Exercise Sheet 1 (due Friday, May 5th, 2017)

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Notes:

- To solve the programming exercises you can use the Prolog interpreter **SWI-Prolog**, available for free at `http://www.swi-prolog.org`. For Debian and Ubuntu it suffices to install the `swi-prolog` package. You can use the command “`swipl`” to start it and use “`[exercise1].`” to load the facts from the file **exercise1.pl** in the current directory.
- Please solve these exercises in **groups of three or four**!
- The solutions must be handed in **directly before (very latest: at the beginning of)** the exercise course on Friday, May 5th, 2017, in lecture hall **AH 2**. Alternatively you can drop your solutions into a box which is located right next to Prof. Giesl’s office (this box is emptied **a few minutes before** the exercise course starts).
- Please write the **names** and **immatriculation numbers** of all students on your solution. Also please staple the individual sheets!
- Please register at `https://aprove.informatik.rwth-aachen.de/lp17/` (https, not http!).

Exercise 1 (Simple Prolog): (1.5 + 2 + 1.5 = 5 points)

Consider the following Prolog program.

```
evolvedFrom(cat,miacis).
evolvedFrom(hyena,miacis).
evolvedFrom(weasel,miacis).
evolvedFrom(cynodictis,miacis).

evolvedFrom(raccoon,cynodictis).
evolvedFrom(bear,cynodictis).
evolvedFrom(tomarctus,cynodictis).

evolvedFrom(fox,tomarctus).
evolvedFrom(wolf,tomarctus).
evolvedFrom(dog,tomarctus).
```

a) Implement a predicate `evolvedFromSameCreature(A,B)` in Prolog which is true if both `A` and `B` evolved from the same creature according to the predicate `evolvedFrom` above. For example, the query `?- evolvedFromSameCreature(fox,wolf)` should yield the answer `true`, whereas `?- evolvedFromSameCreature(cat,dog)` should yield `false`.

b) Implement a predicate `descendsFrom(A,C)` in Prolog which is true if `A` is a descendant of `C`, i.e., `A` either directly evolved from `C` or `A` evolved from a descendant `B` of `C`.

Make sure that the evaluation of all queries `?- descendsFrom(...,...)` terminates.

c) List all answers that Prolog gives for the following queries, in the order that Prolog gives them. Try to solve this part of the exercise without the help of a computer.

1. `?- evolvedFrom(X,tomarctus).`
2. `?- evolvedFromSameCreature(raccoon,X).`
3. `?- descendsFrom(wolf,X).`

Exercise 2 (Syntax): (2 + 1 = 3 points)

Consider the following Prolog program.

```
eats(rabbit,grass).
eats(grasshopper,grass).
eats(mouse,grass).
eats(mouse,corn).
eats(mouse,grasshopper).
eats(fox,rabbit).
eats(fox,mouse).

plant(grass).
plant(corn).
animal(rabbit).
animal(grasshopper).
animal(mouse).
animal(fox).

has_enemy(X) :- animal(X), eats(Y,X).
competitors(X,Y) :- animal(X), animal(Y), eats(X,Z), eats(Y,Z).
```

a) Construct the corresponding sets of formulas, predicate symbols, function symbols, and variables based on the program.

b) Give Prolog queries corresponding to the following questions:

- "Which plants does the mouse eat?"
- "Which competitors of the grasshopper do eat grasshoppers?"

Exercise 3 (Induction): (3 points)

Let t be an arbitrary term. Then the size $|t|$ of t is defined as follows. $|X| = 1$ if X is a variable. Otherwise we have for $n \geq 0$ that $|f(t_1, \dots, t_n)| = 1 + \Sigma_{i=1}^{n} |t_i|$.
Show by structural induction that for every term t and every variable renaming σ we have $|t| = |\sigma(t)|$.

Exercise 4 (Semantics): (3 + 3 + 3 = 9 points)

Let (Σ, Δ) be a signature with $\Sigma = \Sigma_0 = \{2, 6\}, \Delta = \Delta_1 \cup \Delta_3, \Delta_1 = \{\mathsf{even}\},$ and $\Delta_3 = \{\mathsf{plus}\}$.
Moreover, let

- $\Phi = \{\mathsf{even}(2), \forall X, Y, Z \quad \mathsf{even}(X) \wedge \mathsf{even}(Y) \wedge \mathsf{plus}(X, Y, Z) \rightarrow \mathsf{even}(Z)\}$,
- $\varphi = \forall Y \quad \mathsf{plus}(2, Y, 6) \wedge \mathsf{even}(6) \rightarrow \mathsf{even}(Y)$,
- $S = (\mathbb{N}, \alpha)$ with
  - $\alpha_2 = 2$, $\alpha_6 = 6$,
  - $\alpha_{\mathsf{plus}} = \{(x, y, z) \in \mathbb{N}^3 \mid x + y = z\}$,
  - $\alpha_{\mathsf{even}} = \{2 * i \mid i \in \mathbb{N}\}$.

Prove or disprove the following statements.
You may use that addition on natural numbers is commutative.

a) $S \models \varphi$

b) $\models \varphi$

c) $\Phi \models \varphi$