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Notes:

- To solve the programming exercises you can use the Prolog interpreter **SWI-Prolog**, available for free at `http://www.swi-prolog.org`. For Debian and Ubuntu it suffices to install the `swi-prolog` package. You can use the command "`swipl`" to start it and use "`[exercise1].`" to load the facts from file **exercise1.pl** in the current directory.
- Please solve these exercises in **groups of three**!
- The solutions must be handed in **directly before (very latest: at the beginning of)** the exercise course on Wednesday, April 24th, 2013, in lecture hall **AH 2**. Alternatively you can drop your solutions into a box which is located right next to Prof. Giesl's office (this box is emptied **a few minutes before** the exercise course starts).
- Please write the **names** and **immatriculation numbers** of all (three) students on your solution. Also please staple the individual sheets!
- Please register at `https://aprove.informatik.rwth-aachen.de/lp13/` (https, not http!).

Exercise 1 (Simple Prolog): (2 + 1.5 + 1.5 = 5 points)

Consider the following Prolog program, where `indir(DIR, A)` means that `A` is directly contained in the directory `DIR`.

```
indir(home,peter).
indir(home,rene).
indir(home,userlist).

indir(peter,cv).
indir(peter,tetris).
indir(peter,photo).

indir(rene,cv).
indir(rene,mahjongg).
indir(rene,dissertation).

samedir(X1, X2) :- indir(DIR, X1), indir(DIR, X2).
```

a) Implement a predicate `both(DIR, A, B)` in Prolog which is true if both `A` and `B` are directly contained in the directory `DIR`, i.e., `indir(DIR, A)` is true and `indir(DIR, B)` is true.

b) Implement a predicate `contains(DIR, X)` in Prolog which is true if `X` is directly contained in the directory `DIR` or `X` is contained in any subdirectory, subsubdirectory, ... of `DIR`. In other words, `contains(DIR, X)` is true if `indir(DIR, X)` is true or if there are `N` > 0 elements `Y_1`,...,`Y_N` such that the following predicates are true:

- `indir(DIR,Y_1)`
- `indir(Y_N,X)`
- `indir(Y_I,Y_J)` for all $\mathtt{I}, \mathtt{J} \in \{1, \ldots, N\}$ with $\mathtt{J} = \mathtt{I} + 1$.

Make sure that the evaluation of all queries `?- contains(..., ...)` terminates.

c) List all answers that Prolog gives for the following queries, in the order that Prolog gives them. Try to solve this part of the exercise without the help of a computer.

1. `?- indir(X, cv).`

2. `?- samedir(tetris, X).`
3. `?- both(X, cv, dissertation).`

Exercise 2 (Syntax): (2 + 1 = 3 points)

Consider the set of formulas $\Phi = \{$

$$
\begin{aligned}
&\text{part}(\text{menu1, medaillon}),\\
&\text{part}(\text{menu1, sauce}),\\
&\text{part}(\text{menu1, ravioli}),\\
&\text{part}(\text{menu2, topping}),\\
&\text{part}(\text{menu2, ravioli}),\\
&\text{ingredient}(\text{sauce, shallot}),\\
&\text{ingredient}(\text{sauce, redwine}),\\
&\text{ingredient}(\text{ravioli, flour}),\\
&\text{ingredient}(\text{ravioli, cream}),\\
&\text{ingredient}(\text{ravioli, mushrooms}),\\
&\text{ingredient}(\text{medaillon, roastsaddle}),\\
&\text{ingredient}(\text{medaillon, truffle}),\\
&\text{ingredient}(\text{topping, mozzarella}),\\
&\text{ingredient}(\text{topping, onion}),\\
&\text{lactoseingredient}(\text{cream}),\\
&\text{lactoseingredient}(\text{mozzarella}),\\
\forall A, B \quad &\text{contains}(A,\ B) \wedge \text{lactoseingredient}(B) \rightarrow \text{containslactose}(A),\\
\forall A, B, C \quad &\text{part}(A,\ B) \wedge \text{ingredient}(B,\ C) \rightarrow \text{contains}(A,\ C)
\end{aligned}
$$

$\}$ over $\Sigma = \Sigma_0 = \{$menu1, menu2, medaillon, sauce, ravioli, topping, shallot, redwine, flour, cream, mushrooms, roastsaddle, truffle, mozzarella, onion$\}$, $\Delta_2 = \{$part, ingredient, contains$\}$, $\Delta_1 = \{$lactoseingredient, containslactose$\}$, $\Delta = \Delta_1 \cup \Delta_2$, and $\mathcal{V} = \{A, B, C\}$.

a) Construct the corresponding Prolog program based on Φ, Σ, Δ and $\mathcal{V}$, where the order of clauses corresponds to the order of formulas given above.

b) Give Prolog queries corresponding to the following questions:

- "Which ingredients are contained in both menus?"
- "Which ingredients with lactose are contained menu1?"

Exercise 3 (Induction): (4 points)

Let t be an arbitrary term. Then the size $|t|$ of t is defined as follows. $|X| = 1$ if X is a variable. Otherwise we have for $n \geq 0$ that $|f(t_1, \ldots, t_n)| = 1 + \Sigma_{i=1}^{n} |t_i|$.
Show by structural induction that for every term t and every variable renaming σ we have $|t| = |\sigma(t)|$.