Prof. Dr. Jürgen Giesl | Carsten Otto

Notes:

- Solve these exercises in **groups of three**! For other group sizes **less points** are given!
- The solutions must be handed in **directly before (very latest: at the beginning of)** the exercise course on Wednesday, 29.05.2013, in lecture hall **AH 2**. Alternatively you can drop your solutions into a box which is located right next to Prof. Giesl's office (this box is emptied **a few minutes before** the exercise course starts).
- Please write the **names** and **immatriculation numbers** of all (three) students on your solution. Also please staple the individual sheets!

Exercise 1 (Resolution): (3 points)

Consider again the following logic program from Exercise Sheet 2.

```
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
plus(0,Z,Z).
```

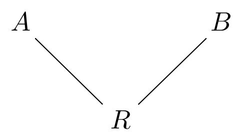
and the query

```
?- plus(s(s(0)),s(0),s(s(s(0)))).
```

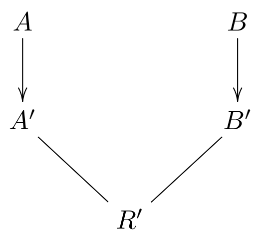
Show that the formulas φ_1 and φ_2 corresponding to the logic program entail the formula φ corresponding to the query (i.e., $\{\varphi_1, \varphi_2\} \models \varphi$) using the resolution algorithm in predicate logic.

Exercise 2 (Lifting Lemma): (3 points)

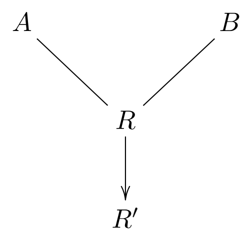
Consider the clauses $\underbrace{\{\neg\mathsf{plus}(X,Y,Z), \mathsf{plus}(\mathsf{s}(X),Y,\mathsf{s}(Z))\}}_{=:A}, \underbrace{\{\neg\mathsf{plus}(\mathsf{s}(U),\mathsf{s}(V),\mathsf{s}(W))\}}_{=:B}$ (based on Exercise 1). These clauses can be resolved to $R := \{\neg\mathsf{plus}(U,\mathsf{s}(V),W)\}$ as follows:



For this resolution step, find all ground instances A', B', and R' of A, B, and R (using ground terms with the function symbols s and 0), such that we have



and by the lifting lemma (Lemma 3.4.8) we get:



If there is an infinite number of such ground instances for A, B, and R, give a suitable finite description of these ground instances.

Exercise 3 (Input and SLD Resolution): (1+4+1=6 points)

Consider the clauses

$$\{\mathsf{p}(\mathsf{a},\mathsf{f}(\mathsf{f}(X)))\}, \{\mathsf{p}(\mathsf{f}(X),\mathsf{f}(Y)), \neg\mathsf{p}(X,\mathsf{f}(Y))\}, \{\neg\mathsf{p}(\mathsf{f}(\mathsf{f}(Z)),\mathsf{f}(\mathsf{f}(Z)))\}, \{\neg\mathsf{p}(\mathsf{f}(\mathsf{a}),\mathsf{f}(\mathsf{f}(\mathsf{b})))\}$$

with $\mathsf{p} \in \Delta_2$, $\mathsf{f} \in \Sigma_1$, and $\mathsf{a}, \mathsf{b} \in \Sigma_0$.

a) Derive the empty clause using input resolution.

b) For both negative clauses, try to derive the empty clause with SLD resolution starting with this negative clause. If you cannot derive the empty clause, continue the derivation as long as possible. For each step denote the substituions. In addition, also give the answer substitution (if you derived the empty clause).

c) Express the above clause set as queries, facts, and rules of a logic program.

Exercise 4 (Procedural Semantics): (3+3=6 points)

Consider the following logic program $\mathcal{P}$:

```
monus(X, 0, X).
monus(X, Y, Z) :- monus(A, B, Z), pred(X, A), pred(Y, B).
pred(s(s(X)), s(Y)) :- pred(s(X), Y).
pred(s(X), X).
```

Also consider the following query:

```
?- monus(s(s(s(0))), s(0), X).
```

a) Show a successful computation for the query above (i.e., a computation of the form $(G, \varnothing) \vdash_{\mathcal{P}}^{+} (\square, \sigma)$ where $G = \{\neg\mathtt{monus}(\mathtt{s}^3(\mathtt{0}), \mathtt{s}(\mathtt{0}), X)\}$). Also give the answer substitution.

b) Indicate an infinite computation for the query above by giving the first few steps. Give enough steps so that it is obvious how the infinite computation looks like.

Example: The query

```
?- pred(s(s(0)), Y).
```

has two successful derivations. Here we use variable renamings to replace the variables X and Y in the rules by X_1 and Y_1 resp. X_2 and Y_2:

$$\begin{aligned}(\{\neg\mathtt{pred}(\mathtt{s}(\mathtt{s}(\mathtt{0})), Y)\}, \emptyset) &\vdash_{\mathcal{P}} (\{\neg\mathtt{pred}(\mathtt{s}(\mathtt{0}), Y_1)\}, \{X_1/\mathtt{0}, Y/\mathtt{s}(Y_1)\}) \\ &\vdash_{\mathcal{P}} (\square, \{X_2/\mathtt{0}, Y_1/\mathtt{0}, X_1/\mathtt{0}, Y/\mathtt{s}(\mathtt{0})\})\end{aligned} \quad (1)$$

$$(\{\neg\mathtt{pred}(\mathtt{s}(\mathtt{s}(\mathtt{0})), Y)\}, \emptyset) \vdash_{\mathcal{P}} (\square, \{X_1/\mathtt{s}(\mathtt{0}), Y/\mathtt{s}(\mathtt{0})\}) \quad (2)$$

The answer subsitution for both derivations is $\{Y/\mathtt{s}(\mathtt{0})\}$

Exercise 5 (Proofs): (2+2=4 points)

a) Please prove that binary resolution is complete for clause sets without variables, i.e., if $\mathcal{K}$ is an unsatisfiable clause set without variables, can one derive $\Box$ by binary resolution? Here, $\mathcal{K}$ may also contain non-Horn clauses.

Hint: Think about the difference between binary and non-binary resolution when regarding clauses without variables.

b) A clause R is an *unrenamed resolvent* of two clauses K_1 and K_2 iff the following two conditions are satisfied:

- There are literals $L_1, \ldots, L_m \in K_1$ and $L'_1, \ldots, L'_n \in K_2$ with $m, n \geq 1$ such that $\{\overline{L_1}, \ldots, \overline{L_m}, L'_1, \ldots, L'_n\}$ is unifiable with some mgu σ.
- $R = \sigma((K_1 \setminus \{L_1, \ldots, L_m\}) \cup (K_2 \setminus \{L'_1, \ldots, L'_n\}))$

Unrenamed resolution is, thus, defined like resolution in predicate logic, but without renaming the clauses first such that they do not have any variables in common.

Please show that unrenamed resolution is incomplete. To this end, give a clause set and derive the empty clause from it with full resolution in predicate logic, but show that you cannot derive the empty clause with unrenamed resolution.

Hint: Think about possible unification failures.