\{L_1, \ldots, L_n\} \text{ is unifiable} \text{ iff there is a } \sigma \text{ with } \sigma(L_1) = \ldots = \sigma(L_n).
\sigma \text{ is mgu iff for every unifier } \sigma' \text{ there is a substitution } \delta \text{ with } \sigma' = \delta \circ \sigma.

**Unification Algorithm**

1. Let \( \sigma = \emptyset \) be the “identical” substitution.

2. If \( |\sigma(K)| = 1 \), then stop and return \( \sigma \).

3. Otherwise, check all \( \sigma(L_i) \) in parallel from left to right, until there are different symbols in two literals.

4. If none of these symbols is a variable, then stop with clash failure.

5. Otherwise, let \( X \) be the variable and \( t \) be the subterm in the other literal. If \( X \) occurs in \( t \), then stop with occur failure.

6. Otherwise, let \( \sigma = \{X/t\} \circ \sigma \) und go back to step 2.
Resolution for Predicate Logic

$R$ is a *resolvent* of $K_1$ and $K_2$ iff

- $\nu_1(K_1)$ and $\nu_2(K_2)$ are variable-disjoint
- $L_1, \ldots, L_m \in \nu_1(K_1)$, $L'_1, \ldots, L'_n \in \nu_2(K_2)$ with $n, m \geq 1$ and $\{\overline{L_1}, \ldots, \overline{L_m}, L'_1, \ldots, L'_n\}$ has mgu $\sigma$
- $R = \sigma((\nu_1(K_1) \setminus \{L_1, \ldots, L_m\}) \cup (\nu_2(K_2) \setminus \{L'_1, \ldots, L'_n\}))$

Example

\[
\{\overline{p(f(X))}, \overline{\neg q(Z)}, \overline{p(Z)}\} \quad \{\overline{\neg p(X)}, \overline{r(g(X))}\}
\]

$\nu_1 = \emptyset$

$\nu_2 = \{X/U, U/X\}$

$\sigma = \{Z/f(X), U/f(X)\}$