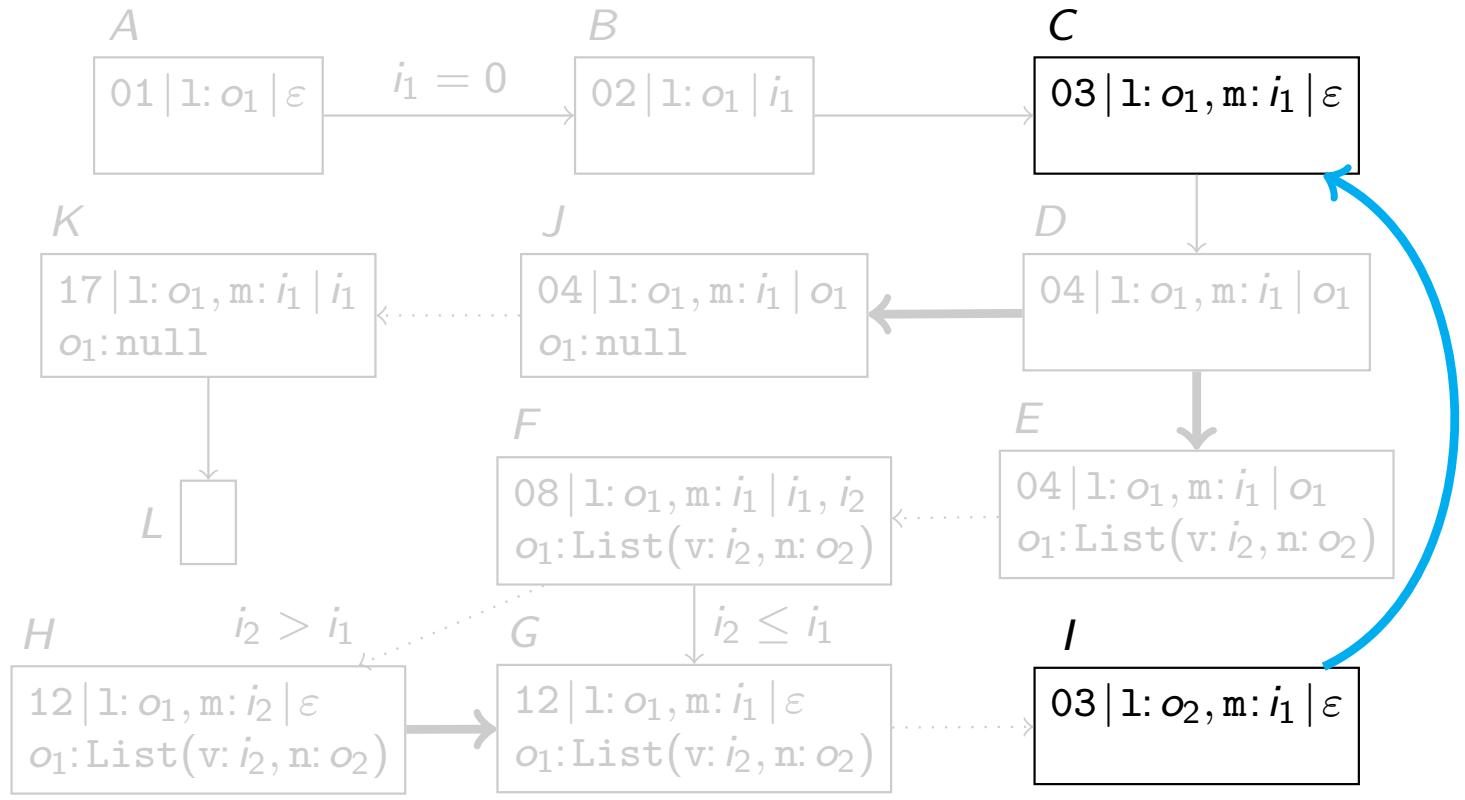

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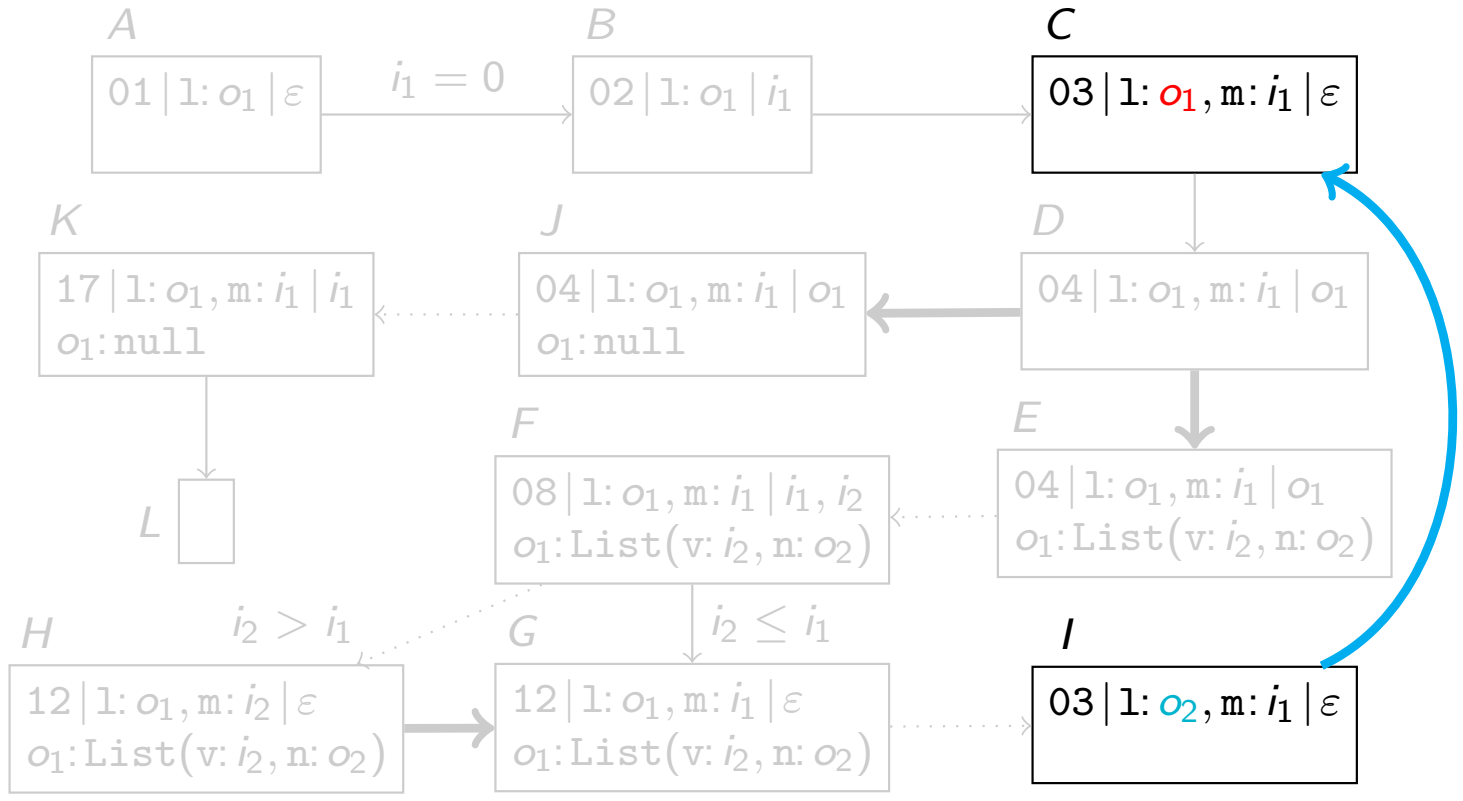

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