Automated Termination Analysis of JAVA BYTECODE by Term Rewriting

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joint work with C. Otto, M. Brockschmidt, C. von Essen

Termination Analysis for TRSs

$$\begin{array}{rrcl} \mathcal{R}: & \mathsf{plus}(x, 0) & \to & x \\ & \mathsf{plus}(x, \mathsf{s}(y)) & \to & \mathsf{s}(\mathsf{plus}(x, y)) \end{array}$$

$\mathcal{R}$ is *terminating* iff there is no infinite evaluation $t_1 \to_{\mathcal{R}} t_2 \to_{\mathcal{R}} \ldots$

Computation of "2 + 1":

$$\begin{array}{rcl} \mathsf{plus}(\mathsf{s}(\mathsf{s}(0)), \mathsf{s}(0)) & \to_{\mathcal{R}} & \mathsf{s}(\mathsf{plus}(\mathsf{s}(\mathsf{s}(0)), 0)) \\ & \to_{\mathcal{R}} & \mathsf{s}(\mathsf{s}(\mathsf{s}(0))) \end{array}$$

- easier / more general than for programs
- suitable for automation
- **But:** halting problem is undecidable!
  $\Rightarrow$ automated termination proofs do not always succeed

Termination Analysis for TRSs

$$\begin{array}{rcl} \mathcal{P}ol(\,\mathsf{plus}(0,y)\,) & > & \mathcal{P}ol(\,y\,) \\ \mathcal{P}ol(\,\mathsf{plus}(\mathsf{s}(x),y)\,) & > & \mathcal{P}ol(\,\mathsf{s}(\mathsf{plus}(x,y))\,) \end{array}$$

$\mathcal{R}$ is *terminating* iff there is no infinite evaluation $t_1 \to_{\mathcal{R}} t_2 \to_{\mathcal{R}} \ldots$

- **Goal:** Find order $\succ$ such that $\ell \succ r$ for all rules $\ell \to r \in \mathcal{R}$
- **Polynomial Order:** $\ell \succ r$ iff $\mathcal{P}ol(\ell) > \mathcal{P}ol(r)$

$$\begin{array}{rcl} \mathcal{P}ol(0) & = & 1 \\ \mathcal{P}ol(\mathsf{s}(t)) & = & 1 + \mathcal{P}ol(t) \\ \mathcal{P}ol(\mathsf{plus}(t_1,t_2)) & = & 2\,\mathcal{P}ol(t_1) + \mathcal{P}ol(t_2) \end{array}$$

Termination Analysis for TRSs

$$\begin{array}{rcl} 2 + \mathcal{P}ol(y) & > & \mathcal{P}ol(y) \\ 2\,(1 + \mathcal{P}ol(x)) + \mathcal{P}ol(y) & > & 1 + 2\,\mathcal{P}ol(x) + \mathcal{P}ol(y) \end{array}$$

$\mathcal{R}$ is *terminating* iff there is no infinite evaluation $t_1 \to_{\mathcal{R}} t_2 \to_{\mathcal{R}} \ldots$

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- can be generated automatically by SAT solving (SAT '07)

Automated Termination Analysis for TRSs

- Classical Techniques (simplification orders)
  - Knuth–Bendix Order *(Knuth & Bendix, 70)*
  - Polynomial Order *(Lankford, 79)*
  - Lexicographic Path Order *(Kamin & Lévy, 80)*
  - Recursive Path Order *(Dershowitz, 82)*
- Recent Techniques (beyond simplification orders)
  - Transformation Order *(Bellegarde & Lescanne, 87)*
  - Semantic Labelling *(Zantema, 95)*
  - Dependency Pairs *(Arts & Giesl, 96)*
  - Monotonic Semantic Path Order *(Borralleras, Ferreira, Rubio, 00)*
  - Match-Bounds *(Geser, Hofbauer, Waldmann, 03)*
  - Matrix Order *(Endrullis, Hofbauer, Waldmann, Zantema, 06)*
- Active area of research: *International Workshop on Termination*

Integer Term Rewriting

Termination of Term Rewriting

- powerful for algorithms on user-defined data structures
(automatic generation of orders to compare arbitrary terms)

- naive handling of pre-defined data structures
(represent data objects by terms)

Representing integers

$$
\begin{aligned}
0 &\equiv \mathsf{pos}(0) \equiv \mathsf{neg}(0)\\
1 &\equiv \mathsf{pos}(\mathsf{s}(0))\\
-1 &\equiv \mathsf{neg}(\mathsf{s}(0))\\
1000 &\equiv \mathsf{pos}(\mathsf{s}(\mathsf{s}(\ldots \mathsf{s}(0)\ldots)))
\end{aligned}
$$

Rules for pre-defined operations

$$
\begin{aligned}
\mathsf{pos}(x) + \mathsf{neg}(y) &\to \mathsf{minus}(x, y)\\
\mathsf{neg}(x) + \mathsf{pos}(y) &\to \mathsf{minus}(y, x)\\
\mathsf{pos}(x) + \mathsf{pos}(y) &\to \mathsf{pos}(\mathsf{plus}(x, y))\\
\mathsf{neg}(x) + \mathsf{neg}(y) &\to \mathsf{neg}(\mathsf{plus}(x, y))\\
\mathsf{minus}(x, 0) &\to \mathsf{pos}(x)\\
\mathsf{minus}(0, y) &\to \mathsf{neg}(y)\\
\mathsf{minus}(\mathsf{s}(x), \mathsf{s}(y)) &\to \mathsf{minus}(x, y)
\end{aligned}
$$

Integer Term Rewriting

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- powerful for algorithms on user-defined data structures
  (automatic generation of orders to compare arbitrary terms)

- naive handling of pre-defined data structures
  (represent data objects by terms)

Integer Term Rewriting (RTA '09)

- integrated pre-defined data structures like $\mathbb{Z}$ into term rewriting
- adapted techniques to prove termination of integer TRSs
- for algorithms on integers: as powerful as direct techniques
- for user-defined data structures: as powerful as before

Integer Term Rewriting

- $\mathcal{F}_{int}$: pre-defined symbols
  - $\mathbb{Z} = \{0, 1, -1, 2, -2, \ldots\}$
  - $\mathbb{B} = \{\mathsf{true}, \mathsf{false}\}$
  - $+, -, *, /, \%$
  - $>, \geq, <, \leq, ==, !=$
  - $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
- $\mathcal{PD}$: pre-defined rules

  $2*21 \to 42 \qquad 42 \geq 23 \to \mathsf{true}$

  $\mathsf{true} \wedge \mathsf{false} \to \mathsf{false} \qquad \ldots$

  $\Rightarrow$ pre-defined operations only evaluated if all arguments are from $\mathbb{Z}$ or $\mathbb{B}$

- ITRS $\mathcal{R}$: finite TRS
  - no pre-defined symbols except $\mathbb{Z}$ and $\mathbb{B}$ in lhs
  - lhs $\notin \mathbb{Z} \cup \mathbb{B}$
  - rewrite relation defined w.r.t. $\mathcal{R} \cup \mathcal{PD}$ (innermost rewriting)

Example ITRS computing $\sum_{i=y}^{x} i$

$$
\begin{aligned}
\mathsf{sum}(x, y) &\to \mathsf{sif}(x \geq y, x, y)\\
\mathsf{sif}(\mathsf{true}, x, y) &\to y + \mathsf{sum}(x, y + 1)\\
\mathsf{sif}(\mathsf{false}, x, y) &\to 0
\end{aligned}
$$

Integer Term Rewriting

$$
\begin{array}{llllll}
\underline{\mathsf{sum}(1,1)} & \to_{\mathcal{R}} & \mathsf{sif}(\underline{1 \geq 1}, 1, 1) & \to_{\mathcal{R}} & \underline{\mathsf{sif}(\mathsf{true}, 1, 1)} \\
 & \to_{\mathcal{R}} & 1 + \mathsf{sum}(1, \underline{1+1}) & \to_{\mathcal{R}} & 1 + \underline{\mathsf{sum}(1, 2)} \\
 & \to_{\mathcal{R}} & 1 + \mathsf{sif}(\underline{1 \geq 2}, 1, 2) & \to_{\mathcal{R}} & 1 + \underline{\mathsf{sif}(\mathsf{false}, 1, 2)} \\
 & \to_{\mathcal{R}} & \underline{1 + 0} & \to_{\mathcal{R}} & 1
\end{array}
$$

- ITRS $\mathcal{R}$: finite TRS
  - no pre-defined symbols except $\mathbb{Z}$ and $\mathbb{B}$ in lhs
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\mathsf{sif}(\mathsf{false}, x, y) & \to & 0
\end{array}
$$

Automated Termination Tools for TRSs

- AProVE *(Aachen)*
- CARIBOO *(Nancy)*
- CiME *(Orsay)*
- Jambox *(Amsterdam)*
- Matchbox *(Leipzig)*
- MU-TERM *(Valencia)*
- MultumNonMulta *(Kassel)*
- TEPARLA *(Eindhoven)*
- Termptation *(Barcelona)*
- TORPA *(Eindhoven)*
- TPA *(Eindhoven)*
- TTT *(Innsbruck)*
- VMTL *(Vienna)*

- Annual *International Competition of Termination Tools*
- well-developed field
- active research
- powerful techniques & tools
- **But:** What about application in practice?

Termination of Programming Languages

Functional Languages

- first-order languages with strict evaluation strategy
  (Walther, 94), (Giesl, 95), (Lee, Jones, Ben-Amram, 01)
- ensuring termination (e.g., by typing)
  (Telford & Turner, 00), (Xi, 02), (Abel, 04), (Barthe et al, 04) etc.
- outermost termination of untyped first-order rewriting
  (Fissore, Gnaedig, Kirchner, 02), (Endrullis & Hendriks, 09), (Raffelsieper & Zantema, 09), (Thiemann, 09)
- automated technique for small HASKELL-like language
  (Panitz & Schmidt-Schauss, 97)

- do not work on full existing languages
- no use of TRS-techniques (stand-alone methods)

Functional Languages

- using TRS-techniques for HASKELL is challenging

  - HASKELL has a lazy evaluation strategy.
    For TRSs, one proves termination of *all* reductions.
  - HASKELL's equations are handled from top to bottom.
    For TRSs, *any* rule may be used for rewriting.
  - HASKELL has polymorphic types.
    TRSs are *untyped*.
  - In HASKELL-programs, often only some functions terminate.
    TRS-methods try to prove termination of *all* terms.
  - HASKELL is a higher-order language.
    Most automatic TRS-methods only handle *first-order* rewriting.

