products and ultraproducts; the notion of a (model theoretic) type does not appear. There is an extensive treatment of Turing machines and of recursive and recursively enumerable sets. The incompleteness theorem is not treated; nor is there any mention of Church's thesis. This book could serve well for a reading course with an extremely strong undergraduate or beginning graduate student. In the United States, its best use may be as a study guide for students preparing for a comprehensive exam in logic.

JOHN T. BALDWIN

Department of Mathematics, Statistics and Computer Science, University of Illinois at Chicago. jbaldwin@uic.edu.

TERESE. ***Term rewriting systems,*** edited by Marc Bezem, Jan Willem Klop, and Roel de Vrijer, Cambridge Tracts in Theoretical Computer Science, vol. 55. Cambridge University Press, 2003, xxii + 884 pp.

Term rewriting is a fundamental concept which historically emerged out of the lambda calculus and combinatory logic. It forms the basis of functional programming languages and has applications in many areas of computer science and mathematics (e.g., algebraic specification, implementation of programming languages, automated theorem proving, recursion theory, computer algebra, etc.).

This book is a collaboration of leading members of the Dutch research community on term rewriting. This community met regularly at a "Term rewriting seminar" at the Free University of Amsterdam for 13 years, which gave rise to the pseudonym "Terese" as the author of the present book. Several researchers from this group contributed separate chapters on different aspects of term rewriting. Some chapters have the nature of a textbook and cover basic material, while others focus on very recent and highly non-trivial research results of their authors. Thus, the level of difficulty and the intended audience varies not only from chapter to chapter, but sometimes also from section to section. Hence, the book is both suitable as a textbook on term rewriting (if one only concentrates on the parts on basic material) and as a valuable reference for the specialist. Each chapter also contains a set of exercises (whose difficulty varies greatly).

The topics covered in this book are the basic subfields of term rewriting (termination, confluence, completion, etc.), several of the main current research topics in rewriting (higher-order rewriting, strategies, etc.), and also several special subfields reflecting the particular research interests of their authors (equivalence of reductions, infinitary rewriting, etc.). Of course, there are also many important topics in rewriting that are not covered or only briefly discussed (e.g., rewriting modulo equivalences, implicit induction, narrowing, Gröbner bases, explicit strategies, etc.). But this is not a drawback, since in this way, most chapters are written by specialists on the respective topic. So naturally, the selection of topics in the book mainly coincides with the research fields of its authors.

Most parts of Chapters 0–7 are an introduction to first-order term rewriting. Starting with several motivating examples, Chapter 1 introduces abstract reduction systems with their main notions (in particular, confluence and termination) and Chapter 2 focuses on the basics of term rewriting which are needed for the rest of the book. These chapters are pleasant to read and serve as a very good introduction into the topic.

Chapter 3 contains several examples of interesting term rewriting systems (such as the system of combinatory logic). In addition, two subsections briefly discuss specializations and extensions of term rewriting (such as string rewriting and conditional rewriting systems).

In Chapter 4, the important class of orthogonal term rewriting systems is introduced and it is proved that orthogonality implies confluence. In addition to the classical proof, several alternative proofs are presented, which introduce advanced proof techniques needed in later chapters. Thus, these parts clearly go beyond the material of a textbook. The chapter also presents first important reduction strategies.

Chapter 5 shows the main (un)decidability results for rewriting (e.g., undecidability of confluence and termination) and the main modularity results for disjoint unions (i.e., unions of rewriting systems over disjoint signatures). However, modularity results for other combinations where constructors or other symbols may be shared are not handled.

Chapter 6 deals with methods to prove termination of term rewriting. Classical techniques like polynomial orders, recursive path orders, and Knuth-Bendix orders are presented and different restricted notions of termination are related to each other. Moreover, powerful transformational methods for termination proofs like semantic labelling and dependency pairs are introduced, giving a good overview also on recent techniques for termination analysis.

The next chapter is on using term rewriting for deduction. It covers Knuth-Bendix completion, a short description on inductive proofs by consistency, and unification modulo equations. The chapter gives a nice overview on the classical results, but it is more in the style of a tutorial without focusing on recent research in this subfield.

Chapter 8 is a very extensive treatment (174 pages) of the concept of equivalent reductions. In contrast to most of the preceding text, this chapter mainly contains new material (which extends existing results beyond orthogonal rewriting systems). The chapter is clearly intended for specialists interested in this specific subfield. For those readers, this chapter will be a very valuable source covering, unifying, and extending the results on this topic in a very convincing way, being the most up-to-date text on this topic.

In Chapter 9, strategies to find normal forms are examined, focusing on the concept of needed reduction steps, i.e., steps that have to be performed in order to reach normal forms. Again, the chapter is on a quite advanced level, giving a very comprehensive and up-to-date treatment of this topic.