weight: 0

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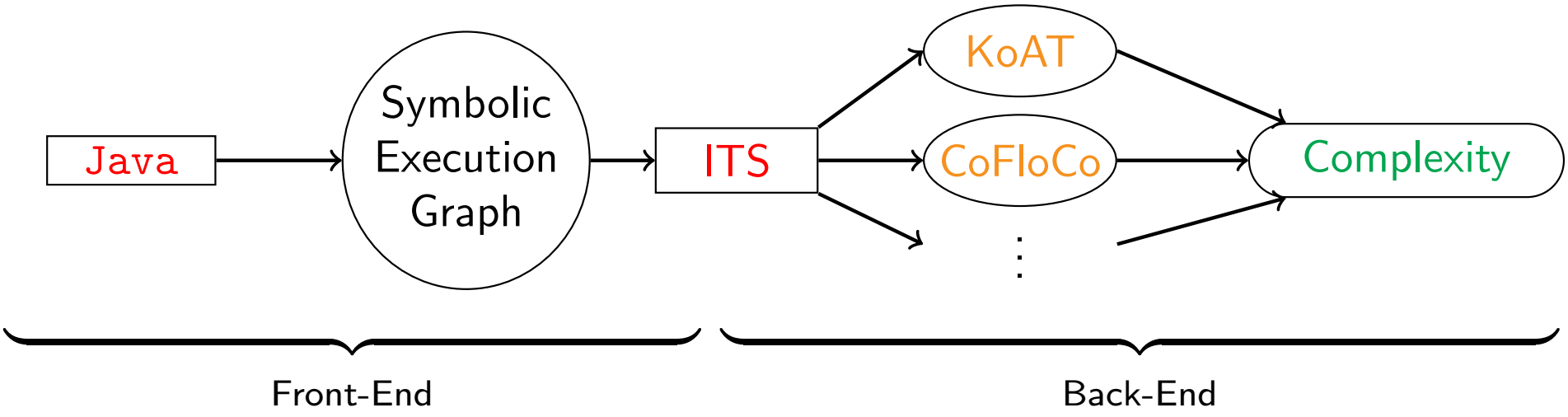
AProVE for Complexity Analysis of Java