Termination of Programming Languages

Functional Languages

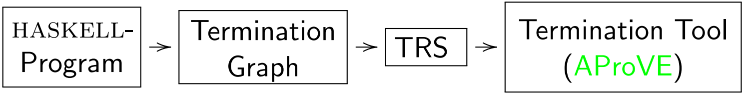
- using TRS-techniques for HASKELL is challenging
- **New approach** (RTA '06),(TOPLAS '10)
  - Frontend
    - evaluate HASKELL a few steps ⇒ termination graph

      termination graph captures evaluation strategy, types, etc.
    - transform termination graph ⇒ TRS
  - Backend
    - prove termination of the resulting TRS
      (using existing techniques & tools)
- implemented in **AProVE**
  - accepts full HASKELL 98 language
  - successfully evaluated with standard HASKELL-libraries
    (succeeds on approx. 80 % of the functions in standard libraries)

Termination of Programming Languages

Functional Languages

- using TRS-techniques for HASKELL is challenging



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Termination of Programming Languages

Logic Languages

- well-developed field *(De Schreye & Decorte, 94) etc.*
- **direct approaches:** work directly on the logic program
  - cTI *(Mesnard et al)*
  - TerminWeb *(Codish et al)*
  - TermiLog *(Lindenstrauss et al)*
  - Polytool *(Nguyen, De Schreye, Giesl, Schneider-Kamp)*

  TRS-techniques can be adapted to work *directly* on the LP
- **transformational approaches:** transform LP to TRS
  - TALP *(Ohlebusch et al)*
  - AProVE *(Giesl et al)*
- only for *definite* LP (without cut)
- not for real PROLOG

Termination of Programming Languages

Logic Languages

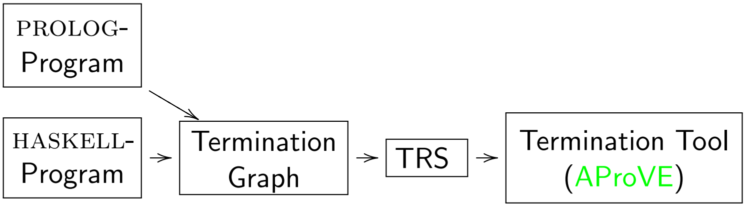
- analyzing PROLOG is challenging due to cuts etc.
- **New approach** (ICLP '10)
  - Frontend
    - evaluate PROLOG a few steps ⇒ termination graph

      termination graph captures evaluation strategy due to cuts etc.
    - transform termination graph ⇒ TRS
  - Backend
    - prove termination of the resulting TRS
      (using existing techniques & tools)
- implemented in **AProVE**
  - successfully evaluated on PROLOG-collections with cuts
  - most powerful termination tool for PROLOG
    (winner of the international *termination competition* for PROLOG)

Termination of Programming Languages

Logic Languages

- analyzing PROLOG is challenging due to cuts etc.



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Termination of Programming Languages

Imperative Languages

- Synthesis of Linear Ranking Functions
  (Colon & Sipma, 01), (Podelski & Rybalchenko, 04)
- Terminator: Termination Analysis by Abstraction & Model Checking
  (Cook, Podelski, Rybalchenko et al., since 05)
- Julia & COSTA: Termination Analysis of JAVA BYTECODE
  (Spoto, Mesnard, Payet, 10),
  (Albert, Arenas, Codish, Genaim, Puebla, Zanardini, 08)
- ...

- used at Microsoft for verifying Windows device drivers
- no use of TRS-techniques (stand-alone methods)

Termination of Programming Languages

Imperative Languages

- analyze JAVA BYTECODE (JBC) instead of JAVA
- using TRS-techniques for JBC is challenging
  - sharing and aliasing
  - side effects
  - cyclic data objects
  - object-orientation
  - ...

Termination of Programming Languages

Imperative Languages

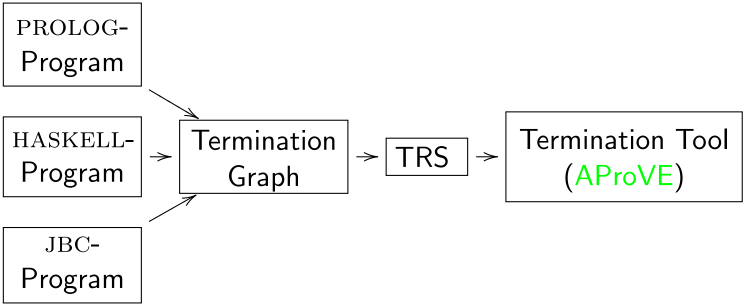
- using TRS-techniques for JBC is challenging
- **New approach** (RTA '10)
  - Frontend
    - evaluate JBC a few steps ⇒ termination graph

      termination graph captures side effects, sharing, cyclic data objects etc.
    - transform termination graph ⇒ TRS
  - Backend
    - prove termination of the resulting TRS
      (using existing techniques & tools)
- implemented in **AProVE**
  - successfully evaluated on JBC-collection
  - most powerful termination tool for JBC
    (winner of the international *termination competition* for JBC)

Termination of Programming Languages

Imperative Languages

- using TRS-techniques for JBC is challenging



- implemented in **AProVE**
  - successfully evaluated on JBC-collection
  - most powerful termination tool for JBC
    (winner of the international *termination competition* for JBC)

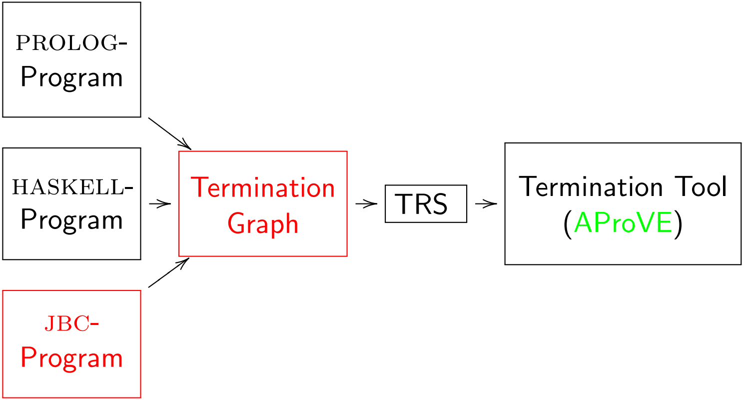
Termination of Programming Languages

- other techniques:
  abstract objects to numbers
  - `IntList`-object representing $[0, 1, 2]$
    is abstracted to length 3

```
public class IntList {
  int value;
  IntList next;
}
```

- our technique:
  abstract objects to terms
  - introduce function symbol for every class
  - `IntList`-object representing $[0, 1, 2]$
    is abstracted to term: $\mathsf{IntList}(0,\ \mathsf{IntList}(1,\ \mathsf{IntList}(2, \mathsf{null})\,)\,)$
  - TRS-techniques generate suitable orders to compare arbitrary terms
  - particularly powerful on user-defined data types
  - powerful on pre-defined data types by using Integer TRSs

From JBC to Termination Graphs



Example

```
00: aload_0       // load num to opstack
01: ifnull 8      // jump to line 8 if top
                  // of opstack is null
04: aload_1       // load limit
05: ifnonnull 9  // jump if not null
08: return
09: aload_0       // load num
10: astore_2      // store into copy
11: aload_0       // load num
12: getfield val // load field val
15: aload_1       // load limit
16: getfield val // load field val
19: if_icmpge 35 // jump if
                  // num.val >= limit.val
22: aload_2       // load copy
23: aload_2       // load copy
24: getfield val // load field val
27: iconst_1      // load constant 1
28: iadd          // add copy.val and 1
29: putfield val // store into copy.val
32: goto 11
35: return
```

```
public class Int {
  // only wrap a primitive int
  private int val;

  // count up to the value
  // in "limit"
  public static void count(
      Int num, Int limit) {

    if (num == null
        || limit == null) {
        return;
    }

    // introduce sharing
    Int copy = num;

    while (num.val < limit.val) {
        copy.val++;
    }
  }
}
```

Abstract States of the JVM

```
00: aload_0       // load num to opstack
01: ifnull 8      // jump to line 8 if top
                  // of opstack is null
04: aload_1       // load limit
05: ifnonnull 9   // jump if not null
08: return
09: aload_0       // load num
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22: aload_2       // load copy
23: aload_2       // load copy
24: getfield val  // load field val
27: iconst_1      // load constant 1
28: iadd          // add copy.val and 1
29: putfield val  // store into copy.val
32: goto 11
35: return
```

$\texttt{ifnull 8} \mid \texttt{n}:o_1, \texttt{l}:o_2 \mid o_1$

$o_1 = \texttt{Int}(\texttt{val} = i_1) \quad i_1 = (-\infty, \infty)$

$o_2 = \texttt{Int}(?)$

4 components

1. next program instruction
2. values of local variables (value of `num` is *reference* o_1)
3. values on the operand stack
4. information about the heap
   - object at address o_2 is `null` or of type `Int`
   - object at o_1 has type `Int`, `val`-field has value i_1
   - i_1 is an arbitrary integer
   - no sharing

From JBC to Termination Graphs

```
00: aload_0
01: ifnull 8
04: aload_1
     ⋮
19: if_icmpge 35
     ⋮
27: iconst_1
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29: putfield val
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35: return
```

A

aload_0 | n : o_1, l : o_2 | ε
o_1 = Int(?) o_2 = Int(?)