In contrast to these more research-oriented chapters, Chapter 10 is a nice tutorial on the lambda calculus which leads to Chapter 11 on higher-order term rewriting. It gives a very good overview on this topic with a focus on results on confluence and termination of higher-order systems.

While higher-order rewriting is one of the main current research areas in term rewriting, the next chapter focuses on a more specialized topic. It summarizes its authors' results on infinitary rewriting where the concept of rewriting is extended to infinite terms and infinite reduction sequences, possibly converging to limits.

Chapter 13 is a short introduction to term graph rewriting, which gives a good survey on this topic and its relation to term rewriting. More detailed treatments of term graph rewriting can be found in several other books that are exclusively devoted to this topic.

Finally, in Chapter 14, advanced specialized results on confluence of abstract reduction systems are presented and there is a short subsection on the important concept of rewriting modulo equivalences.

Chapter 15 is an overview on implementations of rewriting (in programming languages, specification languages, program transformation systems, theorem provers, etc.) which is very helpful to estimate the use and impact of term rewriting. The book concludes with an appendix on the mathematical background required.

Concerning the overall structure of the book, the order of presentation is sometimes confusing, in particular for topics which are only briefly presented, but not treated in detail. For instance, the subsections on specializations and extensions of term rewriting are in the middle of Chapter 3 on examples, instead of Chapter 2 where first-order term rewriting is introduced. As another example, rewriting modulo equivalences is introduced in Chapter 14 on advanced abstract reduction systems (although the most prominent variant of it, AC-rewriting, cannot be demonstrated in this abstract setting). In fact, this concept is already used several times before (e.g., in Chapter 6 on termination). However, the editors and authors have added many references in the text indicating in which sections concepts have been or will be introduced. Since most readers will rather concentrate on certain chapters of the book instead of reading it from the beginning to the end, such references are very helpful and even forward references may not be problematic.

The are two other recent English books on term rewriting: a textbook (F. Baader and T. Nipkow, ***Term rewriting and all that***, Cambridge University Press, 1998) and a book covering more advanced material (E. Ohlebusch, ***Advanced topics in term rewriting***, Springer, New York, 2002). Compared to them, the present book is unique in its mixture of textbook-like and research-oriented parts and by the fact that its chapters have been written by different specialists. Thus, the current book may be less suitable as a textbook than the one of Baader and Nipkow, since it goes far beyond the basics and it may not always be clear to the reader which sections of the book are the "basic" ones. Here, it would have been helpful if the editors and authors would have indicated which parts are suitable as a textbook and which parts are mainly intended for the specialized researcher. Moreover, since the chapters are written by different authors, it is unavoidable that the style of presentation is somewhat heterogeneous. On the other hand, the present book covers much more material than the one of Baader and Nipkow and treats the topics in much more depth. Hence, by selecting suitable parts of it, the book indeed serves very well both as a textbook and as an advanced text. Some of the advanced topics are similar to the ones treated in Ohlebusch's book, but since both Ohlebusch and the authors of the present book mainly report on their own research, the contents of these two books hardly overlap, but rather complement each other. To summarize, I recommend the book both as a textbook with several very nice surveys and as a comprehensive source of advanced research results which are very well presented in a unified setting.

JÜRGEN GIESL

LuFG Informatik II, RWTH Aachen, Ahornstr. 55, 52074 Aachen, Germany.
giesl@informatik.rwth-aachen.de.

ALEXANDER LUBOTZKY and DAN SEGAL. ***Subgroup growth.*** Progress in Mathematics, vol. 212. Birkhäuser Verlag, Basel, 2003, xxvii + 453 pp.

The classification of infinite groups is a distant and perhaps unattainable goal. Finite simple groups have already shown us how immense such a project is likely to be. But this does not mean we cannot make inroads into understanding and sorting these groups. Given that the mathematician is essentially a finite beast at heart, the best way to get to grips with the expanse of infinite groups is to impose finiteness conditions on the group in the hope that they will become more tractable.

This was the approach taken by the Russian school of group theory at the end of the last century. For example what do finitely generated groups look like whose elements all have bounded order, the so-called Burnside Problem? The Tarski monsters created by Olshanskii and Rips in the 1970s showed that such finiteness conditions still leave room for strange creatures. To exclude such monsters, group theorists in the 1980s realized that insisting that the group be residually finite excluded such monsters and made understanding finiteness conditions on infinite groups more reasonable. For example, Zelmanov was awarded his Fields medal for the proof of the restricted Burnside Problem: every finitely generated, residually finite group of finite exponent is in fact finite.

The condition that a group is residually finite means that the intersection of all the normal subgroups of finite index is 1. This can be interpreted as saying that the group has sufficiently many finite index subgroups that one can distinguish between elements in finite quotients of the group. The central question of Lubotzky and Segal's book is how many subgroups of finite index can a residually finite group have. To quantify this question, group theorists began to explore the relationship between the structure of an infinite group and the rate of growth of the number of subgroups of each particular index in the group.