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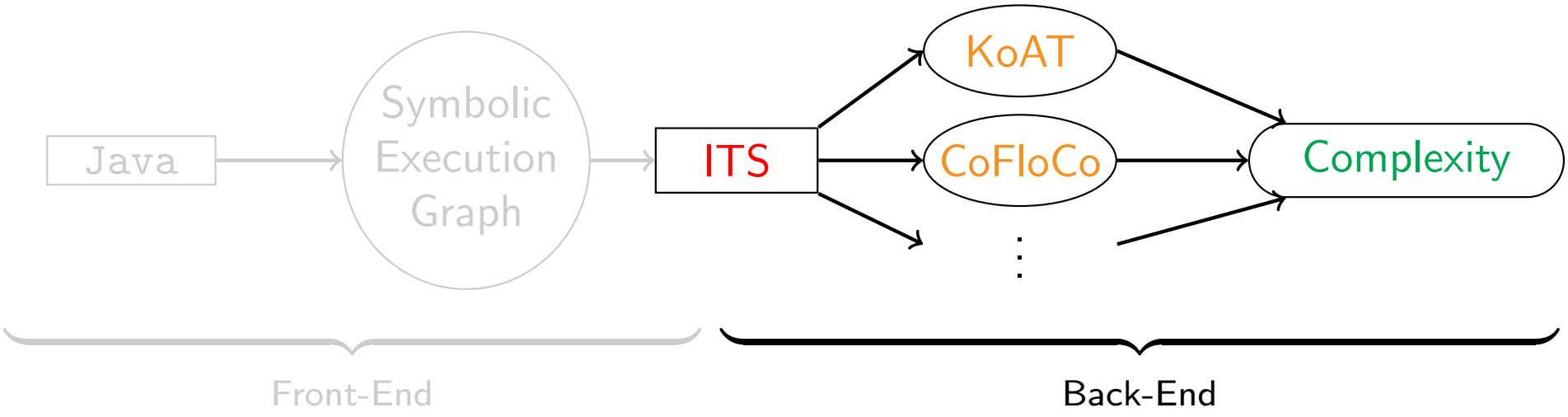
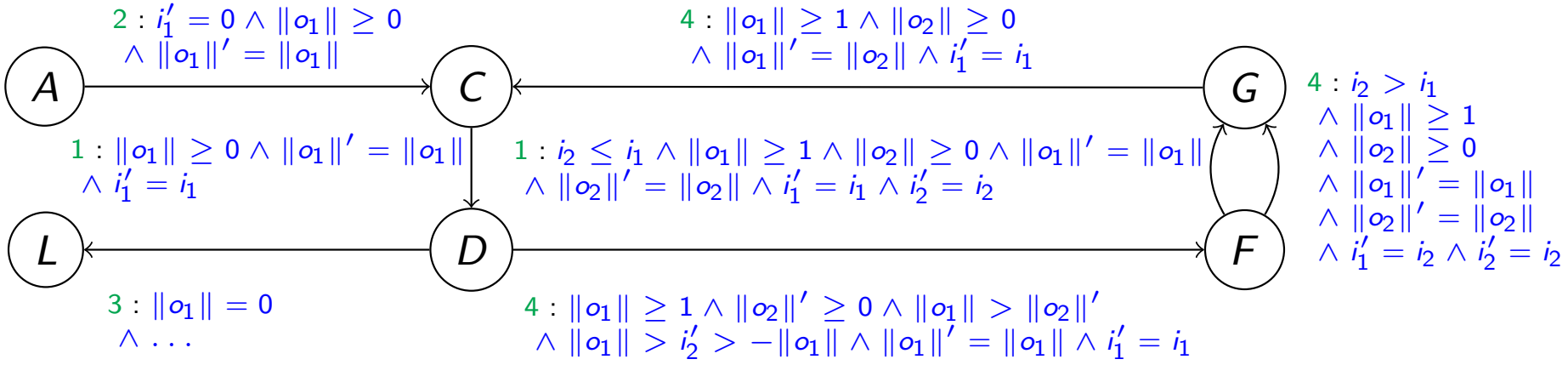


Complexity of Integer Transition Systems



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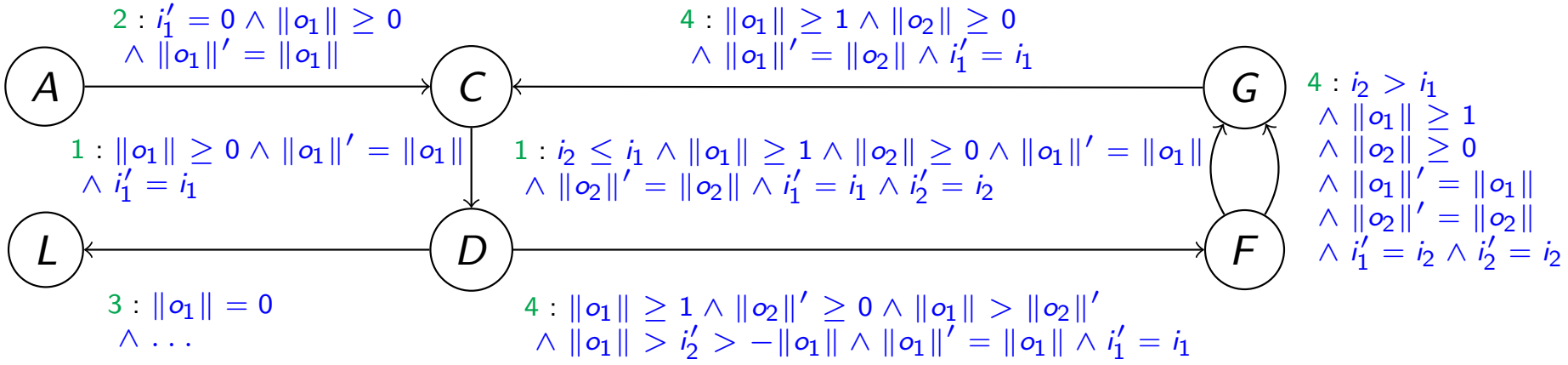

