Prof. Dr. Jürgen Giesl

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Notes:

- Solve these exercises in **groups of three**! For other group sizes **less points** are given!
- The solutions must be handed in **directly before (very latest: at the beginning of)** the exercise course on Wednesday, 15.05.2013, in lecture hall **AH 2**. Alternatively you can drop your solutions into a box which is located right next to Prof. Giesl's office (this box is emptied **a few minutes before** the exercise course starts).
- Please write the **names** and **immatriculation numbers** of all (three) students on your solution. Also please staple the individual sheets!

Exercise 1 (Conjunctive Normal Form): (3 points)

Consider the following formula φ with $\mathsf{p}_1, \dots, \mathsf{p}_6 \in \Delta_0$:

$$\varphi = \neg\left([(\mathsf{p}_1 \vee (\mathsf{p}_2 \to \mathsf{p}_3)) \wedge \neg(\mathsf{p}_4 \vee \mathsf{p}_5)] \vee \mathsf{p}_6 \right)$$

Use the algorithm presented in the proof of Theorem 3.3.2 to convert φ to an equivalent formula in *conjunctive normal form (CNF)*.

Exercise 2 (Multi-Resolution): (1.5+1.5=3 points)

In this exercise we consider an extension of resolution in propositional logic, which we call *multi-resolution*. Let K_1 and K_2 be clauses without variables. Then a clause R is a *multi-resolvent* of K_1 and K_2 iff for some $n > 0$ there are literals $L_1, \dots, L_n$ such that $K_1 = K_1' \uplus \{L_1, \dots, L_n\}$, $K_2 = K_2' \uplus \{\overline{L_1}, \dots, \overline{L_n}\}$, and $R = K_1' \cup K_2'$. Here, $\uplus$ denotes disjoint union. Thus, $K \uplus K'$ stands for the set $K \cup K'$ and it states that $K \cap K' = \emptyset$. The following diagram illustrates a multi-resolution step:

$$\begin{array}{ccc} K_1' \uplus \{L_1, \dots, L_n\} & & K_2' \uplus \{\overline{L_1}, \dots, \overline{L_n}\} \\ & K_1' \cup K_2' & \end{array}$$

Please prove or disprove the following statements:

a) Multi-resolution is *sound*, i.e., there is no satisfiable clause set $\mathcal{K}$ without variables from which one can derive $\Box$ by multi-resolution.

b) Multi-resolution is *complete*, i.e., from any unsatisfiable clause set $\mathcal{K}$ without variables one can derive $\Box$ by multi-resolution.

Exercise 3 (Resolution for propositional logic): (3 points)

Consider the following clause set $\mathcal{K}$ with $p_1, \dots, p_4 \in \Delta_0$:

$$\mathcal{K} = \{\{\mathsf{p}_1, \neg\mathsf{p}_2\}, \{\neg\mathsf{p}_4\}, \{\neg\mathsf{p}_3, \mathsf{p}_4\}, \{\neg\mathsf{p}_1, \mathsf{p}_4\}, \{\mathsf{p}_1, \mathsf{p}_2, \mathsf{p}_3\}\}$$

Please show that $\mathcal{K}$ is unsatisfiable by using resolution for propositional logic (cf. Definition 3.3.4 and Example 3.3.5).

Hint: It suffices to perform five resolution steps.

Exercise 4 (Unification): (1.5 + 1.5 + 1.5 + 1.5 = 6 points)

Consider the signature (Σ, Δ) with $\Sigma_0 = \{\mathsf{a}, \mathsf{b}\}, \Sigma_1 = \{\mathsf{h}\}, \Sigma_2 = \{\mathsf{f}, \mathsf{g}\}$, and $\Delta_3 = \{\mathsf{p}\}$. Use the algorithm from the lecture to decide whether the following clauses are unifiable. To document your application of the algorithm on some clause K, please write down the current substituted clause $\sigma(K)$ whenever the algorithm checks whether $|\sigma(K)| = 1$ and underline the position of the next symbols where the literals are not equal. Additionally, write down the resulting most general unifier (mgu) or the kind of failure (clash or occur) the algorithm returns. To illustrate this exercise, we give a short example for the clause $\{\mathsf{p}(X, Y, Z), \mathsf{p}(Z, \mathsf{a}, \mathsf{b})\}$:

1. $\{\mathsf{p}(\underline{X}, Y, Z), \mathsf{p}(\underline{Z}, \mathsf{a}, \mathsf{b})\}$
2. $\{\mathsf{p}(Z, \underline{Y}, Z), \mathsf{p}(Z, \underline{\mathsf{a}}, \mathsf{b})\}$
3. $\{\mathsf{p}(Z, \mathsf{a}, \underline{Z}), \mathsf{p}(Z, \mathsf{a}, \underline{\mathsf{b}})\}$
4. $\{\mathsf{p}(\mathsf{b}, \mathsf{a}, \mathsf{b})\}$
5. mgu: $\{X/\mathsf{b}, Y/\mathsf{a}, Z/\mathsf{b}\}$

a) $\{\mathsf{p}(X, \mathsf{h}(Z), \mathsf{f}(X, X)), \mathsf{p}(\mathsf{f}(Y, Y), Y, \mathsf{f}(Z, Z))\}$

b) $\{\mathsf{p}(\mathsf{h}(X), X, \mathsf{f}(X, Y)), \mathsf{p}(Y, \mathsf{h}(Z), \mathsf{f}(X, \mathsf{h}(\mathsf{h}(Z))))\}$

c) $\{\mathsf{p}(X, \mathsf{f}(\mathsf{h}(Z), X), \mathsf{a}), \mathsf{p}(\mathsf{h}(Y), \mathsf{f}(X, \mathsf{h}(\mathsf{a})), Z)\}$

d) $\{\mathsf{p}(\mathsf{f}(\mathsf{g}(\mathsf{a}, \mathsf{b}), Z), X, \mathsf{a}), \mathsf{p}(\mathsf{f}(X, \mathsf{a}), \mathsf{g}(Y, Y), Z)\}$