State A:

- do all calls of `count` terminate?
- `num` and `limit` are arbitrary, but distinct `Int`-objects

From JBC to Termination Graphs

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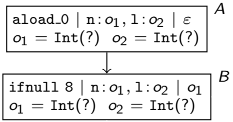

State *B*:

- "aload_0" loads value of num on operand stack
- *A* connected to *B* by *evaluation edge*

From JBC to Termination Graphs

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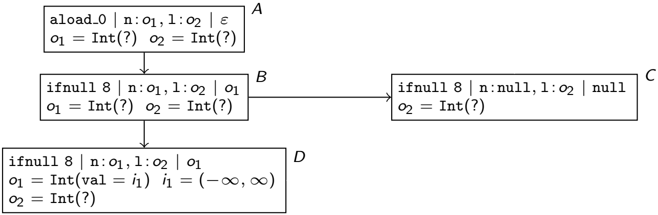

States *C* and *D*:

- "`ifnull 8`" needs to know whether o_1 is `null`
- *refine* information about heap (*refinement edges*)

From JBC to Termination Graphs

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     ⋮
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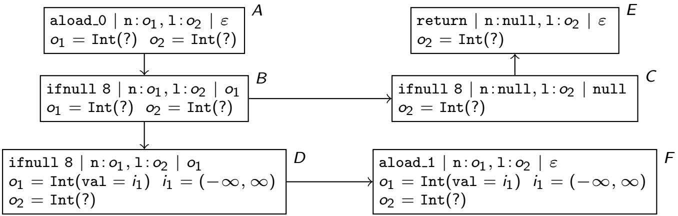

States *E* and *F*:

- evaluate "`ifnull 8`" in *C* and *D*
- *evaluation edges*

From JBC to Termination Graphs

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     :
19: if_icmpge 35
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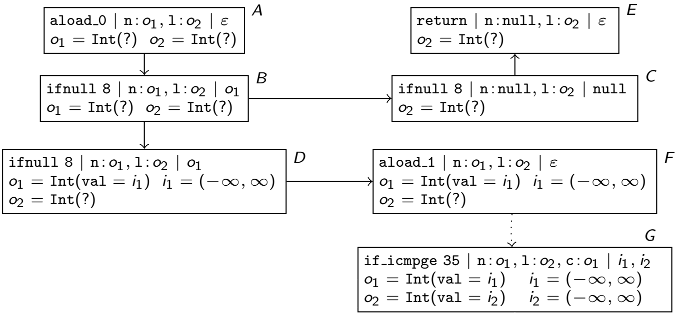

State G:

- in state F, check if `limit` is `null` analogously
- aliasing in G: `num` and `copy` point to the same address o_1
- `val`-fields of `num` and `limit` pushed on operand stack

From JBC to Termination Graphs

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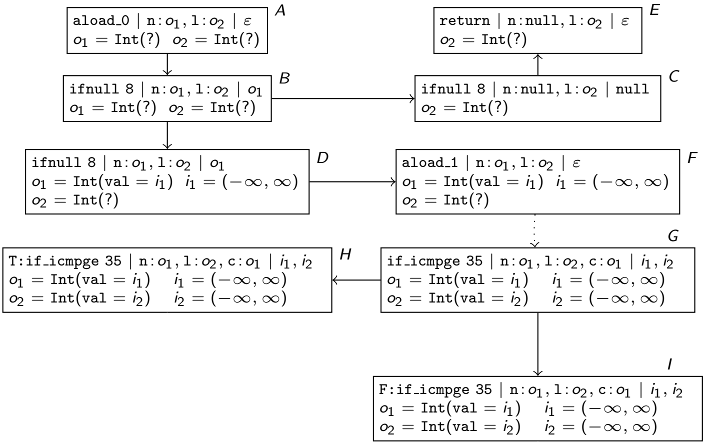

States H and I:

- "`if_icmpge 35`" needs to know whether $i_1 \geq i_2$
- *refine* information about heap (*refinement edges*)

From JBC to Termination Graphs

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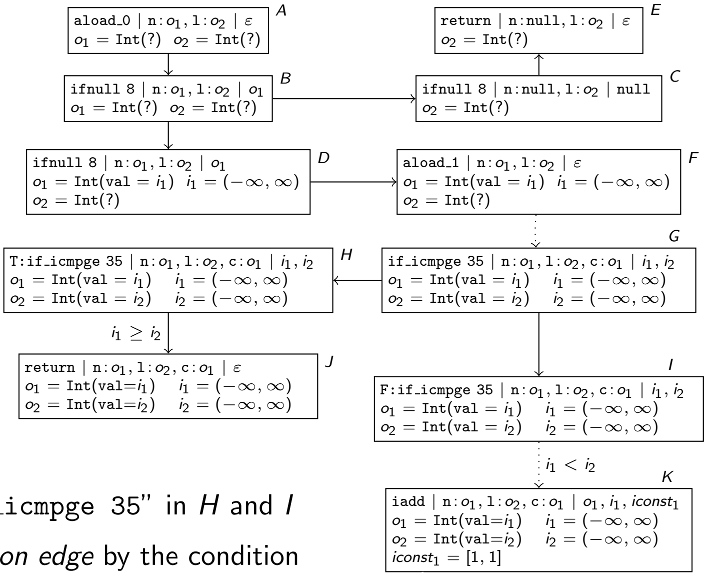

States J and K:

- evaluate "if_icmpge 35" in H and I
- label *evaluation edge* by the condition
- val-field of copy and integer variable with value 1 on operand stack

From JBC to Termination Graphs

```
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01: ifnull 8
04: aload_1
      ⋮
19: if_icmpge 35
      ⋮
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29: putfield val
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35: return
```
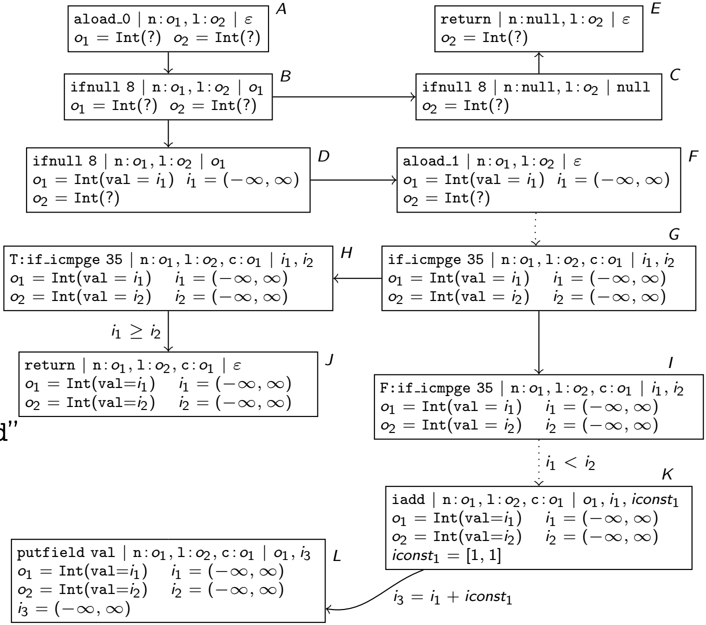

State L:

- evaluate "iadd"
- new variable i_3
- label edge by connection



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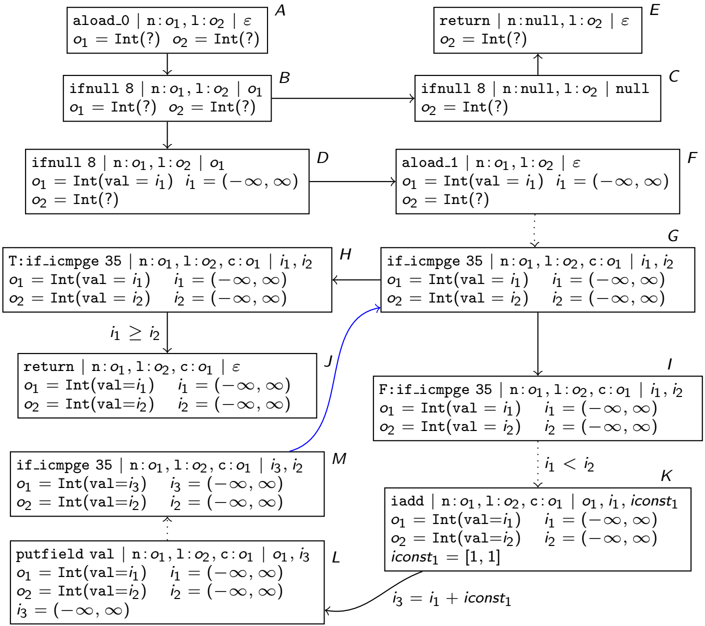


State *M*:

- again reaches "`if_icmpge`"
- *M* *instance* of *G*
- *instantiation edge*

From JBC to Termination Graphs

Termination Graphs

- expand nodes until all leaves correspond to program ends
- by appropriate generalization steps,
  one always reaches a *finite* termination graph
- state s_1 is *instance* of s_2 iff
  every concrete state described by s_1 is also described by s_2

Using Termination Graphs for Termination Proofs

- every JBC-computation of concrete states
  corresponds to a *computation path* in the termination graph
- termination graph is called *terminating* iff
  it has no infinite computation path

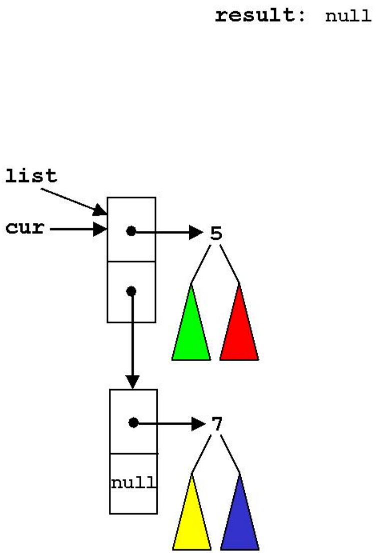
Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
         flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```

```
public class Tree {
  int value;
  Tree left;
  Tree right;
}

public class TreeList {
  Tree value;
  TreeList next;
}

public class IntList {
  int value;
  IntList next;
}
```