Complexity Bound b for location s:

$$b \quad \geq \quad \text{cost of any ITS evaluation starting with } s$$

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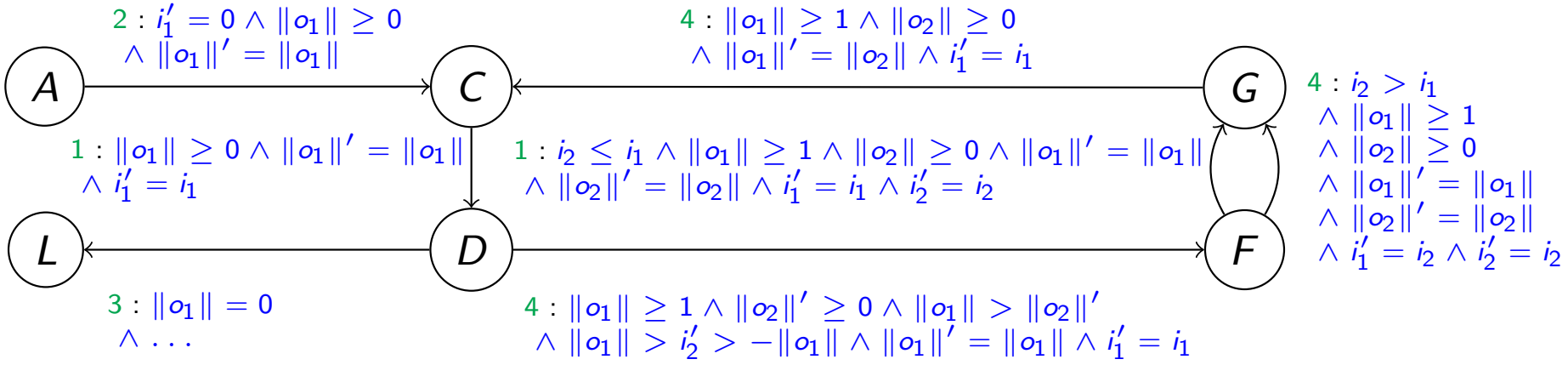

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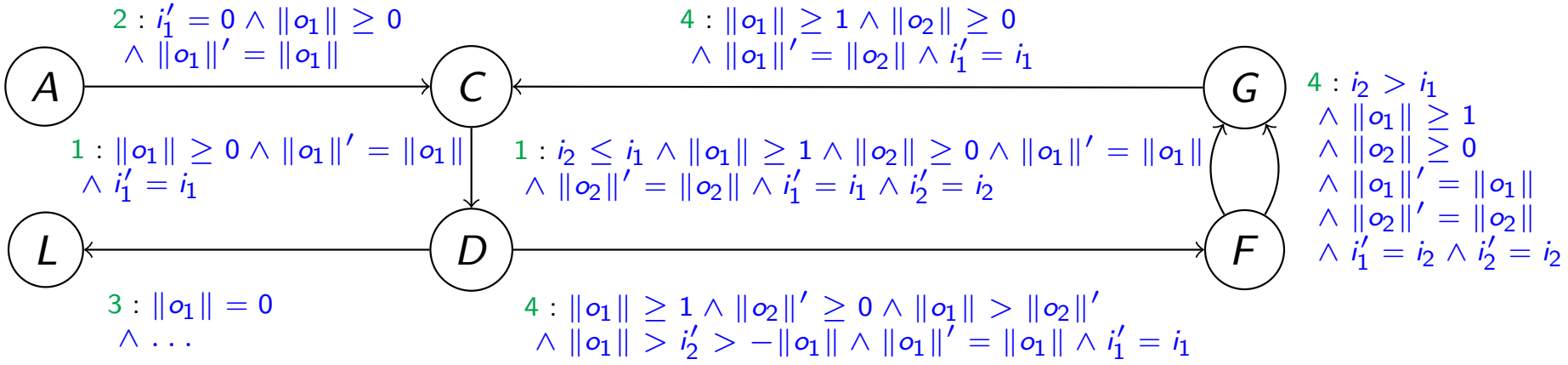
for all instantiations of variables in b and s

