Strategies for Semantical Contractions

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The *Logic of Theory Change* developed by Alchourrón, Gärdenfors and Makinson (AGM) is concerned with the revision of beliefs in the face of new and possibly contradicting information. This nonmonotonic process consists of a contraction and an expansion transforming one set of beliefs into another. Beliefs are represented by consistent deductively closed sets of formulas. To achieve minimal change AGM suggested widely accepted postulates that proper contractions have to fulfill.

In practical applications, e.g. knowledge representation, deductively closed sets of formulas have to be representable in a finite way. Therefore our main interest is in *finite contractions*, i.e. contractions that transform sets of formulas possessing a finite base into finitely representable sets again.

We have formulated a semantical characterization of finite contractions satisfying the AGM-postulates which provides an insight into the true nature of these operations and shows all possibilities to define concrete functions of this kind.

Semantically, a finite contraction of a set Φ by a formula φ means extending the models M of Φ by a set of models of $\neg\varphi$ that has to be uniquely determined by its restriction to a finite subsignature.

Examining the concrete contractions known from literature by this characterization we obtain that they are all defined according to the same semantical strategy: The original set of models M is extended by those models of $\neg\varphi$ that can be obtained by a *"small" change* of M. This strategy results in maintaining those formulas of Φ which belong to the (hopefully maximal) subsignature not affected by that change of M. But as the number of "important" formulas in Φ is not equal for different subsignatures of the same size we argue that this strategy leads to a contraintuitive behaviour[3].

Instead, the syntactical goal of keeping as many important formulas as possible in the contracted set corresponds to the following semantical strategy: M has to be extended by some models I of $\neg\varphi$ such that the *number of "big" changes* of M which result in I is *as large as possible*. Finite contractions defined this way meet the intuitive notion of minimal change.

[3] When restricting ourselves to clauses instead of formulas a clause is the more important the less literals it consists of. If Φ is the deductive closure of $\{a, b \vee c\}$ the subsignatures $\{a, c\}$ and $\{b, c\}$ have the same size, but the most important clause of Φ does not belong to the latter one.