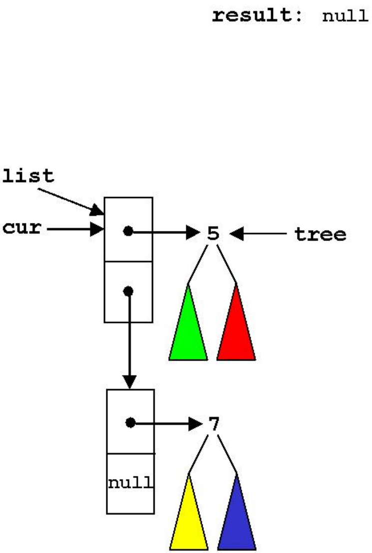
Example with User-Defined Data Type

```java
public class Flatten {
  public static IntList
        flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```



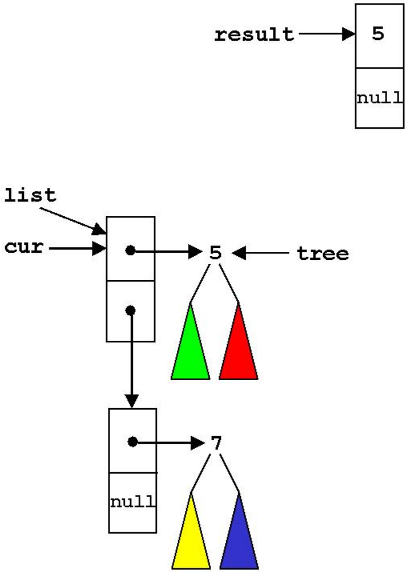
Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
         flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```



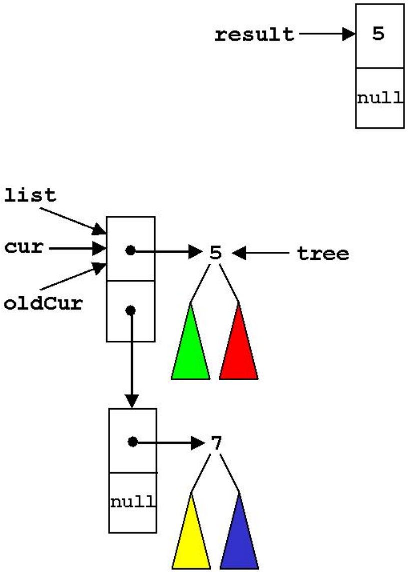
Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
         flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```



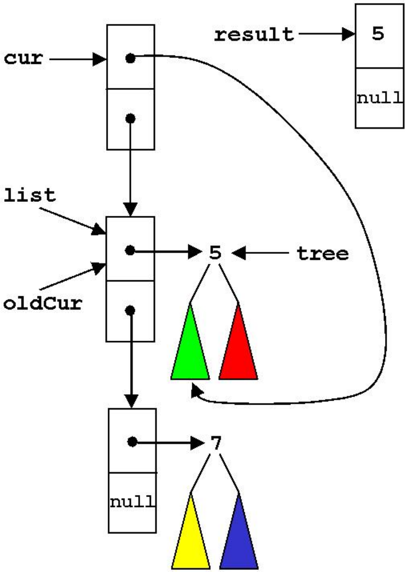
Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
         flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```



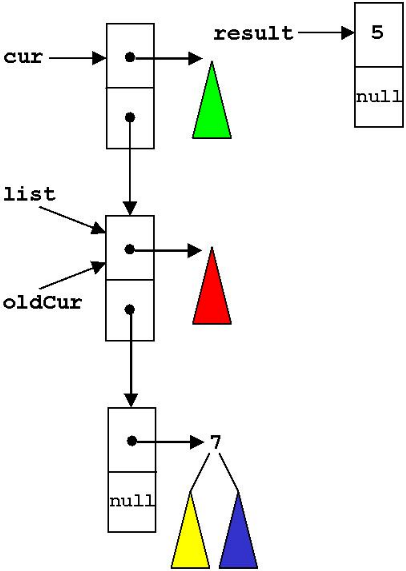
Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
         flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
        IntList oldIntList = result;
        result = new IntList();
        result.value = tree.value;
        result.next = oldIntList;
        TreeList oldCur = cur;
        cur = new TreeList();
        cur.value = tree.left;
        cur.next = oldCur;
        oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```



Example with User-Defined Data Type

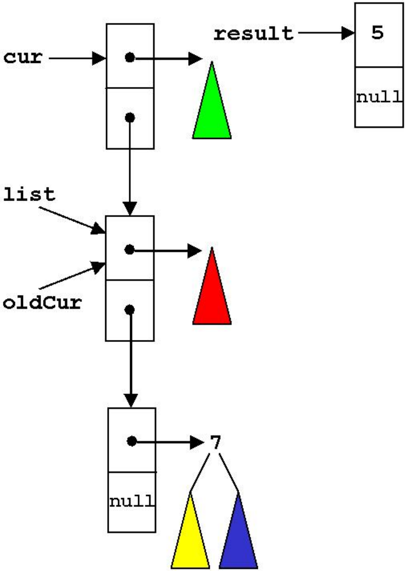
```
public class Flatten {
  public static IntList
         flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```



no termination by *path length*

Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
        flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```



cur and list can be *sharing*

Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
         flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```

General state at beginning of loop body

$\texttt{aload\_1} \mid \texttt{l}:o_1, \texttt{c}:o_2, \texttt{r}:o_3 \mid \varepsilon$
$o_1 = \texttt{TreeList}(?)$ $o_2 = \texttt{TreeList}(?)$
$o_3 = \texttt{IntList}(?)$
$o_1 =^? o_2$ $o_1 \searrow\!\swarrow o_2$

Annotations

- $o_1 =^? o_2$: o_1 and o_2 *may be equal*
- $o_1 \searrow\!\swarrow o_2$: o_1 and o_2 *may join*
  - $o \to o'$ iff object at address o has a field with value o'
  - $o_1 \searrow\!\swarrow o_2$: $o_1 \to^* \circ \leftarrow^+ o_2$ or $o_1 \to^+ \circ \leftarrow^* o_2$
- $o\,!$: *o does not have to be a tree*

Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
         flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```

A

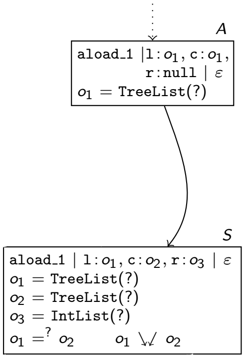
aload_1 | l : o_1, c : o_1, r : null | ε
o_1 = TreeList(?)

State *A*:

- reaches loop condition "`cur != null`" for the first time
- `list` and `cur` (o_1) are equal

Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
        flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```



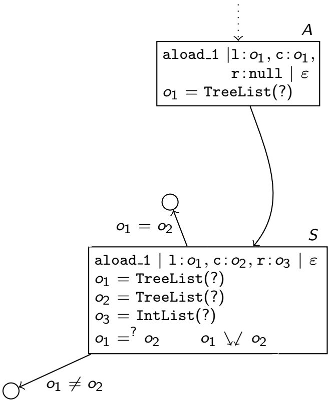
State S:

- *generalize* A to obtain finite termination graph
- `list` (o_1) and `cur` (o_2) may be equal and may join

Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
         flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```

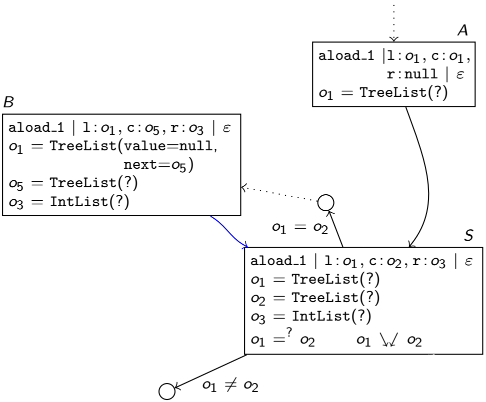
State S:



- *refinement* of annotation $o_1 =^? o_2$

Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
         flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```

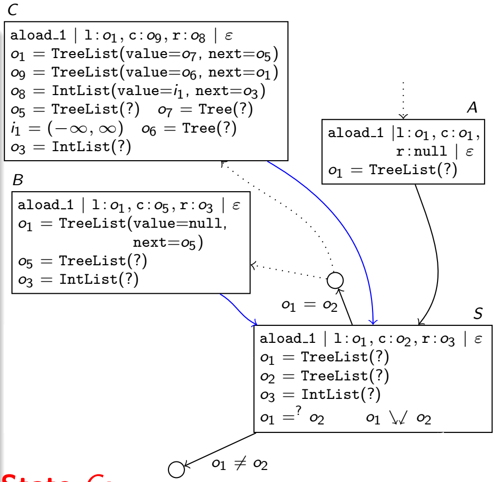


State B:

- reach loop condition if `tree == null`
- `list` $\rightarrow^{+} \circ \leftarrow^{*}$ `cur`
- B is *instance* of S

Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
        flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```

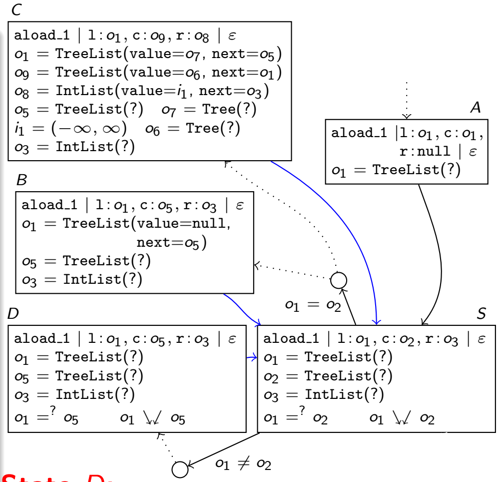


State C:

- $\texttt{Tree(value=}i_1\texttt{, left=}o_6\texttt{, right=}o_7\texttt{)}$
- $\texttt{list} \rightarrow^* \circ \leftarrow^+ \texttt{cur}$
- C is *instance* of S

Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
        flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
         IntList oldIntList = result;
         result = new IntList();
         result.value = tree.value;
         result.next = oldIntList;
         TreeList oldCur = cur;
         cur = new TreeList();
         cur.value = tree.left;
         cur.next = oldCur;
         oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```

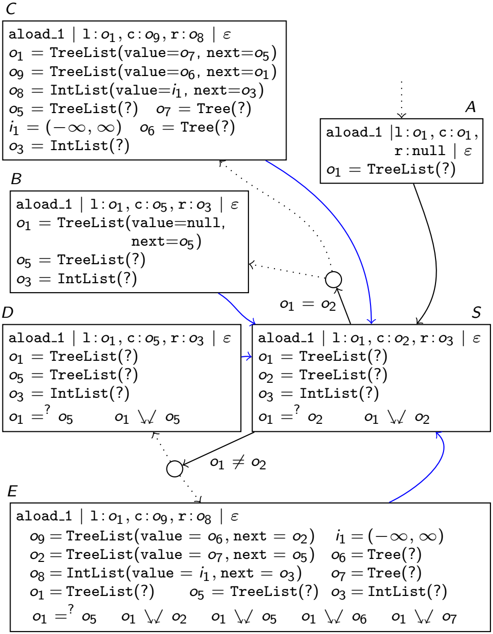


State *D*:

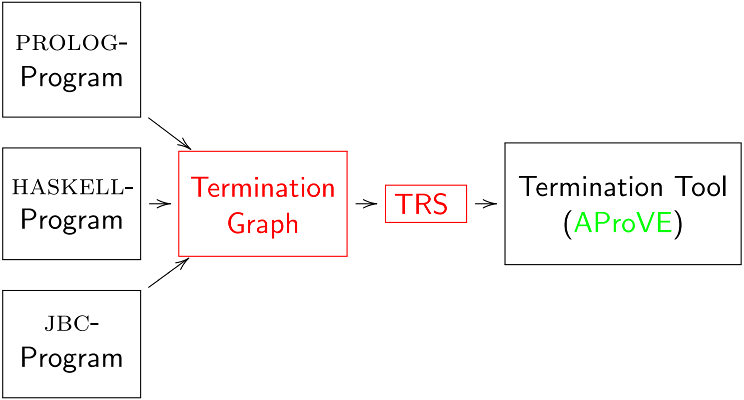
- o_2 = TreeList(value=o_4, next=o_5)
- tree (o_4) is null
- *D* is *instance* of *S*

Example with User-Defined Data Type

```
public class Flatten {
  public static IntList
        flatten(TreeList list) {
    TreeList cur = list;
    IntList result = null;

    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
        IntList oldIntList = result;
        result = new IntList();
        result.value = tree.value;
        result.next = oldIntList;
        TreeList oldCur = cur;
        cur = new TreeList();
        cur.value = tree.left;
        cur.next = oldCur;
        oldCur.value = tree.right;
      } else cur = cur.next;
    }
    return result;
  }
}
```



From Termination Graphs to TRSs



Transforming Objects to Terms

$\texttt{aload\_1} \mid \color{red}{\texttt{l}: o_1}, \texttt{c}: o_9, \texttt{r}: o_8 \mid \varepsilon$

$o_9 = \texttt{TreeList}(\texttt{value} = o_6, \texttt{next} = o_2) \qquad i_1 = (-\infty, \infty)$

$o_2 = \texttt{TreeList}(\texttt{value} = o_7, \texttt{next} = o_5) \qquad o_6 = \texttt{Tree}(?)$

$o_8 = \texttt{IntList}(\texttt{value} = i_1, \texttt{next} = o_3) \qquad o_7 = \texttt{Tree}(?)$

$\color{red}{o_1 = \texttt{TreeList}(?)} \qquad o_5 = \texttt{TreeList}(?) \qquad o_3 = \texttt{IntList}(?)$

$o_1 =^? o_5 \qquad o_1 \searrow\!\swarrow o_2 \qquad o_1 \searrow\!\swarrow o_5 \qquad o_1 \searrow\!\swarrow o_6 \qquad o_1 \searrow\!\swarrow o_7$

For every class C with n fields,
introduce function symbol C with n arguments

- term for o_1: o_1

Transforming Objects to Terms

$\texttt{aload\_1} \mid \texttt{l}: o_1, \texttt{c}: o_9, \texttt{r}: o_8 \mid \varepsilon$

$o_9 = \texttt{TreeList}(\texttt{value} = o_6, \texttt{next} = o_2)$ $\quad i_1 = (-\infty, \infty)$

$o_2 = \texttt{TreeList}(\texttt{value} = o_7, \texttt{next} = o_5)$ $\quad o_6 = \texttt{Tree}(?)$

$o_8 = \texttt{IntList}(\texttt{value} = i_1, \texttt{next} = o_3)$ $\quad o_7 = \texttt{Tree}(?)$

$o_1 = \texttt{TreeList}(?)$ $\quad o_5 = \texttt{TreeList}(?)$ $\quad o_3 = \texttt{IntList}(?)$

$o_1 =^? o_5 \quad o_1 \searrow\!\!\swarrow o_2 \quad o_1 \searrow\!\!\swarrow o_5 \quad o_1 \searrow\!\!\swarrow o_6 \quad o_1 \searrow\!\!\swarrow o_7$

For every class C with n fields,
introduce function symbol C with n arguments

- term for o_1: o_1
- term for o_2: $\mathsf{TL}(o_7, o_5)$

Transforming Objects to Terms

$$
\begin{array}{l}
\texttt{aload\_1} \mid \texttt{l}: o_1, \texttt{c}: o_9, \texttt{r}: o_8 \mid \varepsilon \\
o_9 = \texttt{TreeList}(\texttt{value} = o_6, \texttt{next} = o_2) \quad i_1 = (-\infty, \infty) \\
o_2 = \texttt{TreeList}(\texttt{value} = o_7, \texttt{next} = o_5) \quad o_6 = \texttt{Tree}(?) \\
o_8 = \texttt{IntList}(\texttt{value} = i_1, \texttt{next} = o_3) \quad o_7 = \texttt{Tree}(?) \\
o_1 = \texttt{TreeList}(?) \quad o_5 = \texttt{TreeList}(?) \quad o_3 = \texttt{IntList}(?) \\
o_1 =^? o_5 \quad o_1 \searrow\!\!\swarrow o_2 \quad o_1 \searrow\!\!\swarrow o_5 \quad o_1 \searrow\!\!\swarrow o_6 \quad o_1 \searrow\!\!\swarrow o_7
\end{array}
$$

For every class C with n fields,
introduce function symbol C with n arguments

- term for o_1: o_1
- term for o_2: $\mathsf{TL}(o_7, o_5)$
- term for o_9: $\mathsf{TL}(o_6, \mathsf{TL}(o_7, o_5))$

Transforming Objects to Terms

$\texttt{aload\_1} \mid \texttt{l}: o_1, \texttt{c}: o_9, \texttt{r}: o_8 \mid \varepsilon$

$o_9 = \texttt{TreeList}(\texttt{value} = o_6, \texttt{next} = o_2)$ $\quad i_1 = (-\infty, \infty)$

$o_2 = \texttt{TreeList}(\texttt{value} = o_7, \texttt{next} = o_5)$ $\quad o_6 = \texttt{Tree}(?)$

$o_8 = \texttt{IntList}(\texttt{value} = i_1, \texttt{next} = o_3)$ $\quad o_7 = \texttt{Tree}(?)$

$o_1 = \texttt{TreeList}(?)$ $\quad o_5 = \texttt{TreeList}(?)$ $\quad o_3 = \texttt{IntList}(?)$

$o_1 =^? o_5 \quad o_1 \searrow\!\!\swarrow o_2 \quad o_1 \searrow\!\!\swarrow o_5 \quad o_1 \searrow\!\!\swarrow o_6 \quad o_1 \searrow\!\!\swarrow o_7$

For every class `C` with n fields,
introduce function symbol C with n arguments

- term for o_1: o_1
- term for o_2: $\mathsf{TL}(o_7, o_5)$
- term for o_9: $\mathsf{TL}(o_6, \mathsf{TL}(o_7, o_5))$
- term for o_8: $\mathsf{IL}(i_1, o_3)$

Transforming Objects to Terms

Class Hierarchy

- for every class `C` with n fields, introduce function symbol C with $n+1$ arguments
- first argument: part of the object corresponding to subclasses of `C`

```
public class A {
  int a;
}
```

```
public class B extends A {
  int b;
}
```

```
A x = new A();
x.a = 1;
```

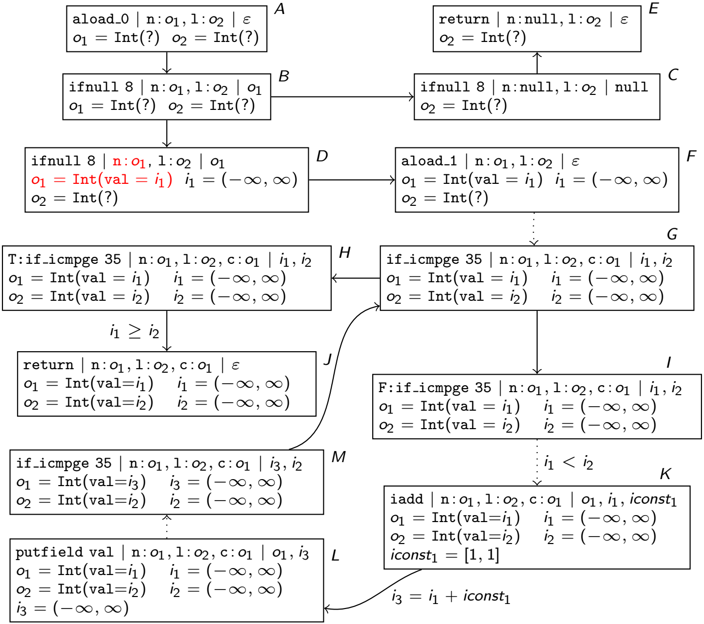
```
B y = new B();
y.a = 2;
y.b = 3;
```

- term for `x`: $\mathsf{jlO}(\mathsf{A}(\mathsf{eoc}, 1))$ (eoc for "end of class")
- term for `y`: $\mathsf{jlO}(\mathsf{A}(\mathsf{B}(\mathsf{eoc}, 3), 2))$ (jlO for "`java.lang.Object`")

Transforming States to Tuples of Terms

Transforming D

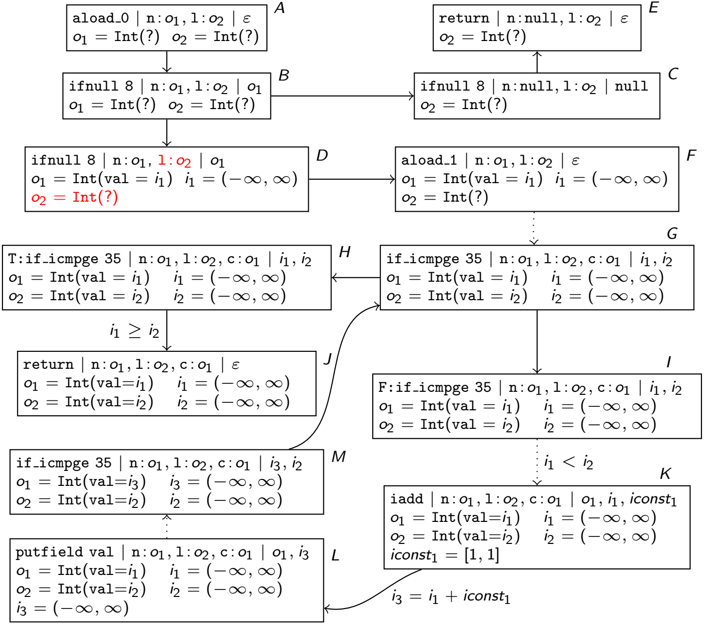
jlO(Int(eoc, i_1))