Complexity of Integer Transition Systems



$13 \cdot \|o_1\| + 6$ is complexity bound for Location A

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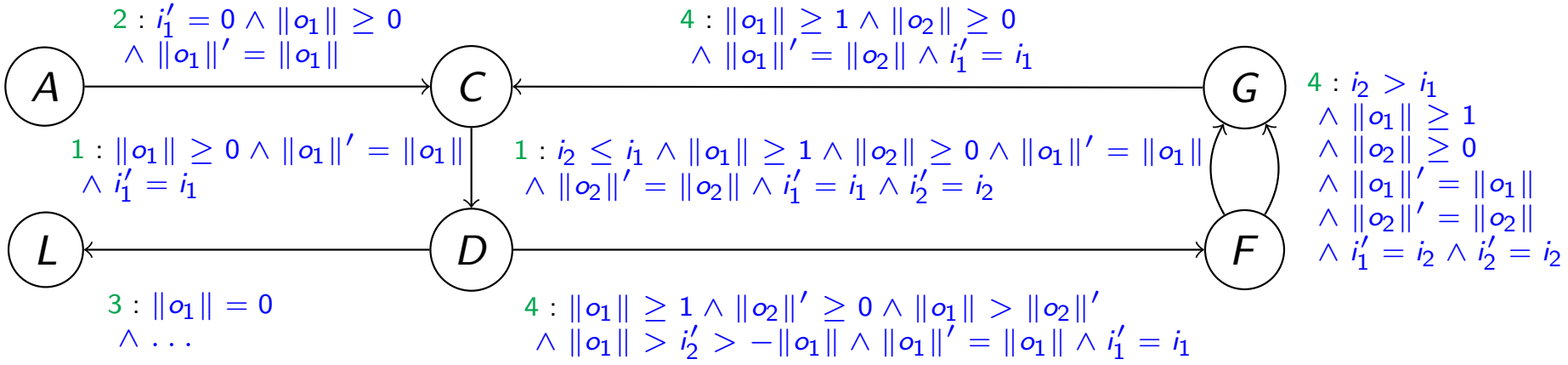

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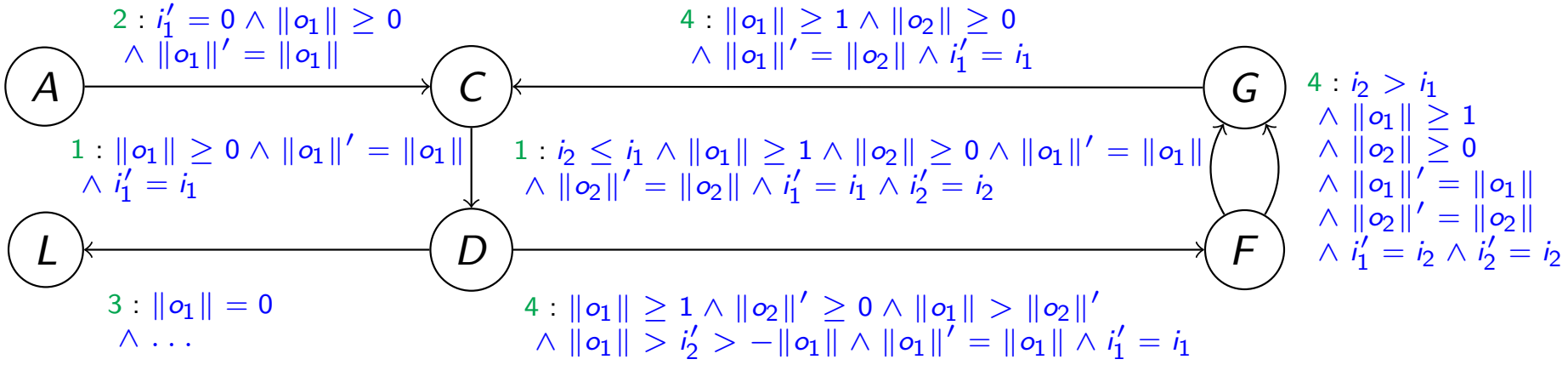

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Soundness Theorem

Complexity bound for location s in ITS is also complexity bound for state s in symbolic execution graph

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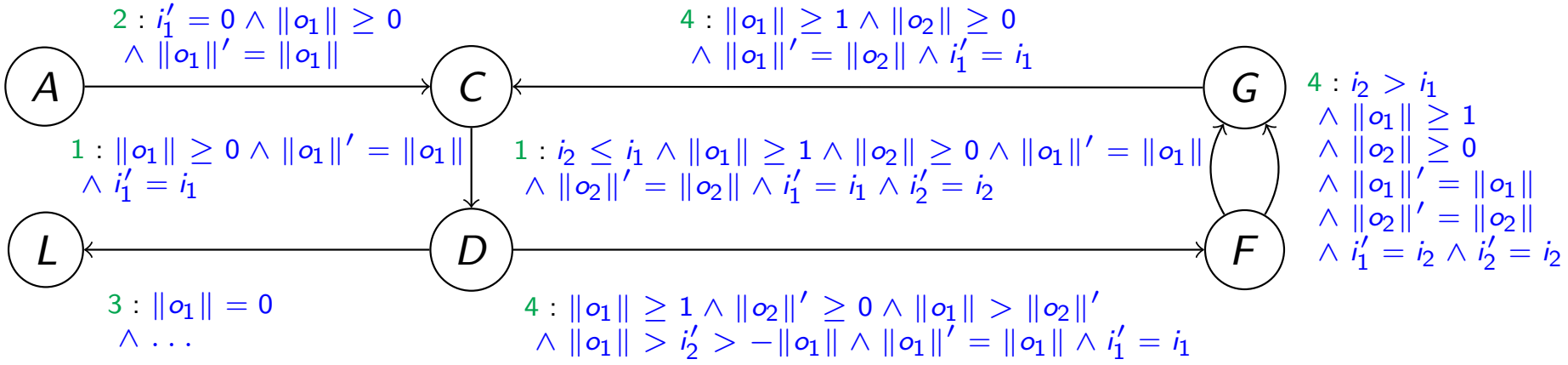

