Notes:

- The solutions must be handed in **directly before (very latest: at the beginning of)** the exercise course on **Friday, 14.06.2013**, in lecture hall **AH 2**. Alternatively you can drop your solutions into a box which is located right next to Prof. Giesl's office (this box is emptied **a few minutes before** the exercise course starts).
- Please write the **names** and **immatriculation numbers** of all students on your solution. Also please staple the individual sheets!

Important: This sheet is only relevant for students attending the **V3M** version of the lecture.

Exercise 1 (μ-recursion): (2+2+1+1+2+2+2=12 points)

Please show that the following six functions `odd`, $\mathtt{f}_b$, $\mathtt{f}_c$, $\mathtt{f}_d$, `predecessor`, and `subtraction` are μ-recursive by expressing each using only the six principles from Def. 4.2.1. Which of these functions are primitive recursive?

Hints:

- To show that a function is primitive recursive, express it using only the first **five** principles.
- To show that a function is not primitive recursive, give a reasonable explanation.
- You may use the predefined μ-recursive functions `plus`, `times`, `p`, `minus`, and `div` from the lecture notes.

a) $\mathtt{odd}(x) = \begin{cases} 1, & \text{if } x \text{ is odd} \\ 0, & \text{otherwise} \end{cases}$ for all $x \in \mathbb{N}$
Hint: You may use the primitive recursive function `minus` from the lecture.

b) Any function $\mathtt{f}_b : \mathbb{N} \to \mathbb{N}$ with $\mathtt{f}_b(x+y) = \mathtt{f}_b(x) + \mathtt{f}_b(y)$ for all $x, y \in \mathbb{N}$
Hint: Work with a fixed, but unknown value α with $\mathtt{f}_b(1) =: \alpha \in \mathbb{N}$.

c) Consider the following two functions $\mathtt{f}_c$ and $\mathtt{g}_c$ with $\mathtt{f}_c : \mathbb{N}^2 \to \mathbb{N}$ and $\mathtt{g}_c : \mathbb{N}^3 \to \mathbb{N}$.
$\mathtt{f}_c(x, y) = z$ iff $\mathtt{g}_c(x, y, z) = 0$ and for all $0 \leq k < z$, $\mathtt{g}_c(x, y, k)$ is defined and $\mathtt{g}_c(x, y, k) > 0$ with
$\mathtt{g}_c(x, y, z) = \begin{cases} y - (z+1), & \text{if } y \geq z+1 \\ 0, & \text{otherwise} \end{cases}$

d) Consider the following two functions $\mathtt{f}_d$ and $\mathtt{g}_d$ with $\mathtt{f}_d : \mathbb{N}^2 \to \mathbb{N}$ and $\mathtt{g}_d : \mathbb{N}^3 \to \mathbb{N}$.
$\mathtt{f}_d(x, y) = z$ iff $\mathtt{g}_d(x, y, z) = 0$ and for all $0 \leq k < z$, $\mathtt{g}_d(x, y, k)$ is defined and $\mathtt{g}_d(x, y, k) > 0$ with
$\mathtt{g}_d(x, y, z) = \begin{cases} 1, & \text{if } z - y < x \\ 0, & \text{otherwise} \end{cases}$

e) $\mathtt{predecessor}(x) = \begin{cases} x - 1, & \text{if } x > 0 \\ \text{undefined}, & \text{otherwise} \end{cases}$ for all $x \in \mathbb{N}$

f) $\mathtt{subtraction}(x, y) = \begin{cases} x - y, & \text{if } x \geq y \\ \text{undefined}, & \text{otherwise} \end{cases}$ for all $x, y \in \mathbb{N}$

g) Write a logic program $\mathcal{P}$ with a predicate $\mathtt{subtraction}(X, Y, Z)$ such that $\mathtt{subtraction}(x, y) = z$ iff $\mathcal{P} \models \mathtt{subtraction}(x, y, z)$ (cf. exercise part **f)**).