Transforming States to Tuples of Terms

Transforming *D*

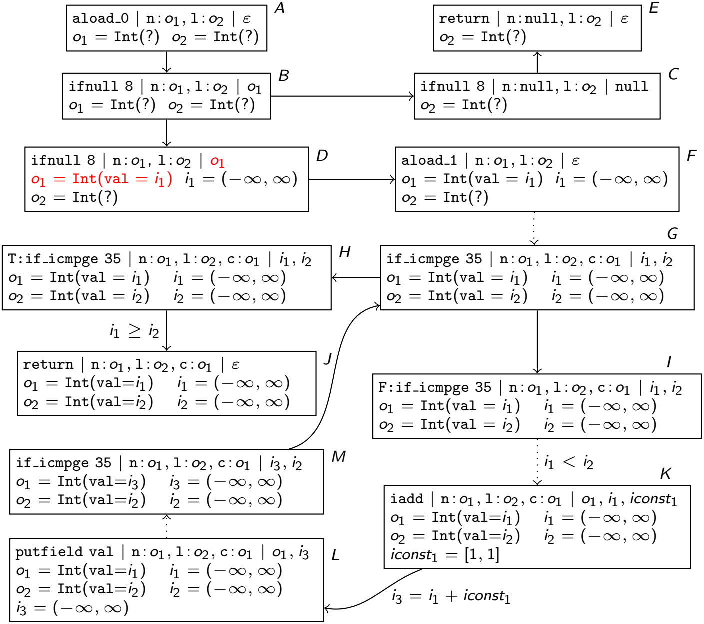
jlO(Int(eoc, i_1)),
o_2



Transforming States to Tuples of Terms

Transforming D

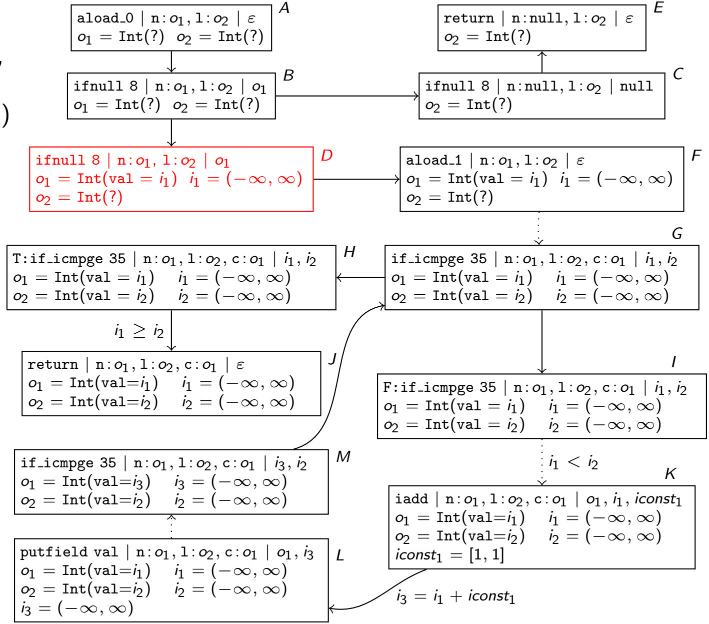
$\mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_1))$,
o_2,
$\mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_1))$



Transforming States to Tuples of Terms

Transforming D

$f_D($ jlO(Int(eoc, i_1)),
    o_2,
    jlO(Int(eoc, i_1)) $)$



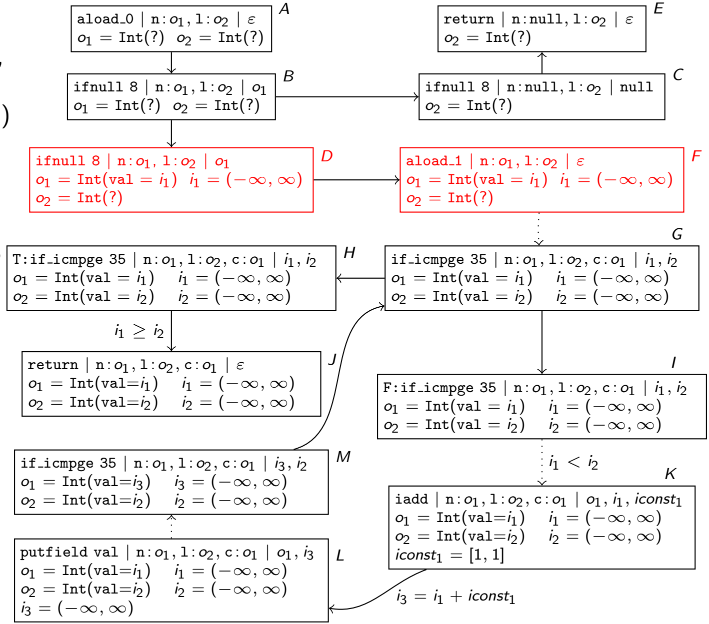
Transforming States to Tuples of Terms

Transforming D

$$f_D(\,\text{jlO}(\text{Int}(\text{eoc}, i_1)),\; o_2,\; \text{jlO}(\text{Int}(\text{eoc}, i_1))\,)$$

Transforming F

$$f_F(\,\text{jlO}(\text{Int}(\text{eoc}, i_1)),\; o_2\,)$$

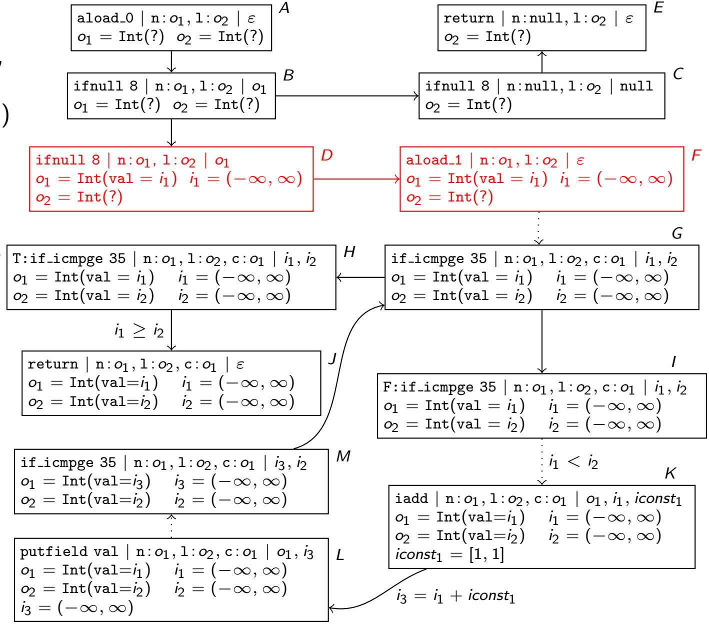


Transforming Edges to Rewrite Rules

$$f_D(\,\mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1)),\ o_2,\ \mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1))\,)$$

$$\rightarrow$$

$$f_F(\,\mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1)),\ o_2\,)$$



Transforming Edges to Rewrite Rules

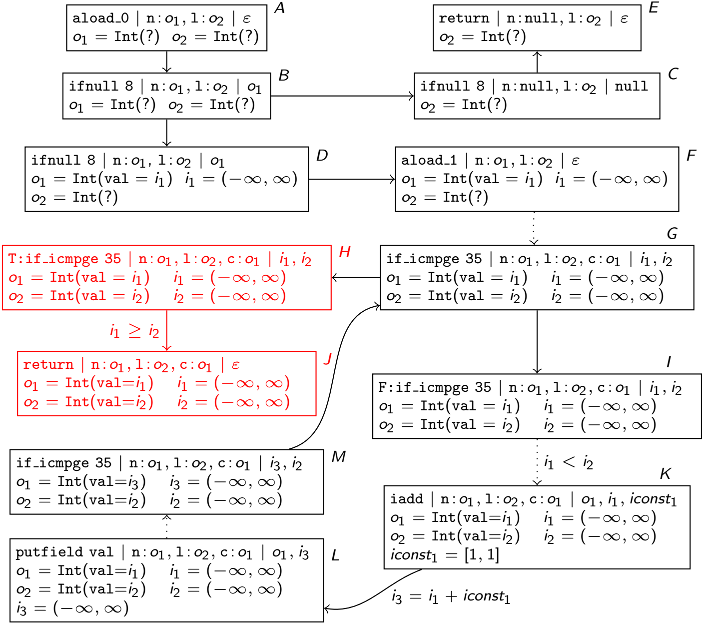
Transforming Evaluation Edges with Conditions

$$f_H(\,\mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1)),\ \mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_2)),\ \mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1)),\ i_1,\ i_2\,)$$

$$\rightarrow$$

$$f_J(\,\mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1)),\ \mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_2)),\ \mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1))\,)$$

$$\mid\ i_1 \geq i_2$$



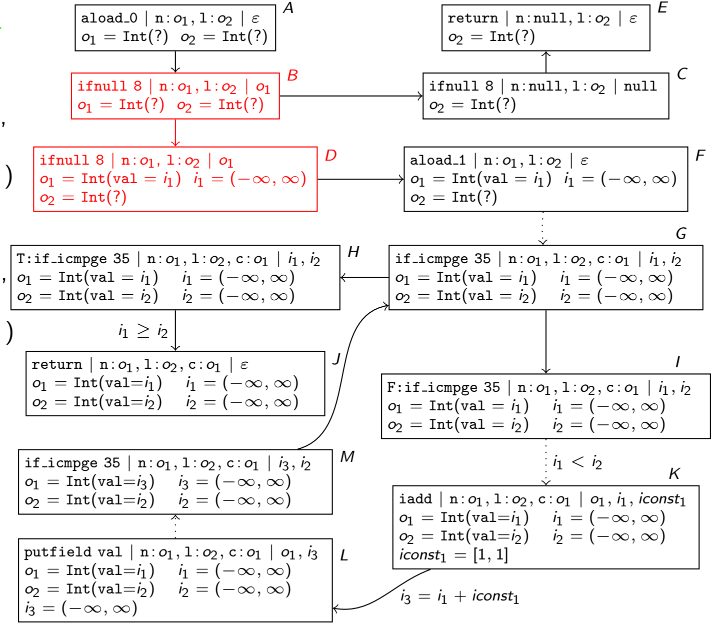
Transforming Edges to Rewrite Rules

Transforming Refinement Edges

$$f_B(\,\mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1)),\; o_2,\; \mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1))\,)$$

$$\longrightarrow$$

$$f_D(\,\mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1)),\; o_2,\; \mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1))\,)$$