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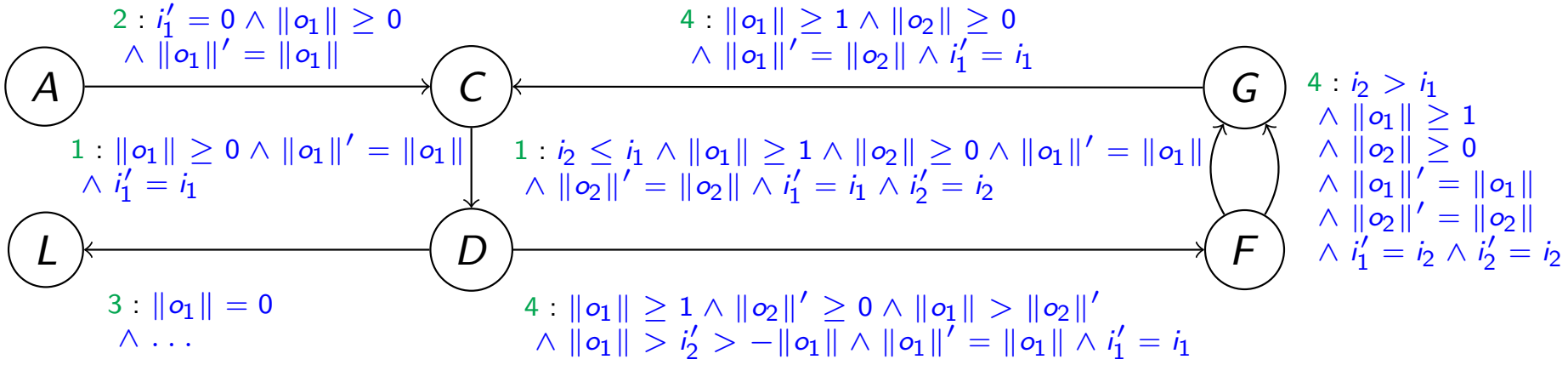


$13 \cdot \|o_1\| + 6$ is complexity bound for State A

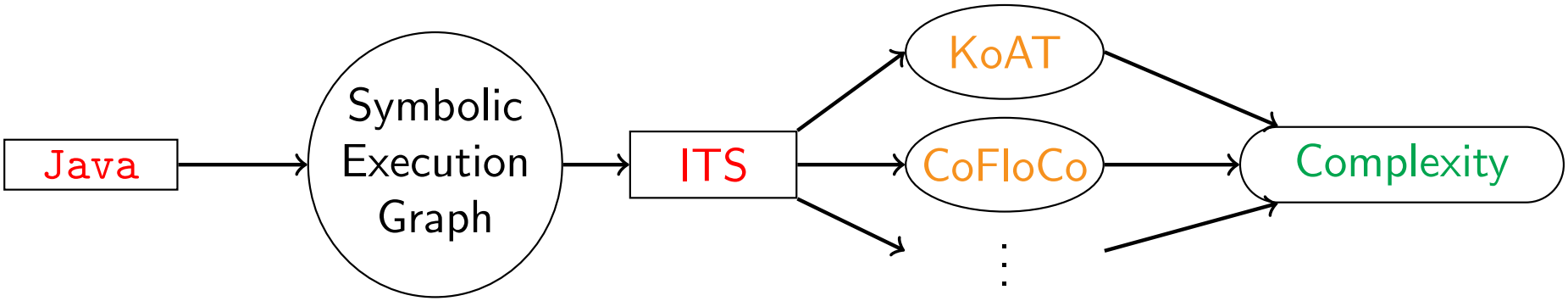
- linear bound determined by KoAT, CoFloCo, ...
- $13 \cdot \|\mathtt{l}\| + 6$ is upper bound for runtime of `max`

```
static int max(List l) {
  int m = 0;
  while (l != null) {
    if (l.v > m) {
      m = l.v;
    }
    l = l.n;
  }
  return m;
}
```

Soundness Theorem

Complexity bound for location s in ITS is also complexity bound for state s in symbolic execution graph

