Exercise 1 (Simple Prolog): 

(2 + 1.5 + 1.5 = 5 points)

Consider the following Prolog program, where `indir(DIR, A)` means that A is directly contained in the directory DIR.

```prolog
indir(home,peter).
indir(home,rene).
indir(home,userlist).

indir(peter,cv).
indir(peter,tetris).
indir(peter,photo).

indir(rene,cv).
indir(rene,mahjongg).
indir(rene,dissertation).

samedir(X1, X2) :- indir(DIR, X1), indir(DIR, X2).
```

a) Implement a predicate `both(DIR, A, B)` in Prolog which is true if both A and B are directly contained in the directory DIR, i.e., `indir(DIR, A)` is true and `indir(DIR, B)` is true.

d) Implement a predicate `contains(DIR, X)` in Prolog which is true if X is directly contained in the directory DIR or X is contained in any subdirectory, subsubdirectory, ... of DIR. In other words, `contains(DIR, X)` is true if `indir(DIR, X)` is true or if there are N > 0 elements \(Y_1, ..., Y_N\) such that the following predicates are true:

- `indir(DIR, Y_1)`
- `indir(Y_i, X)`
- `indir(Y_i, Y_j)` for all \(i, j \in \{1, ..., N\}\) with \(j = i + 1\).

Make sure that the evaluation of all queries `?- contains(\ldots, \ldots)` terminates.

c) List all answers that Prolog gives for the following queries, in the order that Prolog gives them. Try to solve this part of the exercise without the help of a computer.

1. `?- indir(X, cv)`. 
2. \(-\text{samedir}(\text{tetris}, X)\).
3. \(-\text{both}(X, \text{cv}, \text{dissertation})\).

**Exercise 2 (Syntax):**

Consider the set of formulas \(\Phi = \{\)

\[
\begin{align*}
\text{part}(\text{menu1, medaillon}), \\
\text{part}(\text{menu1, sauce}), \\
\text{part}(\text{menu1, ravioli}), \\
\text{part}(\text{menu2, topping}), \\
\text{part}(\text{menu2, ravioli}), \\
\text{ingredient}(\text{sauce, shallot}), \\
\text{ingredient}(\text{sauce, redwine}), \\
\text{ingredient}(\text{ravioli, flour}), \\
\text{ingredient}(\text{ravioli, cream}), \\
\text{ingredient}(\text{ravioli, mushrooms}), \\
\text{ingredient}(\text{medaillon, roastsaddle}), \\
\text{ingredient}(\text{medaillon, truffle}), \\
\text{ingredient}(\text{topping, mozzarella}), \\
\text{ingredient}(\text{topping, onion}), \\
\text{lactoseingredient}(\text{cream}), \\
\text{lactoseingredient}(\text{mozzarella}),
\end{align*}
\]

\(\forall A, B \text{ contains}(A, B) \land \text{lactoseingredient}(B) \rightarrow \text{containslactose}(A), \)

\(\forall A, B, C \text{ part}(A, B) \land \text{ingredient}(B, C) \rightarrow \text{contains}(A, C)\}

\(\)

\(\Sigma = \Sigma_0 = \{\text{menu1, menu2, medaillon, sauce, ravioli, topping, shallot, redwine, flour, cream, mushrooms, roastsaddle, truffle, mozzarella, onion}\}, \)

\(\Delta_2 = \{\text{part, ingredient, contains}\}, \)

\(\Delta_1 = \{\text{lactoseingredient, containslactose}\}, \)

\(\Delta = \Delta_1 \cup \Delta_2, \) and \(\mathcal{V} = \{A, B, C\}\).

a) Construct the corresponding Prolog program based on \(\Phi, \Sigma, \Delta\) and \(\mathcal{V}\), where the order of clauses corresponds to the order of formulas given above.

b) Give Prolog queries corresponding to the following questions:

- “Which ingredients are contained in both menus?”
- “Which ingredients with lactose are contained menu1?”

**Exercise 3 (Induction):**

Let \(t\) be an arbitrary term. Then the size \(|t|\) of \(t\) is defined as follows. \(|X| = 1\) if \(X\) is a variable. Otherwise we have for \(n \geq 0\) that \(|f(t_1, \ldots, t_n)| = 1 + \sum_{i=1}^{n} |t_i|\).

Show by structural induction that for every term \(t\) and every variable renaming \(\sigma\) we have \(|t| = |\sigma(t)|\).