Transforming Edges to Rewrite Rules

Merging Rewrite Rules

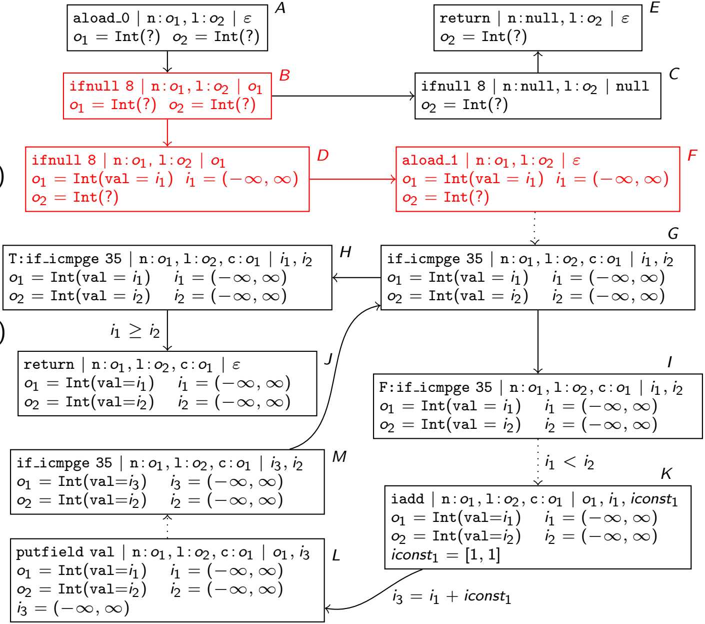
$$f_B(\,\mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1)),\; o_2,\; \mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1))\,)$$

$$\rightarrow$$

$$f_D(\,\mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1)),\; o_2,\; \mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1))\,)$$

$$\rightarrow$$

$$f_F(\,\mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1)),\; o_2\,)$$



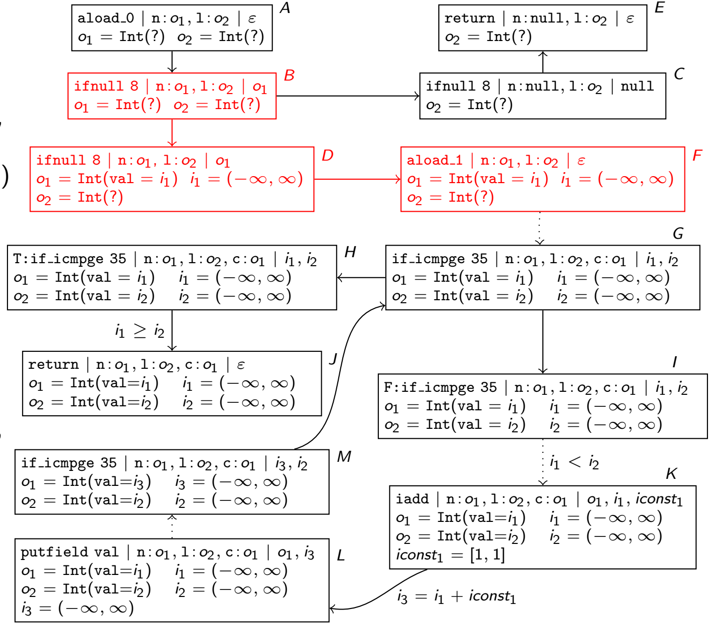
Transforming Edges to Rewrite Rules

Merging Rewrite Rules

$f_B(\,\mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1)),$
$\quad o_2,$
$\quad \mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1))\,)$

$\rightarrow$

$f_F(\,\mathrm{jlO}(\mathrm{Int}(\mathrm{eoc}, i_1)),$
$\quad o_2\,)$



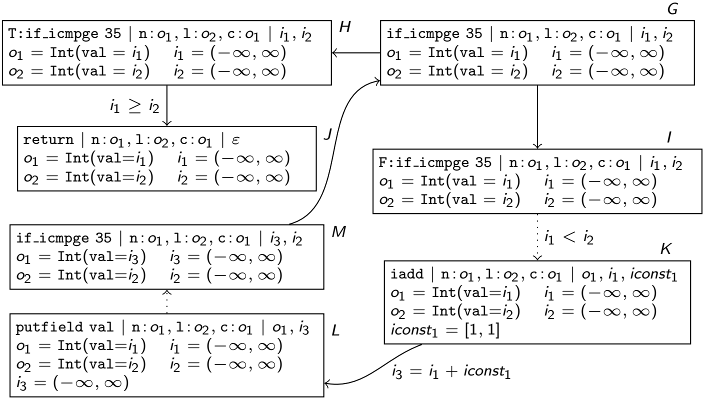
Transforming Edges to Rewrite Rules

TRS for count

$$
\begin{array}{l}
\mathsf{f}_G(\,\mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_1)),\ \mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_2)),\ \mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_1)),\ i_1,\ i_2) \ \to \\
\mathsf{f}_G(\,\mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_1+1)),\ \mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_2)),\ \mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_1+1)),\ i_1+1,\ i_2) \ \mid \ i_1 < i_2
\end{array}
$$

TRS is "*natural*"

termination easy to prove automatically



From Termination Graphs to TRSs

TRS for count

$$\begin{array}{lllllll} \mathsf{f}_G(\,\mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_1)), & \mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_2)), & \mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_1)), & i_1, & i_2) & \rightarrow & \\ \mathsf{f}_G(\,\mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_1+1)), & \mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_2)), & \mathsf{jlO}(\mathsf{Int}(\mathsf{eoc}, i_1+1)), & i_1+1, & i_2) & \mid & i_1 < i_2 \end{array}$$

- every JBC-computation of concrete states corresponds to a *computation path* in the termination graph
- termination graph is called *terminating* iff it has no infinite computation path
- every computation path corresponds to rewrite sequence in TRS

Theorem

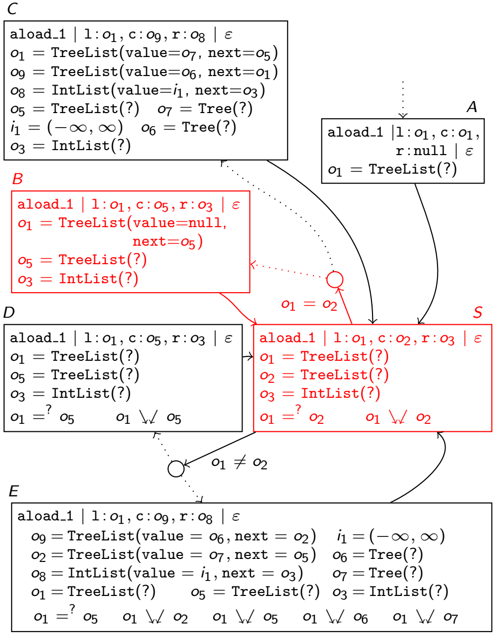
TRS corresponding to termination graph is terminating $\Rightarrow$

termination graph is terminating $\Rightarrow$

JBC-program terminating for all states represented in termination graph

From Termination Graphs to TRSs

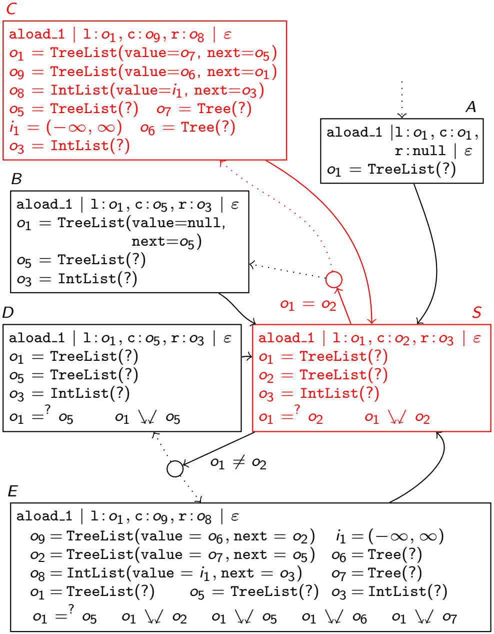
$$f_S(\mathsf{TL}(\mathsf{null}, o_5), \mathsf{TL}(\mathsf{null}, o_5), o_3) \to$$
$$f_S(\mathsf{TL}(\mathsf{null}, o_5), \quad o_5 \ , o_3)$$



From Termination Graphs to TRSs

$$f_S(\mathsf{TL}(\mathsf{null}, o_5), \mathsf{TL}(\mathsf{null}, o_5), o_3) \to$$
$$f_S(\mathsf{TL}(\mathsf{null}, o_5), \quad o_5 \ , o_3)$$

$$f_S(\ldots, \mathsf{TL}(\mathsf{T}(i_1, o_6, o_7), o_5), \quad o_3 \ ) \to$$
$$f_S(\ldots, \mathsf{TL}(\ o_6, \mathsf{TL}(o_7, o_5)), \mathsf{IL}(i_1, o_3))$$

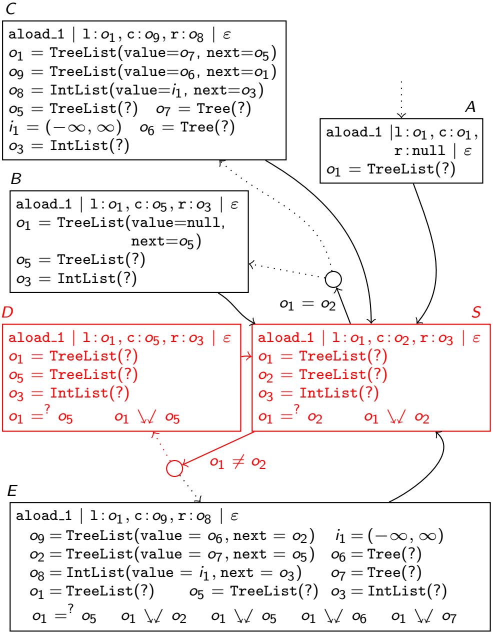


From Termination Graphs to TRSs

$$f_S(\mathsf{TL}(\mathsf{null}, o_5), \mathsf{TL}(\mathsf{null}, o_5), o_3) \to$$
$$f_S(\mathsf{TL}(\mathsf{null}, o_5), \quad o_5 \;, o_3)$$

$$f_S(\ldots, \mathsf{TL}(\mathsf{T}(i_1, o_6, o_7), o_5), \quad o_3 \;) \to$$
$$f_S(\ldots, \mathsf{TL}(\; o_6, \mathsf{TL}(o_7, o_5)), \mathsf{IL}(i_1, o_3))$$

$$f_S(o_1, \mathsf{TL}(\mathsf{null}, o_5), o_3) \to$$
$$f_S(o_1, \quad o_5 \;, o_3)$$