AProVE for Complexity Analysis of Java



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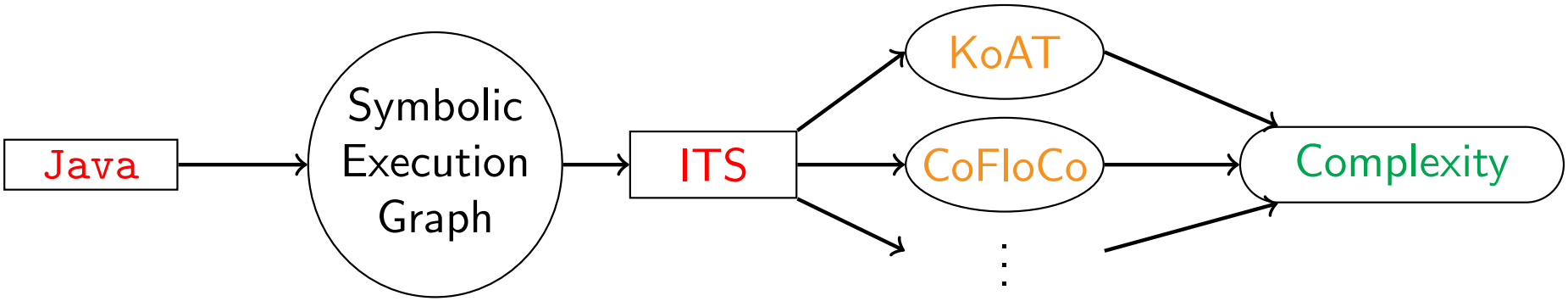

- runtime complexity analysis for heap-manipulating Java programs

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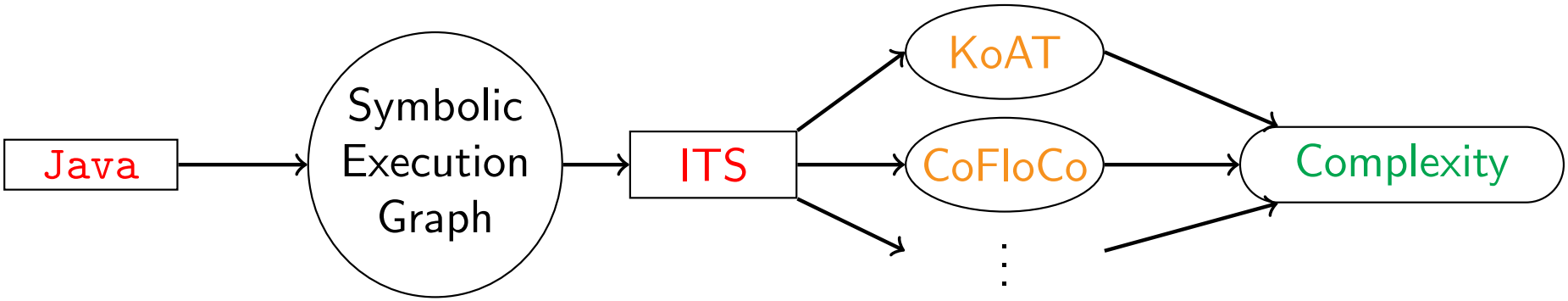

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- adaption for space or size analysis

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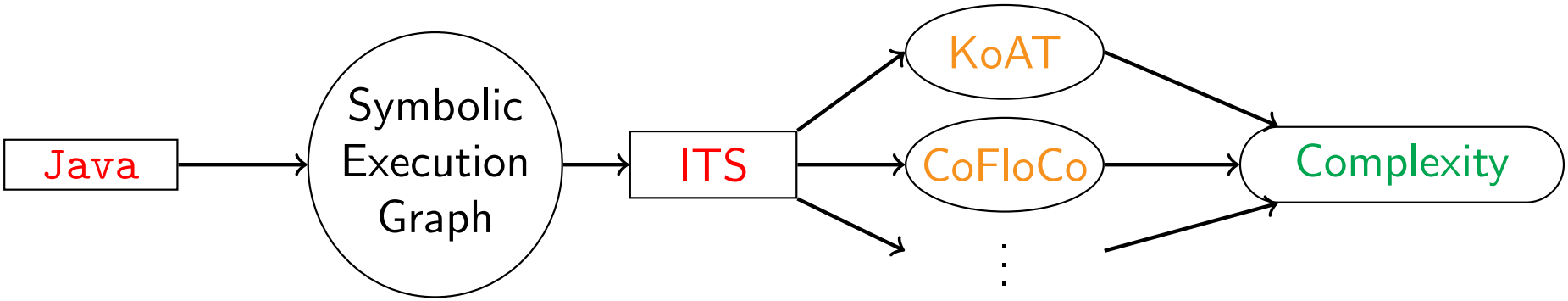

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- modular analysis by using summaries