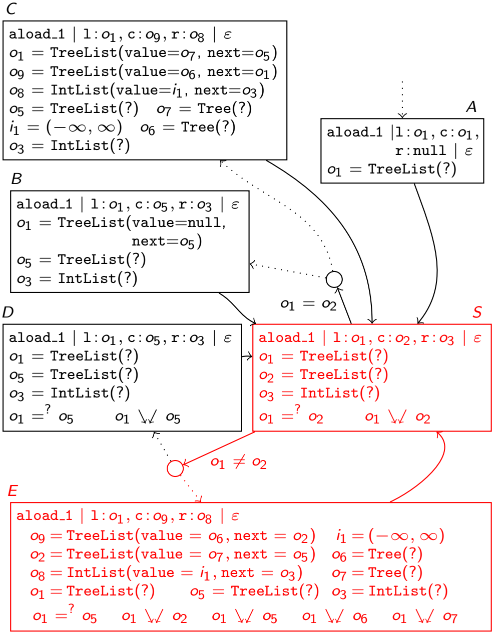
From Termination Graphs to TRSs

$$f_S(\mathsf{TL}(\mathsf{null}, o_5), \mathsf{TL}(\mathsf{null}, o_5), o_3) \to f_S(\mathsf{TL}(\mathsf{null}, o_5), o_5, o_3)$$

$$f_S(\ldots, \mathsf{TL}(\mathsf{T}(i_1, o_6, o_7), o_5), o_3) \to f_S(\ldots, \mathsf{TL}(o_6, \mathsf{TL}(o_7, o_5)), \mathsf{IL}(i_1, o_3))$$

$$f_S(o_1, \mathsf{TL}(\mathsf{null}, o_5), o_3) \to f_S(o_1, o_5, o_3)$$

$$f_S(o_1, \mathsf{TL}(\mathsf{T}(i_1, o_6, o_7), o_5), o_3) \to f_S(o_1', \mathsf{TL}(o_6, \mathsf{TL}(o_7, o_5)), \mathsf{IL}(i_1, o_3))$$



From Termination Graphs to TRSs

$$f_S(\mathsf{TL}(\mathsf{null}, o_5), \mathsf{TL}(\mathsf{null}, o_5), o_3) \to$$
$$f_S(\mathsf{TL}(\mathsf{null}, o_5), \quad o_5 \quad, o_3)$$

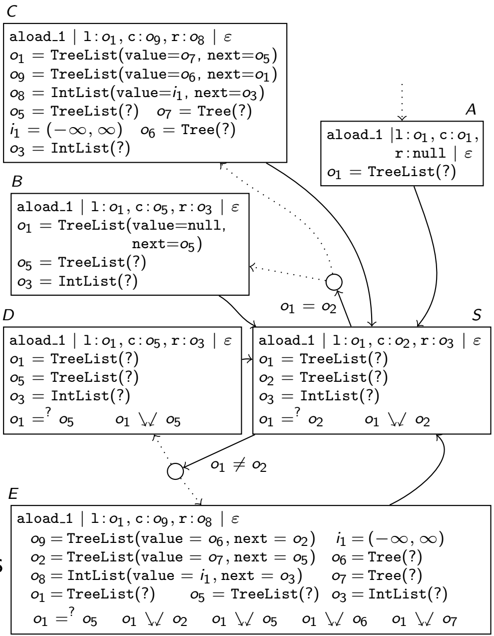
$$f_S(\ldots, \mathsf{TL}(\mathsf{T}(i_1, o_6, o_7), o_5), \quad o_3 \quad) \to$$
$$f_S(\ldots, \mathsf{TL}(\quad o_6, \mathsf{TL}(o_7, o_5)), \mathsf{IL}(i_1, o_3))$$

$$f_S(o_1, \mathsf{TL}(\mathsf{null}, o_5), o_3) \to$$
$$f_S(o_1, \quad o_5 \quad, o_3)$$

$$f_S(o_1, \mathsf{TL}(\mathsf{T}(i_1, o_6, o_7), o_5), \quad o_3 \quad) \to$$
$$f_S(o_1', \mathsf{TL}(\quad o_6, \mathsf{TL}(o_7, o_5)), \mathsf{IL}(i_1, o_3))$$

Rewrite Rules & Annotations

- when writing to a field of o_2 with $o_1 \searrow\!\!\swarrow o_2$: o_1 on lhs, fresh variable o_1' on rhs
- cyclic objects: fresh variable on rhs



From Termination Graphs to TRSs

$f_S(\mathsf{TL}(\mathsf{null}, o_5), \mathsf{TL}(\mathsf{null}, o_5), o_3) \to$

$f_S(\mathsf{TL}(\mathsf{null}, o_5), \quad o_5 \;, o_3)$

$f_S(\ldots, \mathsf{TL}(\mathsf{T}(i_1, o_6, o_7), o_5), \quad o_3 \;) \to$

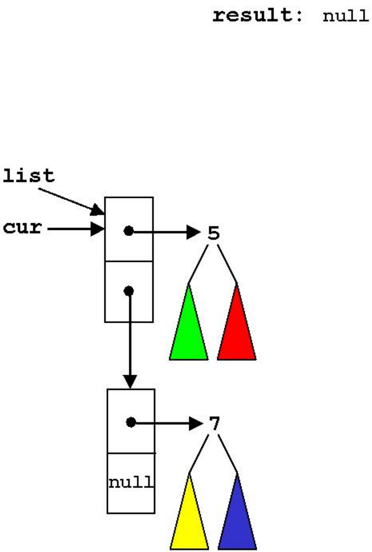
$f_S(\ldots, \mathsf{TL}(\; o_6, \mathsf{TL}(o_7, o_5)), \mathsf{IL}(i_1, o_3))$

$f_S(o_1, \mathsf{TL}(\mathsf{null}, o_5), o_3) \to$

$f_S(o_1, \quad o_5 \;, o_3)$

$f_S(o_1, \mathsf{TL}(\,\mathsf{T}(5, o_6, o_7), o_5), \quad \mathsf{null} \;) \to$
$f_S(o_1', \mathsf{TL}(\; o_6, \mathsf{TL}(o_7, o_5)), \mathsf{IL}(5, \mathsf{null}))$

TRS is “*natural*”



From Termination Graphs to TRSs

$f_S(\mathsf{TL}(\mathsf{null}, o_5), \mathsf{TL}(\mathsf{null}, o_5), o_3) \rightarrow$

$f_S(\mathsf{TL}(\mathsf{null}, o_5), \quad o_5 \ , o_3)$

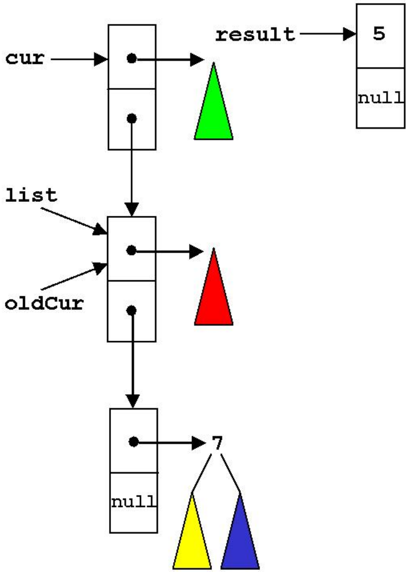
$f_S(\ldots, \mathsf{TL}(\mathsf{T}(i_1, o_6, o_7), o_5), \quad o_3 \ ) \rightarrow$

$f_S(\ldots, \mathsf{TL}(\ o_6, \mathsf{TL}(o_7, o_5)), \mathsf{IL}(i_1, o_3))$

$f_S(o_1, \mathsf{TL}(\mathsf{null}, o_5), o_3) \rightarrow$

$f_S(o_1, \quad o_5 \ , o_3)$

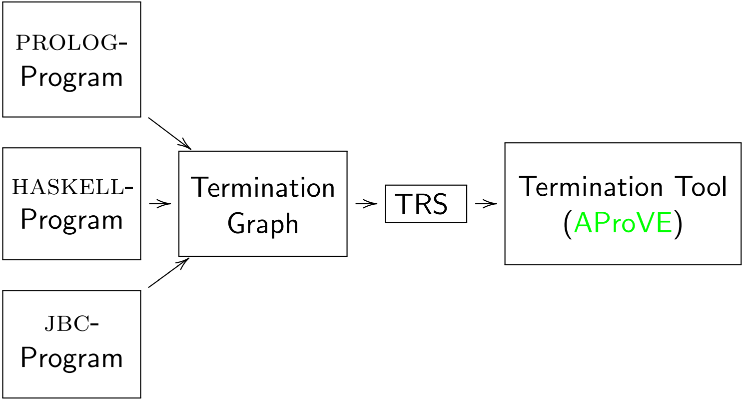
$f_S(o_1, \mathsf{TL}(\ \mathsf{T}(5, o_6, o_7), o_5), \quad \mathsf{null} \ ) \rightarrow$
$f_S(o_1', \mathsf{TL}(\ o_6, \mathsf{TL}(o_7, o_5)), \mathsf{IL}(5, \mathsf{null}))$



TRS is "*natural*"

termination easy
to prove automatically

Automated Termination Analysis of JAVA BYTECODE by Term Rewriting



Automated Termination Analysis of JAVA BYTECODE by Term Rewriting

- implemented in AProVE and evaluated on collection of 125 JBC-programs (*Termination Problem Data Base*)

	Success	Failure	Timeout	Runtime
AProVE	102	6	17	15.0
Julia	91	34	0	2.7
COSTA	72	52	1	4.5

- AProVE winner of the *International Termination Competition* for JBC, HASKELL, PROLOG, term rewriting

- http://aprove.informatik.rwth-aachen.de

- termination of "real" languages can be analyzed automatically, term rewriting is a suitable approach