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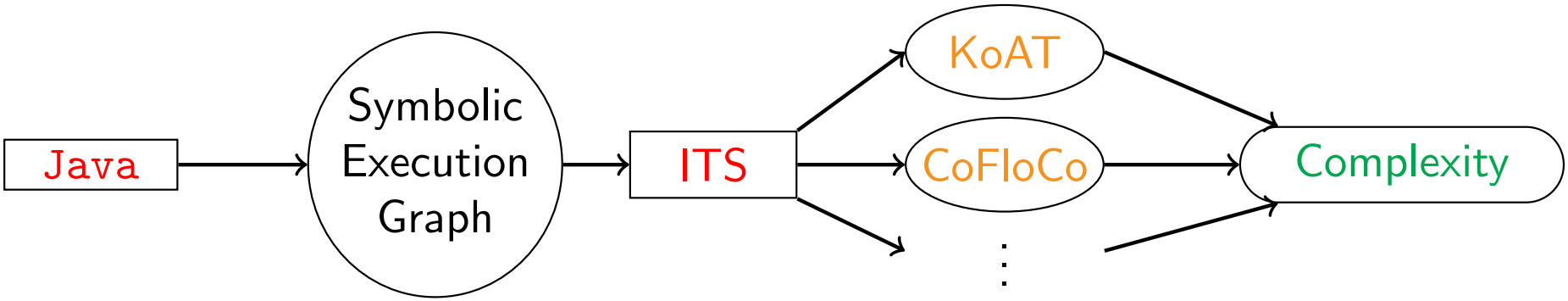

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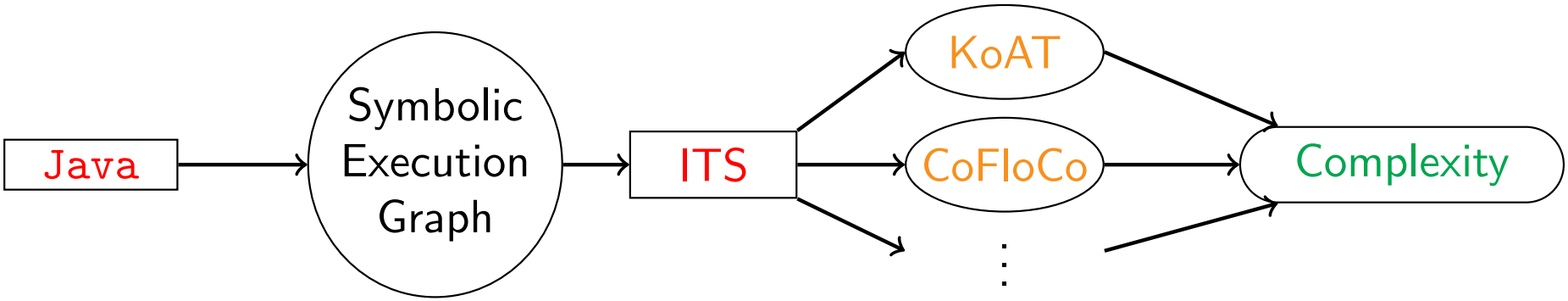
Experiments on 211 programs from the TPDB

	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$	$\mathcal{O}(n)$	$\mathcal{O}(n \cdot \log n)$	$\mathcal{O}(n^2)$	$\mathcal{O}(n^3)$	$\mathcal{O}(n^{>3})$	?	Success
AProVE	28	0	102	0	13	2	4	62	71 %
COSTA	10	4	45	3	5	0	1	143	32 %

AProVE for Complexity Analysis of Java



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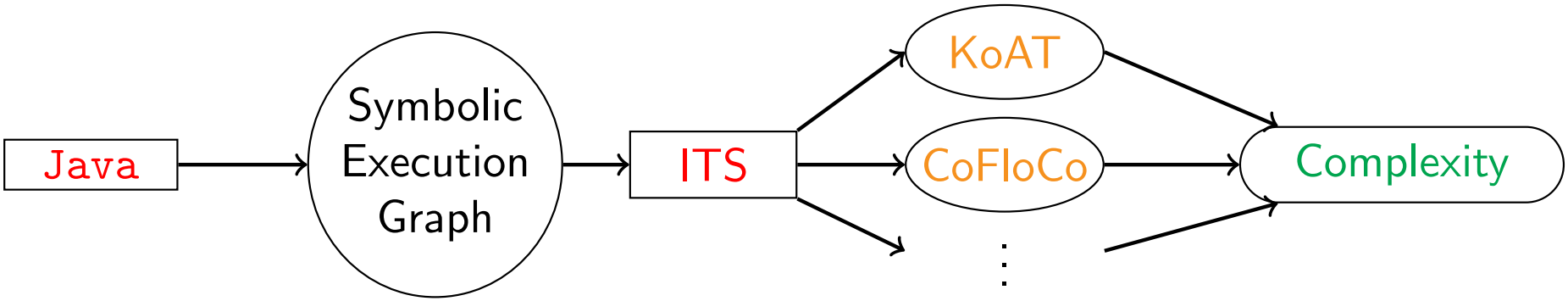
AProVE: *size* considers path length and values
COSTA: *size* considers path length only

AProVE for Complexity Analysis of Java



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STAC Project "Complexity Analysis-Based Guaranteed Execution"

- DARPA program "Space/Time Analysis for Cybersecurity"
- joint project with Draper Inc. and University of Innsbruck
- AProVE and KoAT crucial to detect/prove absence of vulnerabilities for Java programs